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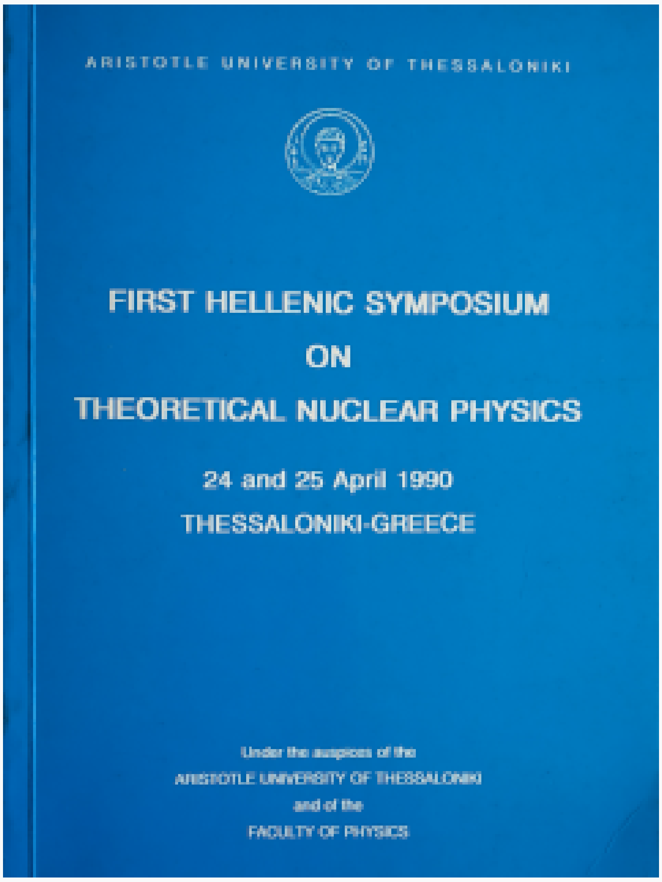

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Tests of Collective Models Based on the Systematics of Experimental Data *

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ABSTRACT: The systematics of energy ratios of successive levels of collective bands in medium and heavy mass even–even nuclei are studied. Applications to ground state, β, γ, octupole, intruder and superdeformed bands, as well as to superbands in cases of backbending are made. Implications for the U(5), SU(3) and O(6) limits of the Interacting Boson Model are discussed.

The introduction of the Interacting Boson Model (IBM) (Arima and Iachello (1976, 1978, 1979); for recent overviews see Iachello and Arima (1987), Bonatsos (1988a)) has stimulated a large number of numerical calculations aiming to compare the results given by the model to the experimental data. Although in some cases efforts have been made to describe a large number of nuclei with constant IBM parameters or smoothly varying ones (Rikovska and Bonatsos 1988), in most cases the comparisons to experiment involve one or a few nuclei. An alternative approach is to try to deduce from the systematics of experimental data general rules, against which the various limits of the model, i.e. the U(5) (vibrational), SU(3) (rotational) and O(6) (γ-unstable) limits, can be tested. In some cases these tests will be parameter independent, so that a version of the model either is found adequate to describe the effect under discussion, or its weaknesses are pointed out. In the latter case one can then try to improve the model in order to obtain the required behaviour. In other cases it will not be possible to eliminate all the parameters of the model. One then obtains constraints which the otherwise free parameters of the model must satisfy.

* Presented by Dennis Bonatsos

In this paper we report recent efforts to deduce useful tests for the IBM model from systematics of energy ratios in collective bands of medium and heavy nuclei. An extended account of this work will be given elsewhere (Bonatsos, Skouras and Rikovska 1990).

Energy ratios of the form $R(J/2) = E(J)/E(2)$, where $E(J)$ is the energy of the level with angular momentum J, have received attention since their introduction by Mallmann (1959). Their universal behaviour for vibrational, rotational and transitional nuclei led to the introduction of the Variable Moment of Inertia (VMI) model (Mariscotti, Scharff–Goldhaber and Buck 1969) and its extended versions, like the Variable Anharmonic Vibrator Model (VAVM) (Bonatsos and Klein 1984b). The ratio $R(4/2)$, in particular, has been widely used as an indicator of collectivity, having the value 2 in the vibrational limit, 10/3 in the rotational limit, and values around 2.5 for transitional nuclei.

In this paper we study the series of ratios $R(J+2/J)$, $J = 2, 4, 6, \ldots$ Using this series we construct a quantity showing distinctly different behaviour in the vibrational, rotational and γ-unstable limits. Therefore this series can be used for the safe determination of the character of a collective band, especially in nuclei where mixing of different bands occurs, in which case the $R(4/2)$ ratio might be seriously affected. In addition to ground state bands, we demonstrate the applicability and usefulness of the criterion for β and γ bands (Sakai 1984), octupole bands (Leander *et al.* 1982), intruder bands (Heyde 1989), superdeformed bands (Twin 1986), as well as in cases where backbending (Bonatsos 1985) occurs.

We start with the study of ground state bands. In the rotational limit the excitation energy of the members of a band is given by

$$E(J) = AJ(J+1), \tag{1}$$

so that their ratios obtain the limiting values

$$R(J+2/J)_{rot} = \frac{(J+2)(J+3)}{J(J+1)}. \tag{2}$$

In the vibrational limit the energy formula is

$$E(J) = BJ, \tag{3}$$

so that the ratios obtain the vibrational limiting values

$$R(J+2/J)_{vib} = \frac{J+2}{J}. \tag{4}$$

For the same J the rotational limiting value is always higher than the corresponding vibrational limiting value, the difference being

$$R(J+2/J)_{rot} - R(J+2/J)_{vib} = \frac{2(J+2)}{J(J+1)}. \tag{5}$$

One can then use the quantity

$$r(J+2/J) = \frac{R(J+2/J)_{exp} - R(J+2/J)_{vib}}{R(J+2/J)_{rot} - R(J+2/J)_{vib}}, \tag{6}$$

where $R(J+2/J)_{exp}$ are the experimental ratios. $r(J+2/J)$ must be close to 1 for a rotational nucleus, close to zero for a vibrational nucleus, and obtain intermediate values for a transitional nucleus.

It is clear that these systematics hold for Yrast bands up to the point of backbending, which can be read from the tables of Bonatsos and Klein (1984a). Beyond this point the bandhead energy must be taken away. The bandhead energy must be taken away also in the case of β and γ bands. In the latter, it is preferable to study the even and odd levels separately, because of the staggering effect (Bonatsos 1988b). If a bandhead energy has to be taken away, or if it is desirable to start the analysis of a band from a spin J_s higher than zero (as it is the case in the superdeformed bands, as well as in the superbands in cases of backbending, for example), the following modified formulae must be used

$$R(J+2/J)_{rot} = \frac{(J+2)(J+3) - J_s(J_s+1)}{J(J+1) - J_s(J_s+1)}, \tag{7}$$

$$R(J+2/J)_{vib} = \frac{J+2-J_s}{J-J_s}, \tag{8}$$

$$R(J+2/J)_{exp} = \frac{E(J+2) - E(J_s)}{E(J) - E(J_s)}, \tag{9}$$

where J_s is the angular momentum of the level used as the bandhead or of the level from which the analysis is started. In the case of β bands, $J_s = 0$, while in the case of even (odd) levels of γ bands, $J_s = 2$ ($J_s = 3$). In cases of backbending, the first or second level after the bandcrossing can be used for this purpose, since it is known (Bonatsos 1985) that backbending occurs only in cases in which the interaction between bands is weak, so that only the levels closest to the crossing point are significantly affected.

The $r(J+2/J)$ ratios have been analyzed for a large number of ground state bands (data taken from Sakai (1984)). Distinctly different behaviour is found in the vibrational, transitional and rotational limits. In particular

i) The magnitude is confined in the region 0.1 to 0.4 in the vibrational limit, takes values between 0.4 to 0.6 for γ-unstable nuclei, and lies in the area between 1.0 and 0.6 in the rotational limit.

ii) More importantly, the ratios as functions of J increase in the vibrational limit, show the opposite behaviour, i.e. decrease, in the rotational limit, while they exhibit intermediate behaviour (first increasing and then decreasing) in the γ-unstable case.

A first qualitative understanding of the different behaviour in the rotational and vibrational limits can be obtained as follows. Rotational bands are known to be well described by an expansion of the form (Xu, Wu and Zeng 1989)

$$E(J) = AJ(J+1) + B(J(J+1))^2 + C(J(J+1))^3 + D(J(J+1))^4 + \dots, \qquad (10)$$

in which A is positive, B is negative and roughly 3 orders of magnitude smaller than A, C is positive and roughly 6 orders of magnitude smaller than A and D is negative and roughly 9 orders of magnitude smaller than A. Using typical values of A, B, C, D (as the ones given in Xu, Wu and Zeng (1989)) in eq. (10) and then using the results in eq. (6), it is easy to verify that $r(J+2/J)$ ratios decreasing with increasing J are obtained. It is well known that stretching is the physical mechanism making the higher order terms in the expansion necessary. Therefore the decrease of the ratios in the rotational limit as functions of J is due to stretching.

In the vibrational limit one can use the formula

$$E(J) = AJ + BJ^2 + \dots. \qquad (11)$$

In this case one can easily check that B is positive and roughly 1 to 2 orders of magnitude smaller than A. Using such values of the parameters in eq. (11) and then using the results in eq. (6) one obtains a sequence of ratios increasing with J. Anharmonicities is therefore the physical reason behind the increase of the ratios as a function of J in the vibrational limit.

Several kinds of bands have been studied using the above method. The main results are summarized below:

i) β_1 and γ_1 bands (Sakai 1984) are in most nuclei of character similar to that of the ground state band.

ii) Superbands (in cases where backbending occurs) are of rotational character, as expected (Bonatsos 1985).

iii) Octupole bands in the Th (Schüler *et al.* 1986) and Ba (Phillips *et al.* 1986) isotopes are of rotational character.

iv) Intruder bands in the Hg region (Heyde 1989) are of rotational character. (The study of intruder bands in other series of isotopes is particularly interesting.)

v) In superdeformed bands (Twin 1986) in the A=130 and A=150 regions the ratios get closer to the rotational limiting values with increasing spin, favoring the assumption that the nucleus tends to behave as a rigid rotor at high spin, while in bands of normal deformation the deviation from the rotational limiting values increases with spin as a result of stretching.

In addition, the predictions of the three limiting symmetries of IBM-1 have been tested against these systematics. The results are summarized here:

i) In the U(5) (vibrational) limit a condition among the otherwise free parameters of the Hamiltonian is found. When this condition is exactly fulfilled, equidistant (i.e. ideal vibrational) spectra are obtained. If the condition is slightly violated, the deviations from the vibrational limit, encountered in the study of the systematics of the experimental data, can be described.

ii) In the SU(3) (rotational) limit it is known that the Hamiltonian which contains only one-body and two-body terms cannot describe the breaking of the degeneracy between the β_1 and γ_1 bands. SU(3) symmetry conserving three-body terms (Vanden Berghe, De Meyer and Van Isacker 1985) can account for the breaking of the degeneracy between the β_1 and γ_1 bands without destroying the rotational limiting values of the energy ratios.

iii) In the O(6) (γ-unstable) limit the Hamiltonian which contains only one-body and two-body terms cannot describe the dependence of the experimental ratios on J (first increasing, then decreasing). In contrast, it predicts a continuous increase. This indicates the need of including degrees of freedom neglected in this version of the model, like the g-boson, the finite N effects, or the cubic (three-body) terms.

The extension of the ratio systematics to odd mass and odd-odd nuclei, and its application to superdeformed bands of odd nuclei in particular, are of interest and will be pursued.

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