ARISTOTLE UNIVERSITY OF THESSALONIKI

FIRST HELLENIC SYMPOSIUM

ON

THEORETICAL NUCLEAR PHYSICS

24 and 25 April 1990

THESSALONIKI-GREECE

Under the auspices of the
ARISTOTLE UNIVERSITY OF THESSALONIKI
and of the
FACULTY OF PHYSICS

On the Foundations of Statistical Nuclear Physics

C. Syros

doi: 10.12681/hnps.2836

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C.Syros

Laboratory of Nuclear Technology,

University of Patras, 26110 Patras,Greece

ABSTRACT: The density matrix for the nucleus, the canonical ensemble in second quantized representation and a quantum mechanical definition of the temperature have been derived from Quantum Field Theory. The evolution operator takes conservative or dissipative form and is ergodic.

1 Introduction

Classical Mechanics has always been considered as the most fundamental example of determinism in Physics and Philosophy. However, in the last decade Hamiltonian Mechanics became the most prolific source of examples with chaotic mechanical behavior . Meanwhile Chaos has expanded to a field of research to which many branches of science-not only Physics- have contributed important developments.

Chaotic phenomena do not occur only in Classical Physics. There have been discovered in recent years quantum chaotical systems (Seligman and Nashioka 1986) although not as many as in Classical Mechanics.

The subject of this paper is to present arguments for the view that stochastic behavior, dissipation and irreversibility in Nuclear Physics are direct consequences of Quantum Field Theory.

Before addressing this question it is worth pointing out that there is a clear relationship between Chaos situations and stochastic behavior. Also Statistical Nuclear Physics has been based by Bohr (1936), Wigner (1958), Ericson (1966) and by others both in structure and reaction theory (Porter 1965) on Classical Statistical Mechanics or on first principles.

The importance of Statistical Mechanics for the description of definite aspects of nuclear structure and reaction theory is best illustrated by the well-known level spacing distribution.

To show the intrinsic relationship between the statistical
nature of some nuclear observables and the dynamics of the
nucleus,the following examples may prove expedient.
1.The level density which is directly related to the mean
level spacing can be calculated from the projections of the
system Hamiltonian onto the suitable model subspaces of
the appropriate Hilbert space (Bohigas et al. 1983).

2.The mean neutron strength function follows from the optical
potential which is in fact a mean field quantity related to
the dynamics of the nucleus.

3.The mean value of the scattering matrix for ion induced
reactions at energies below a few tens MeV and for moderate
mass numbers is obtained by the coupled channel method.This
again is based on the dynamics of the mean field.These three
classes of nuclear observables exhibit fluctuation properties
suggesting that the nucleus is a quantum system with
stochastic behavior,at least at excitation energies above
several MeV (Bohigas and Weidenmueller,1988).

The role of the chaos hypothesis concerning the velocities of
the atoms in a gas is well-known from Classical Statistical
Mechanics. This role is to be compared with the role of the
nuclear matter in the theory of Statistical Nuclear Physics.

This short introduction makes sufficiently clear the scope of
this paper.It will be shown that the statistical theory of
nuclear structure and reactions is a direct consequence of
Quantum Field Theory in conjunction with the theory of the
generalized random fields (Gel'fand and Vilenkin 1964).This
purpose will be achieved with the help of path integrals in
sect. 2. In sect. 3 a new form of the evolution operator will
be given starting from the usual form obeying the Schroedinger
equation.By acting with this operator on the second quantized
state vector we shall obtain the finite temperature density
matrix for the nucleus in sect. 4. Finally, some conclusions
and a discussion will be given in the last section.

2 The path integral

The path integral is characterized by the two quantities $\mathcal{D}p.\mathcal{D}q$
determining the measure in the Feynman path integral:

$$F = (1/2\pi) \int \mathcal{D}p\ \mathcal{D}q\ \exp(i\hbar^{-1} \int dt [pq - H(p,q)]).$$

We shall not use this form of path integral in QFT.One reason
is that the product $d\pi(x).d\phi(x)$ would violate the Uncertainty
Principle.The elimination of p via a Gauss integral is
possible only if the Hamiltonian is of the particular form
(Abers and Lee,1973)

$$H(t) = p^2/2m + V(q). \tag{2.1}$$

This condition represents a restriction for the set of
admissible Lagrangian densities in field theory due to the
relationship

$$\mathcal{L}(\phi(x), \partial_\mu \phi(x))) = \partial_t \phi(x)\pi(x) - \mathcal{H}(\phi(x), \pi(x)),$$

where $\pi(x) = \partial \phi(x)$ and $\mathcal{H}(x)$ is the Hamiltonian density. This restriction holds true also for the propagators of the field. One consequence may be for example that it does not give the most general quantization in gauge field theory. The fact that the expression

$$\int \mathcal{D}[x(t)] = \int \prod_t dx(t)$$

is not countably additive (Caratheodory,1963), (Halmos,1970) may imply serious mathematical and physical shortcomings in a theory (Klauder,1986).

In order to exclude this event one should demonstrate that the lack of the necessary property of the measure does not preclude the description of any class of physical phenomena.
The measure we shall derive from the evolution operator itself is:

$$\mu(t) = \int d^3x \int d\phi(\vec{x}, t)\pi(\vec{x}, t), \qquad (\vec{x}, t) \in \mathbb{M}^4$$

$$= \partial_t \int d^3x \int \lim [\phi(\vec{x}, t'; h') - \phi(\vec{x}, t'; h)]\phi(\vec{x}, t; h)$$

$$= \partial_t \left\{ \lim_{h' \to h} \int \left[\int d^3x \phi(\vec{x}, t'; h')\phi(\vec{x}, t; h) \right.\right.$$

$$\left.\left. - \int d^3\vec{x}\phi(\vec{x}, t'; h)\phi(\vec{x}, t; h) \right] \right\}$$

with $\phi(x; h) \in \mathfrak{H}$ = Hilbert space of the field equation solutions, $\mathbb{M}^4$=Minkowski space, (h)=space of the symmetry group parameters in accordance with the Noether theorem and $\vec{x} \in \mathbb{R}^9$.

If,
$$\mu''(t, t'; h, h') = \int d^3\vec{x} \, \phi(\vec{x}, t'; h')\phi(\vec{x}, t; h)$$
and
$$\mu'(t, t'; h, h) = \int d^3\vec{x} \, \phi(\vec{x}, t'; h)\phi(\vec{x}, t; h).$$
then
$$\left| \int \partial_t \lim_{h' \to h} [\mu''(t, t'; h, h') - \mu'(t, t'; h, h)] \right|$$

$$= \left| \int \partial_t d\mu (t, t'; h) \right|$$

$$= \left| \mu (t, t'; h) \right| < \infty.$$

The measure $\mu(t)$ of our integral is obtained from $\mu(t, t')$ by putting

$$\mu(t; h) = \mu(t, t'; h)|_{t'=t}.$$

This well-behaved measure will be the basis of what follows.

3 The evolution operator

By summing a series of path integrals (including a limiting case corresponding to the Feynman path integral for n=∞) which results from a development of the evolution operator, the following expression has been derived by Syros (1990):

$$U(t,t') = exp[-\hbar^{-1}\int d^3x \int d\phi(x)\pi(x)sin[\hbar^{-1}\int d\ x(\mathcal{H} - \pi(x)\partial^t\phi(x))]$$

$$-i\hbar^{-1}\int d^3x\int d\phi(x)\pi(x)cos[\hbar^{-1}\int dx(\mathcal{H} - \pi(x)\partial^t\phi(x))]). \qquad (3.1)$$

The expression $\pi(x)\partial^t\phi(x))$ in (3.1), proportional to the number of nucleons,is related to the chemical potential.

One of the main features of our evolution operator is that it encompasses both possibilities: It may be conservative or dissipative (Halmos,1956). It can be shown that in both cases there are conditions under which ergodic behavior spontaneously appears.We introduce the quantization conditions

$$\Lambda(n,\sigma) = \hbar^{-1}\int d^4x \ (\mathcal{H} - \partial_t \phi(x)\partial^t\phi(x)), \qquad (3.2)$$

with

$$\Lambda(n,\sigma) = \begin{cases} n + 1/2 & ,\sigma = 1 \\ n & ,\sigma = 2 \end{cases} . \qquad (3.3)$$

With the definitions of (3.2) and (3.3) the evolution operator becomes

$$U(t,t') = \begin{cases} exp[-\hbar^{-1}\int_{t'}^{t} dt H(t)+\Lambda(n,1)], & \text{dissipative (a)} \\ exp[-i\hbar^{-1},\int_{t'}^{t} dt\ H(t)+\Lambda(n,1)], & \text{conservative (b),} \end{cases} \qquad (3.4)$$

where $H(t)=\int d^3\vec{x}\ \mathcal{H}(x)$.

The following remarks may be made:

1. The measure in eq.(3.1) is countably additive.

2. A Feynman path-integral-like expression summed-up in the series has a zero contribution due to the zero weight factor following from the series.

3. Integration over the momentum space in the path integral does not follow from QFT.

4. Hamiltonians not quadratic in the momentum can also be used.

5. Spontaneous dissipation and irreversibility is implied for Hermitian Hamiltonians,if the quantization condition is fulfilled

$$\int_{t'}^{t} dt\ H(t) = \pi j\hbar, \ j = 1/2,3/2,... \ .$$

6. If on the contrary, the condition is applied,then

$$\int_{t'}^{t} dt\ H(t) = \pi j\hbar , \ j = 1,2,...$$

no dissipation appears.

The deeper significance of this quantization is not yet clear.

7.Let us define $H_{nm}=\langle n|H|m\rangle$ and $\langle H\rangle ,\bar{\tau}$ by

$$\langle H\rangle = \Sigma\Sigma H_{nm}/M, \qquad \bar{\tau} = \Sigma\Sigma H_{nm}\tau_{nm}/(M.\langle H\rangle)$$

and

$$\Sigma_n H_{nm}.\tau_{nm} = \int_{t'}^{t} H(t)dt , \ n = 1,2,...,m=\text{any microstate}$$

where $\tau_{nm} = \tau_{\text{life of } n} + \tau_{n \to m}$, $M = N^2$.

Then, the absolute temperature of the nucleus in state m is

$$T_{nm} = \hbar / (\tau_{nm} \cdot k_B),$$

where k_B is the Boltzmann constant.

8. Thermodynamic equilibrium establishes itself, if $\langle H \rangle$=constant in time and $\langle H \rangle . \tau$ = constant.

M is the number of states involved in the transitions during the interval $[t', t]$. From this the equilibrium temperature of the nucleus follows

$$T = \text{constant}.$$

4 The density matrix in the second quantized method

Assuming that the transition time, τ_{nm}, is very short we may decompose $[t', t]$ into short time steps corresponding to the individual microscopic transitions. This may be done by using the group property of the U:

$$U(t + \tau_{nm}, t') = U(t + \tau_{nm}, t) U(t, t').$$

The average transition time plus the sojourn time of a nucleon in each state is a measure for the magnitude of $1/T_m$ with T_m the temperature. This requires writing $U(t, t')$ in the form

$$U(t, t') = \prod_{\nu=1}^{n} U(t_\nu, t_{\nu-1}), \; t_n = t \text{ and } t_o = t'.$$

This acts on the state vector

$$\Psi = \sum_{n=0}^{\infty} (n!)^{-1/2} \prod_{\nu=0}^{n} \int dk_\nu^3 C_n(\vec{k}_1, \ldots \vec{k}_n; t) a^+(\vec{k}_\nu; t) |0\rangle \qquad (4.1)$$

Each term in eq. (4.1) represents an n-nucleon wave function with probability amplitude proportional to $C_n(\vec{k}_1, \ldots \vec{k}_n; t)$. It satisfies

$$i\hbar \partial_t \tilde{C}_n = H \tilde{C}_n \quad , n=1, 2, \ldots \text{with } C_n = \mathfrak{F}\{\tilde{C}_n\}.$$

$\mathfrak{F}\{\}$ is the Fourier transformation operator in the configuration space and $\{\tilde{C}_n\}$ are antisymmetrized. In the

state vector , (4.1), we have

$$\Psi(x) = \sum_{n_f} \tilde{C}(\ldots n_f \ldots; t) \sum_{(P)} \Psi_{\ldots n_f \ldots}(x)$$

$$\Psi_{\ldots n_f \ldots}(x) = ([\prod_{n_f}(n!)]/N!)^{1/2} \sum_{(P)} \phi_f(x_1) \ldots \phi_f(x_N) .$$

To study the action of $U(t, t')$ on Ψ we shall study it in the case of the operator function

$$C_n(\vec{k}_1,\ldots\vec{k}_n;t) \overset{n}{\underset{\nu}{\prod}} a^+(\vec{k}_\nu;t)|0\rangle \ . \tag{4.2}$$

of the generic term of (4.1).$|0\rangle$ is the vacuum state vector.

Since every transition changes either the first or the second or both of these factors in (4.2),the Hamiltonian in U is expressed as matrix element,H_{nm},and integrated over τ.We get from (4.2)

$$U(t+\underset{\nu}{\Sigma}\tau_{\nu m},t') \underset{n}{C}(\vec{k}_1,\ldots\vec{k}_n;t') \overset{n}{\underset{\nu}{\prod}}a^+(\vec{k}_\nu;t')|0\rangle$$

$$= \underset{n}{C}(\vec{k}_1,\ldots\vec{k}_n;t'+\underset{\nu}{\Sigma}\tau_{\nu m}) \overset{n}{\underset{\nu=0}{\prod}} a^+(\vec{k}_\nu;t')exp[-\Sigma H_{\nu m}\tau_{\nu m}$$

$$+\underset{n}{\Sigma}\underset{m}{\Sigma} \Lambda(n,o)\delta_{nm}]|0\rangle.$$

The exponent depends on the number,N,of terms corresponding to the states involved in the transitions.

Consequently, for a number,N, of transitions the time integral can be written as

$$\overset{N}{\underset{n}{\Sigma}}\overset{N}{\underset{m}{\Sigma}} H_{nm}\tau_{nm} \ .$$

This sum is repeated in the course of the time with its terms in different orders,if H attains a time independent value for $t>t''$.
Up to this point we are discussing in terms of many particles in QFT.Statistical Nuclear Physics enters the play if we decide to introduce a hypothetical nucleon exhibiting on the average the behavior of the nucleons.

Taking the geometric average is equivalent to subdividing the time variable in M parts:

$$\overset{M-1}{\underset{\nu=0}{\prod}} U(t_{\nu+1},t_\nu) = \overset{M-1}{\underset{\nu=0}{U}} (\Sigma t_{\nu+1},t_0)$$

$$= U(t,t') \ ,t_M= t \text{ and } t_0= t'$$

and

$$[U(t,t')]^{1/M} = exp(-[(o)/M \ [\int_{t'}^{t} dtH(t) - \overset{N}{\underset{n}{\Sigma}} \Lambda(n,o)]). \tag{4.3}$$

If H is time independent ,then the first term in the exponent of (4.3) becomes $H(t-t')/M$.Appropriate choice $\Sigma\Lambda$ puts to zero the exponent,makes U isometric and the system ergodic.

If,however,the Hamiltonian depends on the time,averaging the time integral of the Hamiltonian is equivalent to mixing the various time contributions of the evolution. From (4.3) one finds for the average factor of the n-th component of the state vector:

$$\langle \int d^3\vec{k}_1 \ldots d^3\vec{k}_n \, C\,(\vec{k}_n, \ldots \vec{k}_1 \, ; t' + \sum_{\nu}^{N} \sum_{m}^{N} \tau_{\nu m}) \, \prod_{\nu} a^{+}(\vec{k}_{\nu} \, ; t')\rangle$$

$$. exp(-\zeta(\sigma)/M \, [\sum_{\nu}^{N} \sum_{m}^{N} H_{\nu m} \tau_{\nu m} - \sum_{n} \cap (n, \sigma)]) \qquad (4.4)$$

Expression (4.4) is the canonical ensemble under the condition of a constant number of nucleons.

We shall now construct the density matrix for non-constant number of particles. The usual definition of the density matrix requires the *ad hoc* introduction of an *infinite* set of weight coefficients.

These are usually calculated in the framework of the Boltzmann or the Gibbs theories. In the present case these coefficients follow directly from the evolution operator making the theory selfcontained.

Next, we define the Schwinger operator, $\hat{S}(x, x')$ in the Schrödinger picture by

$$\hat{S}(x, x') = \sum_{n}^{\infty} |\Psi_n(\vec{x} \, ; t) \rangle \langle \Psi_n(\vec{x}' \, ; t')|$$

and let the evolution operator $U(t, t')$ act on $\hat{S}$ as it is given by (4.3). Then,

$$U(t, t')\hat{S}(x, x') = \sum_{\nu}^{\infty} \mathcal{E}_{\nu} \, exp[-\zeta(\sigma)\langle H\rangle/(\hbar/\tau)]$$

$$. |\Psi_{\nu}(\vec{x} \, ; t \,)\rangle \langle \Psi_{\nu}(\vec{x}' \, ; t)\rangle| \, . \qquad (4.5)$$

$$=: \hat{\rho}(\vec{x}, t \, ; \vec{x}', t)$$

The coefficients, $\langle \mathcal{E}_{\nu} \rangle$, are defined by

$\mathcal{E}_{\nu} = exp[\zeta(\sigma)/(2M)\cap(\nu, \sigma)]$. $\mathcal{E}_{\nu}$ tends to unity, if the number of

states, M, involved in the system transitions tends to infinity. From (4.5) the partition function follows as the matrix element of $\hat{\rho}$.

It has, thus, been shown that Statistical Nuclear Physics which was derived on first principles by the pioneers of Nuclear Theory, is a direct consequence of the quantum theory, provided one assumes that the Lagrangian density is a random field.

5 Discussion and conclusions

The statistical behavior of nuclei is justified in the framework of the Quantum Field Theory by the recognition that the Lagrangian density represents a random field. This fact was already used tacitly by Feynman in the construction of his path integral. The deeper meaning of the quantization condition in (3.2) has not yet been fully understood.

From our analysis it follows that the Feynman path integral corresponds to the term of infinite order in the series development of the evolution operator. The nonexistence of the measure $\mathcal{D}p(t)\mathcal{D}q(t)$ is outweighed in our case by the factor $1/n!$ with $n=\infty$ and its contribution is put to zero.

An essential difference is that in our case, which follows from the Schroedinger equation, a product of differentials in the momentum and path functions space does not appear in the measure of the integrals. What appears, instead, is a product of the momentum itself and a differential of the path space.

This product does not interfere with the Uncertainty Principle, for the integral of infinite order corresponding to the Feynman path integral vanishes due to a factor $1/(n!)$ with $n = \infty$. Another point is that the path functions space variations must for consistency be in accordance with the Noether theorem.

The density matrix and the partition function for the nucleus follow in a unified way by application of the evolution operator on the second quantization state vector representing the nucleus. From this all statistical properties of the nucleus can be derived.

References

Abers S and Lee B W 1973 Phys Rep **9C 1**

Bogolubov N N 1967 Lectures on Quantum Statistics Vol I (London:Mac-Donald Technical and Scientific) p 62

Bogolubov N N ibid pp 44

Bohigas O Haq R O and Pandey A 1983 Nuclear Data for Science and Technology (Ed Boeckhoff) (Dodrecht: Reidel) p 809

Bohigas O and Weidenmueller A 1988 Ann Nucl Part Sci 38 421

Bohr N 1936 Nature 137 344

Caratheodory C 1963 Measure and Integration (New York:Chelsea) p 189

Ericson T and Mayer-Kuckuck T. 1966 Ann. Rev. Nucl.Sci 16 183

Gel'fand I M, and Vilenkin N Ya 1964 Generalized Functions Vol 4 (New York:Academic) p 283

Halmos R P 1970 Measure Theory (New York:Springer) p 154

Halmos R P 1956 Ergodic Theory (New York:Chelsea) p 25

Hao Bai Lin 1984 Chaos (Singapore:World Scientific)

Itzykson C and Zuber J B 1985 Quantum Field Theory (New York :MacGraw-Hill) p 176

Klauder J R 1986 Progress in Quantum Field Theory Eds Ezawa E, and Kamefuzi S (Amsterdam:North-Holland)p 51

Nishijima K 1966 Particles and Fields (New York: Benjamin) p 16

Porter C E (Ed) 1965 Statistical Theory of Spectra: Fluctuations (New York:Academic)

Schulman L S 1981 Techniques and Applications of Path Integration (New York:J Wiley & Sons) p 303

Seligman T H and Nishioka H (Eds)1986 Quantum Chaos and Statistical Nuclear Physics Vol 263 (Berlin:Springer-Verlag)

Syros C 1990 Modern Physics Letters B,17 1089

Wigner E P 1958 Ann Math 67 325