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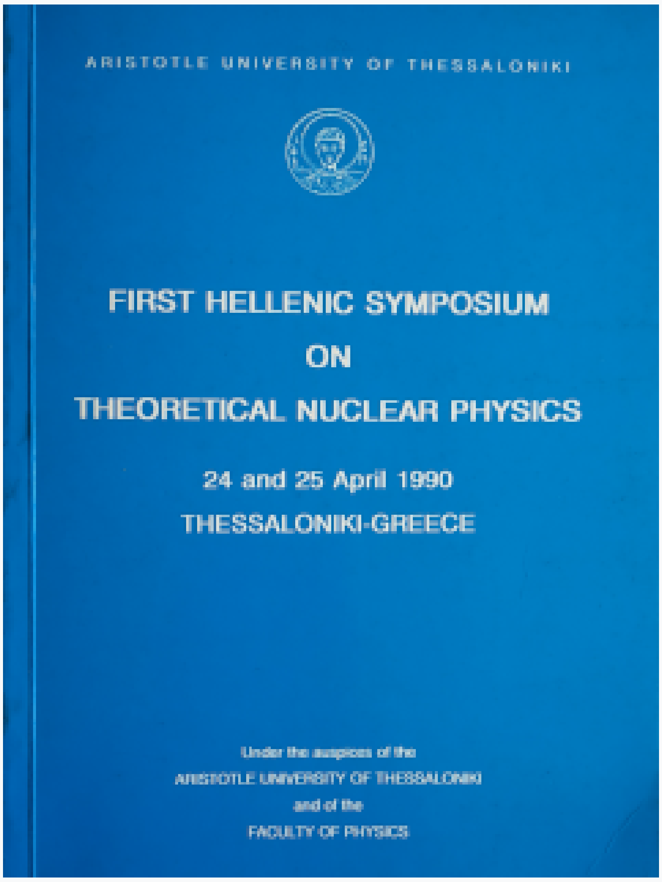

Equivalent Local Potentials for the coupled channel system and the optical potential

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Equivalent Local Potentials for the coupled channel system and the optical potential

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Abstract: The explicit formulae of the equivalent local potentials for a coupled channel problem are calculated. We prove that the equivalent local potential of the coupled channel system coincides with the equivalent local potential of the Feshbach optical potential.

1. Introduction

There are different methods of constructing an Equivalent Local Potential (ELP), which is equivalent to the single channel Schrödinger equation with non-local interactions. One method, for the single channel Schrödinger equation, was proposed by Fiedeldey [1]. In this method, each pair of independent solutions of the single channel problem corresponds to an equivalent local potential. The solutions of the Schrödinger equation with the ELP are wave proportional to the solutions of the original equation with non-local potentials. The Fiedeldey method was used with success in the nucleon-nucleon problems[2], in the $\alpha\alpha$ interaction [3], and also in the case of the quark cluster interpretation of the NN interaction[4].

The WKB method for the single channel equation with a non-local potential was introduced by H.Horiuchi [5]. In this method the non local potential is replaced by a local one, using the Wigner transform of the non-local operators. If the approximate equivalent local potential is treated by the WKB method, its WKB solutions are wave proportional to the WKB solutions of the original problem with the non-local potential. This method was successfully used for the quark structure investigations of the NN interaction [6] The WKB treatment of the coupled channel equation with non-local potentials was introduced by Yabana and Horiuchi[7]. The WKB method of constructing the single channel equivalent potential was generalised for the coupled channel Schrödinger equation by Yabana and Horiuchi [8]. In this method, the WKB solutions of the coupled channel problem with non-local potentials are wave proportional to the WKB solutions of the equivalent system with local potentials. In these papers the WKB-equivalent coupled channel problem is constructed with local potentials which are linearly dependent on the momentum, if the non-local potentials are not symmetric.

Mackellar and Coz [9] proposed also a generalisation of the Fiedeldey method for the coupled channel problem.

In part 2 of this paper we study the Fiedeldey-Mackellar-Coz method and we give explicit expressions for the ELP in the elastic channel. In part 3 we study the properties of the coupled channel system and we examine the relation between the Fiedeldey-Mackellar-Coz procedure and the Feshbach optical potential.We prove that the ELP constructed by the Fiedeldey method for a coupled channel system coincides with the ELP of the Feshbach optical potential. So the Fiedeldey method is compatible with the Feshbach optical potential. In part 4, we summarize our results.

2. Construction of the Equivalent Local Potential

The Fiedeldey theory [1] for the single channel problem is summarized as follows:

Fiedeldey ELP. *If $\mu(r)$ and $\nu(r)$ are two independent solutions of the equation*

$$\left(\frac{d^2}{dr^2}-\frac{\ell(\ell+1)}{r^2}\right)\psi(r)=\frac{2m}{\hbar^2}\left(V(r)-E\right)\psi(r)+\int_0^\infty U(r,r')\psi(r')dr' \tag{1}$$

we define the function:

$$F(r)=f^2(r)=\mu'\nu-\nu'\mu \tag{2}$$

then the functions:

$$u(r)=\frac{\mu(r)}{f(r)}\quad\text{and}\quad w(r)=\frac{\nu(r)}{f(r)} \tag{3}$$

are the two independent solutions of the Schrödinger equation:

$$-\frac{\hbar^2}{2m}\left(\frac{d^2}{dr^2}-\frac{\ell(\ell+1)}{r^2}\right)\psi(r)+\left(V^{ELP}(r)-E\right)\psi(r)=0$$

where $V^{ELP}(r)$ is the ELP, given by the formula:

$$\frac{2m}{\hbar^2}V^{ELP}(r)=\frac{2m}{\hbar^2}V(r)+\frac{1}{2}\frac{F''}{F}-\frac{3}{4}\left(\frac{F'}{F}\right)^2+\frac{\int_0^\infty U(r,r')\left(\mu'(r)\nu(r')-\nu'(r)\mu(r')\right)dr'}{F} \tag{4}$$

We consider now the system of coupled differential equations:

$$\left(\frac{d^2}{dr^2}-\frac{\ell(\ell+1)}{r^2}\right)\psi_i(r)-\sum_{m=1}^{n}(2m_i/\hbar^2)\left(V_{im}(r)-E_i\delta_{im}\right)\psi_m(r)=$$

$$=\sum_{m=1}^{n}\int_0^\infty U_{im}(r,r')\psi_m(r')dr' \tag{5}$$

If $\{\mu_i(r)\}$ and $\{\nu_i(r)\}$, with $i = 1 \ldots n$, are two independent set of solutions of the system (5), the wronskians

$$F_i(r) = f_i^2(r) = \mu_i' \nu_i - \nu_i' \mu_i$$

satisfy the system of equations:

$$\frac{dF_i}{dr} = \frac{2m_i}{\hbar} \sum_{m=1}^{n} V_{im}(r) \left(\mu_m(r)\nu_i(r) - \nu_m(r)\mu_i(r)\right) +$$

$$+ \sum_{m=1}^{n} \int_0^\infty U_{im}(r,r') \left(\mu_m(r')\nu_i(r) - \nu_m(r')\mu_i(r)\right) dr' \quad i = 1 \ldots n \tag{6}$$

We can define the functions

$$g_i(r) = \frac{\mu_i(r)}{f_i(r)} \quad \text{and} \quad h_i(r) = \frac{\nu_i(r)}{f_i(r)} \quad i = 1 \ldots n \tag{7}$$

these functions constitute a pair of solutions for the system of equations:

$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2}\right) \phi_i(r) + \chi_i(r) \frac{d\phi_i(r)}{dr} = \frac{2m_i}{\hbar^2} \sum_{m=1}^{n} \left(f_i^{-1}(r) V_{im}(r) f_m(r) - E_i \delta_{im}\right) \phi_m(r) +$$

$$\left[\frac{1}{2}\frac{F_i''}{F_i} - \frac{3}{4}\left(\frac{F_i'}{F_i}\right)^2 + \frac{\int_0^\infty \sum_{m=1}^n U_{im}(r,r') \left(\mu_i'(r)\nu_m(r') - \nu_i'(r)\mu_m(r')\right) dr'}{F_i}\right] \phi_i(r) \tag{8}$$

where

$$\chi_i(r) = \frac{2m_i}{\hbar^2} \frac{\sum_{m=1}^n V_{im}(r) \left(\mu_i(r)\nu_m(r) - \nu_i(r)\mu_m(r)\right)}{F_i} \tag{9}$$

Equation (8) can be written in a more compact form:

$$-\frac{\hbar^2}{2m_i}\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2}\right) \phi_i(r) + \sum_{m=1}^{n} \left(V_{im}^{ELP}(r,\hat{p}) - E_i \delta_{im}\right) \phi_m(r) = 0$$

where the potential $V_{im}^{ELP}(r,\hat{p})$ is a momentum dependent local potential:

$$\frac{2m_i}{\hbar^2} V_{im}^{ELP}(r,\hat{p}) = \left[-\frac{d\chi_i(r)}{dr} + \frac{1}{2}\frac{F_i''}{F_i} - \frac{3}{4}\left(\frac{F_i'}{F_i}\right)^2 + \frac{i}{2\hbar}\left(\chi_i(r) \circ \hat{p} + \hat{p} \circ \chi_i(r)\right)\right] \delta_{im} +$$

$$+ \frac{2m_i}{\hbar^2} f_i^{-1}(r) V_{im}(r) f_m(r) + \frac{\sum_{k=1}^n \int_0^\infty U_{ik}(r,r') \left(\mu_i'(r)\nu_k(r') - \nu_i'(r)\mu_k(r')\right) dr'}{F_i} \delta_{im} \tag{10}$$

where

$$\hat{p} \equiv \frac{\hbar}{i}\frac{\partial}{\partial r}$$

The above local potential contain momentum-dependent terms. We must notice that in the case of WKB method the equivalent local potentials are also momentum-dependent ones (see ref.[8]), but the momentum terms appear in the coupling channel potentials $V_{im}^{WKB}(r)$, $(i \neq m)$. In our case, these terms can be suppressed by a further reduction of the local potentials. Equation (8) can be reduced to a usual Schrödinger equation without derivatives, even if the local potentials are not symmetric. We define the transformation:

$$\psi_i(r) = \phi_i(r) \exp\left[\frac{1}{2}\int \chi_i(r)dr\right] \tag{11}$$

After the application of this transformation, the generalisation of the Fiedeldey theory for a coupled channel problem is summarised by the following proposition:

ELP for the coupled channel problem. *If* $\{\mu_i(r)\}$ *and* $\{\nu_i(r)\}$, $i = 1 \ldots n$ *are two independent solutions of the equation (5) then the functions:*

$$u_i(r) = \exp\left[\frac{1}{2}\int \chi_i(r)dr\right]\mu_i(r)/f_i(r)$$

and

$$w_i(r) = \exp\left[\frac{1}{2}\int \chi_i(r)dr\right]\nu_i(r)/f_i(r) \quad i = 1 \ldots n$$

are two independent solutions of the Schrödinger coupled channel system:

$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2}\right)\psi_i(r) - \sum_{m=1}^{n}(2m_i/\hbar^2)\left(V_{im}^{ELP}(r) - E_i\delta_{im}\right)\psi_m(r) = 0$$

where the $V_{im}^{ELP}(r)$ *is given by the following formula:*

$$\frac{2m_i}{\hbar^2}V_{im}^{ELP}(r) = \frac{2m_i}{\hbar^2}\frac{\exp\left[\frac{1}{2}\int \chi_i(r)dr\right]}{f_i(r)}V_{im}(r)\exp\left[-\frac{1}{2}\int \chi_m(r)dr\right]f_m(r) +$$
$$+\left(\frac{1}{2}\frac{d\chi_i}{dr} + \frac{1}{2}\frac{F_i''}{F_i} - \frac{3}{4}\left(\frac{F_i'}{F_i}\right)^2\right)\delta_{im}$$
$$+\frac{\int_0^\infty \sum_{k=1}^n U_{ik}(r,r')\left(\mu_i'(r)\nu_k(r') - \nu_i'(r)\mu_k(r')\right)dr'}{F_i}\delta_{im} \tag{12}$$

From the relation (6) and (9) we can find the exact form of the equivalent local potential. After a little algebra we find:

$$\frac{2m_i}{\hbar^2}V_{im}^{ELP}(r) = \frac{2m_i}{\hbar^2}\frac{\exp\left[\frac{1}{2}\int \chi_i(r)dr\right]}{f_i(r)}V_{im}(r)\exp\left[-\frac{1}{2}\int \chi_m(r)dr\right]f_m(r) + V_i^{diag}(r)\delta_{im}$$

where

$$V_i^{diag}(r) = -\frac{3}{4}\left(\frac{F_i'}{F_i}\right)^2 - \frac{1}{2}\frac{\int_0^\infty \sum_{k=1}^n U_{ik}(r,r')\left(\mu_i'(r)\nu_k(r') - \nu_i'(r)\mu_k(r')\right)dr'}{F_i}$$

$$+\frac{1}{2}\frac{\int_0^\infty \sum_{k=1}^n \frac{\partial U_{ik}(r,r')}{\partial r}\left(\mu_i(r)\nu_k(r') - \nu_i(r)\mu_k(r')\right)dr'}{F_i} \tag{13}$$

We consider the coupled channel problem, without non-local potentials:

$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2}\right)\psi_i(r) - \sum_{m=1}^{n}(2m_i/\hbar^2)\left(W_{im}(r) - E_i\delta_{im}\right)\psi_m(r) = 0 \tag{14}$$

This system can be compared to the system (5), if:

$$V_{im}(r) = 0$$

and

$$U_{im}(r,r') = W_{im}(r)\delta(r-r')$$

In this case we can compute the functions $dF_i(r)/dr$ in eqn.(6), and the final form of the ELP is given by the following proposition:

ELP for a system of coupled local equations. *Let $\mu_i(r)$ and $\nu_i(r)$ two independent solutions of the system (14), with local potentials. Then we define*

$$F_i(r) = \begin{vmatrix} \mu_i'(r) & \nu_i'(r) \\ \mu_i(r) & \nu_i(r) \end{vmatrix}, \quad i = 1, \ldots, n$$

where the functions $u_i(r)$ and $w_i(r)$ are defined as follows:

$$u(r) = \frac{\mu_i(r)}{\sqrt{F_i(r)}} \quad \text{and} \quad w(r) = \frac{\nu_i(r)}{\sqrt{F_i(r)}}$$

are two independent solutions of the uncoupled Schrödinger equation:

$$-\frac{\hbar^2}{2m_P}\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2}\right)\psi_i(r) + \left(V_i^{ELP}(r) - E_i\right)\psi_(r) = 0$$

where $V_i^{ELP}(r)$ is the i-th channel ELP, given by the formula:

$$V_i^{ELP}(r) = -\frac{3}{4}\frac{2m_i}{\hbar^2}\left(\frac{\sum_{m=1}^n W_{im}(r)\begin{vmatrix} \mu_i(r) & \nu_i(r) \\ \mu_m(r) & \nu_m(r) \end{vmatrix}}{F_i(r)}\right)^2 +$$

$$+\frac{1}{2}\frac{\sum_{m=1}^{n}\frac{dW_{im}}{dr}\begin{vmatrix}\mu_i(r) & \nu_i(r)\\ \mu_m(r) & \nu_m(r)\end{vmatrix}}{F_i(r)}$$

$$+\frac{\sum_{m=1}^{n}W_{im}(r)\left(\begin{vmatrix}\mu_i(r) & \nu_i(r)\\ \mu'_m(r) & \nu'_m(r)\end{vmatrix}-\begin{vmatrix}\mu'_i(r) & \nu'_i(r)\\ \mu_m(r) & \nu_m(r)\end{vmatrix}\right)}{F_i(r)} \qquad (15)$$

If all the potentials $W_{im}(r)$ have the same range then the final potentials have also the same range. If these ranges are different for the different channels then the ELP is a combination of potentials with these ranges.

3. The Feshbach optical potential

The number of the independent solutions of the system (5) is 2^n. For every pair of linear independent solutions, we can find a system of local equivalent potentials. Each ELP depends on the boundary conditions imposed on the initial pair of the solutions. The set of the equivalent local potentials to a system is characterised by the number of pairs of independent solutions. All the components of the ELP in eqn (15) contain ratios of the determinants of the components for a given pair of solutions, thus the set of possible equivalent potentials is parametrised by $\binom{2^n}{2}-1$ independent complex parameters. In the case of the single channel problem system we have only 2 independent solutions, so there is only one ELP.

Usually the coupled channel Schrödinger equation describes the reactions with many output (inelastic) channels but only one input (elastic) channel. The particles in an inelastic channel move out from the centre of the interaction and they behave as independent particles after a certain time, which is sufficiently large. Let i=1 be the elastic channel and $i = 2, \ldots, n$ are the output inelastic channels. In this case the boundary conditions imposed on all the inelastic channels are the following ones

$$\psi_i(r) \sim \exp\left[ik_i r\right] \ \text{ if } r \to \infty\ , \quad k_i = \frac{\sqrt{2m_i E_i}}{\hbar} \quad i = 2, \ldots, n \qquad (16)$$

If the above restriction is imposed, the number of the independent solutions of the system (5) is exactly two. If there are Coulomb potentials in the channel potentials $V_{ii}(r)$ then the exponential function in eqn (16) must be replaced by the corresponding asymptotic form of the Coulomb function. Therefore for a given elastic channel we have only one ELP, corresponding to outgoing waves in the inelastic channels.

The asymptotic condition (16) can be satisfied if the coupling potentials $V_{im}(r)$ and $U_{im}(r,r')$, $(i \neq m)$ have a finite range R. That means that these potentials must be zero for $r > R$. In the theory of nuclear reactions the basic interactions are the Coulomb interaction combined with the strong interaction between nucleons. The Coulomb interaction has not a finite range but is appears only in the channel local potentials $V_{ii}(r)$. The coupling potentials are derived from combinations of microscopic nucleon-nucleon potentials which have a finite range. In this case, for $r > R$, the initial system (5) is written in an uncoupled form. Solutions satisfying the condition (16) exist and the method can be applied but all the definite integrals in eqn.(13) should be calculated from 0 to R. Eqn(15) is valid when $r < R$. For $r > R$ the ELP are equal to the local parts of the channel potentials. The existence of the outgoing inelastic wave functions is related to the finite range of the coupling potentials.

In the Fiedeldey method the equivalent potentials have two solutions which are wave proportional to the corresponding two solutions of the initial non-local system. In the WKB method the equivalent potentials have WKB solutions which are wave proportional to the WKB solutions of the initial non-local system. Therefore the WKB coupled channel method corresponds to the WKB treatment of the Fiedeldey-Mackellar-Coz method.

Let Ω_n be the set of all systems with n-equations, Ω_n^{loc} is the subset of Ω_n, which contains all the coupled systems of n-equtions with local kernels. For every system x in Ω_n, we attach a linear 2^n–dimensional space, which is the space of the solutions of the coupled differential equations. We choose two fixed solutions $\mu_i(r)$ and $\nu_i(r)$ of the system x, then the Fiedeldey method is an application from Ω_n into Ω_n^{loc}

$$\Omega_n \ni x \rightarrow Fied(x) \in \Omega_n^{loc}$$

The equivalent local system $Fied(x)$ has two solutions $\{u_i(r)\}$ and $\{w_i(r)\}$, which have constant wronskians $W[u_i, w_i] = 1$, for every i. The Fiedeldey procedure was based essenially on the hypothesis that the wronskians of the chosen solutions $\{\mu_i(r)\}$ and $\{\nu_i(r)\}$ are *not* constant functions, for all i. Generally, a system of n-equations can be reduced to another system of equations with local potentials, but the new system cannot be furhter reduced by the same procedure, i.e. after choosing the corresponding couple of independent solutions. Symbolically we can put

$$Fied(Fied(x)) = Fied(x) \tag{17}$$

More generally, the Fiedeldey transform behaves like a projection:

$$Fied(Fied(\Omega_n)) = Fied(\Omega_n)$$

The set of the n uncoupled systems in our formulation is $[\Omega_1]^n$. Any uncoupled local system does not change when the Fiedeldey procedure is applied, therefore:

$$Fied\left(\left[\Omega_1^{loc}\right]^n\right) = \left[\Omega_1^{loc}\right]^n$$

This property is compatible with eqn.(17)

An interesting topic is the relation between the Fiedeldey method and the Feshbach procedure of constructing the optical potential of a coupled channel system [10]. We consider the system:

$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2}\right)\psi_i(r) - (2m_i/\hbar^2)(V_{ii}(r) - E_i)\psi_i(r) - \int_0^\infty U_{ii}(r,r')\psi_i(r')dr' =$$

$$\sum_{\substack{m=1\dots n\\ m\neq i}} \left((2m_i/\hbar^2)\,(V_{im}(r) - E_i\delta_{im})\,\psi_m(r) + \int_0^\infty U_{im}(r,r')\psi_m(r')dr' \right) \tag{18}$$

Let $G_n(E_n, r, r')$ be the Green function of the n-th homogeneous equation:

$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} \right) G_n(E_n, r, r') - (2m_n/\hbar^2)\,(V_{ii}(r) - E_n)\,G_n(E_n, r, r') +$$

$$- \int_0^\infty U_{nn}(r,r')G_n(E, r'', r')dr'' = \delta(r - r')$$

The Green function satisfies the asymptotic conditions:

$$G_n(E_n, r, r') \sim \mu_n(r)\ ,\ \text{for } r \to 0\ ,\ \text{and}\ \ r' = \text{const.}$$

$$G_n(E_n, r, r') \sim \nu_n(r)\ ,\ \text{for } r \to \infty\ ,\ \text{and}\ \ r' = \text{const.} \tag{19}$$

The n-th equation, in system (5) can be solved:

$$\psi_n(r) = \frac{2m_n}{\hbar^2} \sum_{k=1}^{n-1} \Big[\int_0^\infty dr' G_n(E_n, r, r')V_{nk}(r')\psi_k(r')$$

$$+ \int_0^\infty dr' \int_0^\infty dr'' G_n(E_n, r, r'')U_{nk}(r'', r')\psi_k(r') \Big] \tag{20}$$

After replacing ψ_n in all equations we arrive at the "optical" system, with n-1 equations:

$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} \right) \psi_i(r) - (2m_i/\hbar^2)\,(V_{ii}(r) - E_i)\,\psi_i(r) - \int_0^\infty U_{ii}(r,r')\psi_i(r')dr' =$$

$$= \sum_{\substack{m=1\dots n-1\\ m\neq i}} \left((2m_i/\hbar^2)\,(V_{im}(r) - E_i\delta_{im})\,\psi_m(r) + \int_0^\infty \widehat{U}_{im}(r,r')\psi_m(r')dr' \right) \tag{21}$$

where the potential $\widehat{U}_{ik}$ is defined as follows:

$$\widehat{U}_{ik}(r,r') = U_{ik}(r,r') + \frac{4m_i m_k}{\hbar^4} V_{in}(r)G_n(E_n, r, r')V_{nk}(r')$$

$$+ \frac{2m_i}{\hbar^2} V_{in}(r) \int_0^\infty dw G_n(E_n, r, w)U_{nk}(w, r') + \frac{2m_k}{\hbar^2} \int_0^\infty ds U_{in}(r, s)G_n(E_n, s, r')V_{nk}(r')$$

$$+ \int_0^\infty ds \int_0^\infty dw U_{in}(r, s)G_n(E_n, s, w)U_{nk}(w, r') \tag{22}$$

We notice that the initial system (5) of n-equations is reduced to a system with n-1 equations (21). If we repeat this procedure n-1 times we find the Feshbach local potential as it is described in ref[10]. Symbolically this reduction of the rank is expressed by the function:

$$F:\ \Omega_n \ni x \to F(x) \in \Omega_{n-1} \tag{23}$$

where $F(x)$ represents the system derived by the Feshbach procedure. The exact definition of this function pressuposes the choice of the solutions $\mu_i(r)$ and $\nu_i(r)$, $i = 1, \dots, n$. This choice defines uniquely the channel Green functions $G_i(E_i, r, r')$ with the appropriate boundary conditions, as it occurs in eqn.(4). The solutions $\mu_i(r)$ and $\nu_i(r)$, $i = 1, \dots, n-1$ of both systems coincide for the first n-1 channels.

An interesting property of the Fiedeldey method is its compatibility with the Feshbach reduction scheme. The following proposition is true:

Proposition I. *For every system* $x \in \Omega_n$

$$F\,(Fied(x)) = Fied\,(F(x)) \tag{24}$$

We consider now the case with $V_{im} = 0$ and two solutions $\mu_i(r)$ and $\nu_i(r)$, $i, \dots,$ are two solutions of the system (5). The first n-1 functions of this kind are solutions of the system (22). Consequently the corresponding wronskians coincide:

$$F_i(r) = \widehat{F}_i$$

.

Also we see that:

$$\int_0^\infty \sum_{m=1}^{n-1} \widehat{U}_{im}(r,r')\,(\mu_i'(r)\nu_m(r') - \nu_i'(r)\mu_m(r'))\,dr' =$$

$$= \int_0^\infty \sum_{m=1}^{n-1} U_{im}(r,r')\,(\mu_i'(r)\nu_m(r') - \nu_i'(r)\mu_m(r'))\,dr' +$$

$$\sum_{m=1}^{n-1} \int_0^\infty dr' \int_0^\infty ds \int_0^\infty dw U_{in}(r,s) G_n(E_n,s,w) U_{nm}(w,r')\,(\mu_i'(r)\nu_m(r') - \nu_i'(r)\mu_m(r'))$$

$$= \int_0^\infty \sum_{m=1}^{n} U_{im}(r,r')\,(\mu_i'(r)\nu_m(r') - \nu_i'(r)\mu_m(r'))\,dr'$$

Therefore the ELP's in both cases are the same.

An obvious generalization of the proposition I is the folowing corrolary:

Corrolary. *For every system* $x \in \Omega_n$

$$F^p\,(Fied(x)) = Fied\,(F^p(x)) \tag{25}$$

This proposition means that that if we apply the Fiedeldey procedure for a Feshbach optical potential in the elastic channel, the ELP should be the same to the Feshbach optical potential derived by the (uncoupled) system constructed by the Fiedeldey method. We can say that the Feshbach and Fiedeldey procedures are manipulations on the differential systems and they "anticommute"

4. Summary

In this paper we study the Fiedeldey method for the coupled channel case. For every pair of solutions of the coupled channel problem, we can define equivalent potentials, such that the coupled channel problem with non-local potentials is transformed to a coupled problem with local potentials. The ELP's in the coupled channel case depend on the boundary conditions imposed on the solutions of the exact problem. In the case of the coupling potentials with finite range we can construct one ELP in the case of outgoing inelastic wave functions. We give the explicit formula for the ELP in the coupled channel case. The Fiedeldey method is a procedure for transforming a coupled channel system to an uncoupled one. The Feshbach optical potential method is a procedure to reduce the rank of the system. In this paper we proved that the Fiedeldey method is related to the Feshbach procedure. The two methods can be viewed as manipulations on the differential systems, which anticommute.

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