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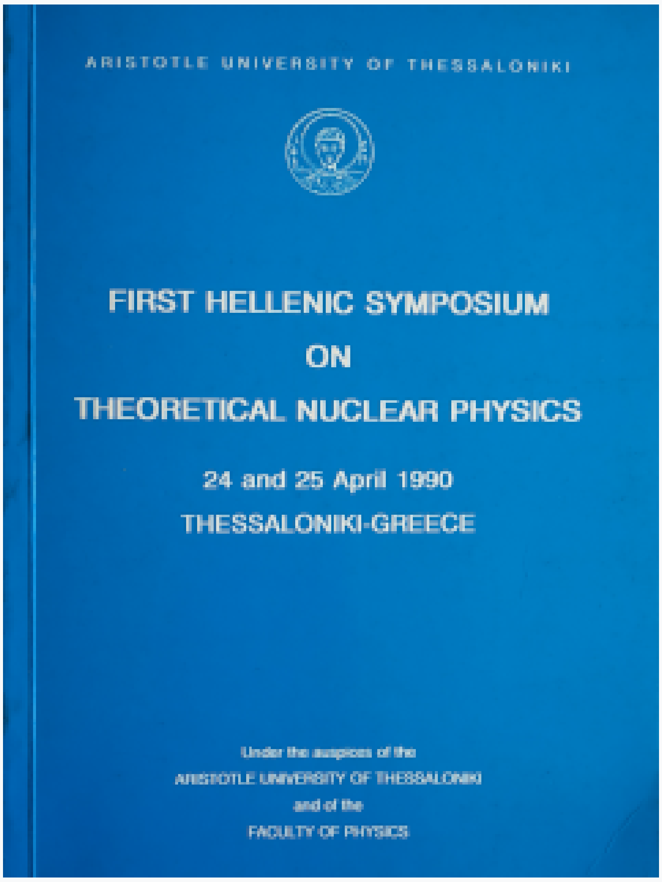

Study of energy quantities for a hyperon in hypernuclei using a single particle potential

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Study of energy quantities for a hyperon in hypernuclei using a single particle potential*

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A single particle hyperon-nucleus potential is adopted for the study of various energy quantities of a hyperon (Y) in hypernuclei.Approximate semi-empirical formulae for the ground state (g.s.) binding energy and for the oscillator spacing $\hbar\omega_\Lambda$ of a Λ in hypernuclei are proposed. The region of their validity is discussed.The g.s. binding energies of the Ξ^- hyperon in the few known Ξ^- hypernuclei are also analyzed and a comparison of the volume integrals of the Ξ^- nucleon and Λ nucleon potentials $|\bar{V}_{\Xi^- N}|$ and $|\bar{V}_{\Lambda N}|$ is made. The value of the ratio $\gamma = \bar{V}_{\Xi^- N}/\bar{V}_{\Lambda N}$ is found to be $\lesssim$ 0.8.Such a conclusion is also obtained by using in the same way other potential models such as the Woods-Saxon one.

1. Introduction

The binding energy of a hyperon in hypernuclei is a fundamental quantity in hypernuclear Physics.Various expressions,which describe the A dependence of the ground state (g.s.) binding energy and of the lowest energy level spacing or of the oscillator spacing of a hyperon have been proposed in the past [1,2]. However, most of these expressions have the disadvantage, that they are not appropriate for the lighter hypernuclei for which experimental data are mainly available.

Recently, through the studies of the associated production reaction (π^+, K^+) [3] it has become possible to track the evolution of Λ binding energies (ground and excited states) as a function of the mass number A up to A $\simeq$ 90.This outcome has increased the interest for the derivation of analytic expressions which reproduce the variation with A of various

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energy quantities of a hyperon in hypernuclei, such as the g.s. binding energy and the energy level spacing of the Λ particle.The latter problem is related to the variation with A of the Λ-oscillator spacing $\hbar\omega_\Lambda$, which approximates the lowest (or the lower) energy level spacing(s) for the Λ.This subject has received considerable attention in the past [2]. Recently [4-6], we have examined such problems by adopting a rather suitable single particle potential for the motion of a hyperon in relatively light hypernuclei.Our attention was mainly focused on Λ hypernclei , but we have also analysed the g.s. binding energy of the Ξ^- hyperon in the few known Ξ^- hypernuclei. Here we report the main results of our investigation. In sec 2 we describe the potential model and the analytic expressions for various energy quantities. In sec.3 we present some of our numerical results with comments, while in sec.4 the main conclusions are summarized.

2. The potential model and analytic expressions for various energy quantities

The hyperon-nucleus interaction is approximated by a (spin- averaged) Y-nucleus potential of the form

$$V_{Y-A} = -D/cosh^2 \frac{r}{R}, \qquad 0 \leq r < \infty \tag{1}$$

where $D > 0$ is the potential depth and R the distance from the origin, at which the value of the potential becomes -0.42D. It is therefore a little larger than the " half-depth " radius. This potential model which seems to be suitable mainly for relatively light hypernuclei, has some worth-mentioning features. It approximates well the harmonic oscillator potential for $r \ll R$ but unlike this potential which becomes infinite for $r \to \infty$, its range is finite. Its volume integral $|J|$ can be obtained analytically. This fact makes it possible to obtain an expression of the radius parameter r_0 in terms of the volume integral of the Y-nucleon potential $|\bar{V}_{YN}|$ and of the depth D, which follows from the well known rigid-core model relation $|J| = A|\bar{V}_{YN}|$, valid because of the expression of the hyperon-nucleus potential in terms of a convolution [7]. We have therefore

$$R = r_0 A^{1/3}, \qquad r_0 = \frac{1}{\pi}\left(3|\bar{V}_{\Lambda N}|/D\right)^{1/3} \tag{2}$$

where A is the mass number of the core nucleus ($A \equiv A_c$) .

We may note that potential (1) falls off exponentially for large r, as does the usual Woods-Saxon one, but its surface region is very extended. This fact makes it suitable for comparatively light hypernuclei, but it is not (in general) appropriate for the intermediate and heavy ones.

Finally, the main advantage in using this potential is that the Schrödinger eigenvalue problem can be solved analytically for states with l=0.(See N. Bessis et al [8], where this eigenvalue problem is also discussed). The radial wave functions are given by the expression

$$\Psi_{n0}(r) = N_{n0} cosh^{-2\lambda}\frac{r}{R} sinh\frac{r}{R}\, {}_2F_1\left(-n, n-2\lambda+1; \frac{3}{2}; -sinh^2\frac{r}{R}\right) \tag{3}$$

where N_{n0} is the normalization constant, which may be expressed in terms of the Γ function [6] as follows:

$$N_{n0} = \left[\frac{8\Gamma(n+3/2)\Gamma(2\lambda-n+1/2)(2\lambda-2n-1)}{R\pi\Gamma(n+1)\Gamma(2\lambda-n)}\right]^{1/2} \tag{4}$$

The parameter λ is given by the expression

$$\lambda = \frac{1}{4}\left[\left(1+8\mu DR^2/\hbar^2\right)^{1/2} - 1\right] \tag{5}$$

where μ is the Y-core reduced mass. The corresponding energy eigenvalues are :

$$E_{no} = \frac{-\hbar^2}{2\mu R^2}\left[\frac{1}{2}\sqrt{\frac{8\mu DR^2}{\hbar^2}+1} - \left(2n+\frac{3}{2}\right)\right]^2 \tag{6}$$

Using (6), the g.s. binding energy of the Λ may be written in the form

$$B_\Lambda = -E_\Lambda = D\left[\left(1+d^2A^{-2/3}\right)^{1/2} - 3dA^{-1/3}\right]^2 \tag{7}$$

where

$$d = \left(\hbar^2/8\mu Dr_0^2\right)^{1/2} \tag{8}$$

We may note that an expansion of (7) in powers of A can be derived

$$B_\Lambda = D\left[1 - 6d_0A^{-1/3} + 10d_0^2A^{-2/3} - 3d_0^3A^{-1} - 3d_0\frac{m_\Lambda}{m_N}A^{-4/3} + \dots\right], d_0 = \left(\frac{\hbar^2}{8m_\Lambda Dr_0^2}\right)^{1/2} \tag{9}$$

It is interesting to note that its first term, apart from the one which is independent of A, is proportional to $A^{-1/3}$ and not to $A^{-2/3}$ as is the case for the square well potential and the Woods-Saxon one[1]. Such a behaviour of B_Λ is observed for the first time, to our knowledge, on the basis of a potential model(with D and r_0 independent of A).

Expression (7) is a rather simple "semi-empirical" formula $B_\Lambda = B_\Lambda(A)$ which reproduces the average trend of the variation of B_Λ with the mass number A and which is obtained from the corresponding energy eigenvalue equation without additional approximation apart from the assumption that the parameters D and r_0 are independent of A.

We may also point out that formula (7) may be solved for A analytically (in an approximate way).Thus, a number of approximate expressions are derived, which may be used in estimating analytically the mass number of a hypernucleus if the corresponding B_Λ value is known.For details and the relevant expressions see ref. 6.

An expression analogous to (7) may also be used for an estimate of the nuclear part $(B_{\Xi^-})_{Nucl}$ of the Ξ^- g.s. binding energy in Ξ^- hypernuclei.For an estimate of the Coulomb part of the B_{Ξ^-} in which the diffuseness of the surface is taken into account, the following approximate formula is used, which follows directly from the analogous formula in the nuclear case [9], on the basis of the corresponding expression of ref.10 for the Ξ^- Coulomb energy in the case of the uniform distribution:

$$E_C = -\frac{6e^2}{5}\frac{Z_C}{c}\left[1 - \frac{7\pi^2}{6\,(4ln3)^2}\left(\frac{t}{c}\right)^2\right] \qquad (10)$$

where Z_C is the atomic number of the corresponding core nucleus, t the skin thickness and c the half density radius. Note that for t=0 , expression (2.1) of ref.10 is obtained.

Unfortunately, there are no available data for the cores of all Ξ^- hypernuclei.The charge distributions of neighbouring nuclei have been used instead.The values of $|E_C|$ obtained with a uniform distribution are bigger than those obtained by taking into account the diffuseness of the surface.This fact leads however to very small differences in the final result.

Another advantage in using the considered potential model is the possibility of obtaining easily, as one should expect, analytic expressions for the expectation values of the potential and kinetic energies of a hyperon in its ground state.Thus, one finds analytically the dependence of these quantities on A.The relevant expressions are [5,6]:

$$< V_Y >= -D + \frac{3D}{(4\lambda+1)} \tag{11}$$

$$< T_Y >= \frac{\hbar^2}{2\mu R^2} \frac{3(4\lambda-2)(4\lambda+2/3)}{4(4\lambda+1)} \tag{12}$$

We may therefore calculate the g.s. potential and kinetic energy of a Λ as well as the expectation values of the kinetic and potential energy part of $(B_{\Xi^-})_{Nucl}$.

Expression (12) is particularly useful in deriving an approximate expression for the Λ oscillator spacing by applying the virial theorem:

$$\begin{aligned}\hbar\omega_\Lambda &= \frac{4}{3} < T >_{h.o.} \simeq \frac{4}{3} < T_\Lambda >_{1s} \\ &= 4Dd_0A^{-1/3} - \frac{40}{3}Dd_0^2A^{-2/3} + 6d_0^3A^{-1} + 2Dd_0\frac{m_\Lambda}{m_N}A^{-4/3} + \ldots (13)\end{aligned}$$

where h.o. means that the expectation value is calculated with the (g.s.) h.o. wave function.

This expression for $\hbar\omega_\Lambda$ which is derived with the approximation $< T >_{h.o.} \approx < T_\Lambda >_{1s}$ gives the energy spacing of the oscillator for which the g.s. Λ kinetic energy equals the g.s. kinetic energy for potential (1). Such an expression for $\hbar\omega_\Lambda$ gives larger values than the values of the lowest energy spacing $E_{1p} - E_{1s}$ for the Λ particle moving in potential (1), because of the inequalities [11]

$$E_{1p} - E_{1s} \leq \frac{3\hbar^2}{2\mu} \frac{1}{< r^2 >_{1s}} \leq \frac{4}{3} < T >_{1s} \tag{14}$$

It turns out, however, that the difference between $E_{1p} - E_{1s}$ and $\hbar\omega_\Lambda$ is quite small for $A \gtrsim 16$.It is therefore suitable for estimates of $E_{1p} - E_{1s}$ in the region of the validity of the model. The leading term of (13) is proportional to $A^{-1/3}$. Such a behaviour should be attributed to the "surface effects" introduced by the potential we used, which has extended surface. However, according to our numerical results, the contribution of the next term $A^{-2/3}$ is quite significant.

Finally, in the framework of this approach, approximate analytic expressions of $< r_\Lambda^2 >$ and of the r.m.s radius for the g.s. orbit of the Λ may also be derived [5,6].

3. Numerical results and comments

The parameters D and r_0 of the Λ-nucleus potential were determined by least square fitting of the analytic expression for the g.s. energy of the Λ to the experimental values of the hypernuclei ${}^{10}_{\Lambda}B, {}^{13}_{\Lambda}C, {}^{15}_{\Lambda}N, {}^{16}_{\Lambda}O, {}^{32}_{\Lambda}S$ [12] together with the upper limits of the binding energies corresponding to A = 63,72,80,93 and 103 [13]. Our best fit values are D=38.93 MeV , r_0=0.986 fm. These were used for the results of Table 1.

It is seen from Table 1. that the values of $\hbar\omega_\Lambda$ obtained using the first two terms of the expansion (13)

$$\hbar\omega_\Lambda \simeq 4Dd_0A^{-1/3} - \frac{40}{3}Dd_0^2A^{-2/3} = 52.86A^{-1/3} - 59.82A^{-2/3} \tag{15}$$

are quite close to those of the energy level spacing $E_{1p} - E_{1s}$. Therefore, it is preferable expression (15) to be used in estimating the lowest Λ-excitation spacing in the region $12 \lesssim A \lesssim 40$ where it is expected to be valid and where there is also satisfactory agreement with the few available "experimental values" [14].

A comparison with the quite realistic (for A$\gtrsim$16) Woods-Saxon potential shows that the values of $E_{1p} - E_{1s}$ obtained with the two potentials agree fairly well for A$\lesssim$40.Therefore, the estimates of $\hbar\omega_\Lambda$ with model (1) are not expected to be appropriate for A$\gtrsim$40.

It should be noted, however, that the values of E_{1s} obtained with the two potentials agree fairly well for $16 \lesssim A \lesssim 140$. In addition the estimated values of B_Λ with potential (1) are quite close to the available experimental Λ binding energies, known from the nuclear emulsions and the $(K^-,\pi^-),(\pi^+,K^+)$ reactions. Thus, formula (7) seems to be rather accurate for quite a wide range of mass numbers ($12 \lesssim A \lesssim 140$).

Table 1. Energy quantities (in MeV) for various Λ hypernuclei

	B_Λ	$< T_\Lambda >$	$< V_\Lambda >$	$\hbar\omega_\Lambda$ Expres. 13	$\hbar\omega_\Lambda$ Expres .15	$E_{1p} - E_{1s}$
${}^{13}_{\Lambda}C$	11.59	9.40	-20.99	12.53	11.68	10.99
${}^{17}_{\Lambda}O$	13.59	9.19	-22.78	12.26	11.56	11.24
${}^{33}_{\Lambda}S$	17.96	8.33	-26.29	11.10	10.72	10.68
${}^{41}_{\Lambda}Ca$	19.24	7.99	-27.23	10.66	10.34	10.33
${}^{57}_{\Lambda}Ni$	21.04	7.47	-28.50	9.96	9.73	9.73
${}^{91}_{\Lambda}Zr$	23.33	6.72	-30.05	8.96	8.82	8.82
${}^{141}_{\Lambda}Ce$	25.24	6.04	-31.28	8.05	7.96	7.97

Finally expression (15) for $\hbar\omega_\Lambda$ may be used in estimating the relative probability for the recoilless Λ production in the (K^-, π^-) strangeness exchange reaction in relatively light nuclei, by means of the nuclear Debye-Waller factor:

$$P(n_i, n_i) = exp\left[-(2n_i+1)\frac{\hbar^2 q^2}{2m_\Lambda}\frac{1}{\hbar\omega_\Lambda}\right] \tag{16}$$

as it was suggested by Povh [15] for this reaction in analogy to the Mössbauer effect [16]. Povh had used the "experimental values" [12] of $\hbar\omega_\Lambda$ in applying this expression to light hypernuclei. However, it is preferable to avoid reference to the experimental values of $\hbar\omega_\Lambda$ and use a theoretical expression like the one we propose, which is suitable for relatively light hypernuclei.Recently [17] improved expressions for the Debye-Waller factor have been obtained, where $\hbar\omega_\Lambda$ also appears.

Computations have also been performed in the case of Ξ^- hypernuclei by using several sets of values for the parameters D and r_0. It should be clear that there is a considerable uncertainty concerning the accuracy of the parameters which is due to the poor experimental data. Here we report (Table 2.) the results obtained with r_o=0.986 fm and D=30 Mev, which seems to be the more appropriate among those considered. The value of r_0 was assumed to be the same with that of the Λ hypernuclei. The value of D was determined by least square fitting of the analytic expression for $(B_{\Xi^-})_{Nucl}$ to the available experimental values of Ξ^- binding energy [18] having subtracted from the latter the binding energy due to Coulomb force.

Table 2. **Energy quantities (in MeV) and r.m.s. radii (in fm) of a Ξ^- hyperon for a number of Ξ^- hypernuclei**

A_c	E_C	$(B_{\Xi^-})_{Nucl}$	B_{Ξ^-}	$< V_{\Xi^-} >$	$< T_{\Xi^-} >$	$< r^2_{\Xi^-} >^{1/2}$
8	1.7	5.8	7.5	-13.0	7.2	2.46
12	3.4	8.1	11.5	-15.4	7.3	2.33
16	3.7	9.7	13.4	-16.9	7.2	2.29
20	4.0	10.9	14.9	-17.9	7.0	2.29
24	4.3	11.8	16.1	-18.7	6.9	2.30
28	4.7	12.6	17.3	-19.3	6.7	2.31
32	5.2	13.2	18.4	-19.8	6.6	2.32
40	6.9	14.2	21.1	-20.6	6.3	2.35

Moreover, a comparison has been made of the volume integrals of the Ξ^- nucleon and Λ nucleon potentials. The value of the ratio γ of the two volume integrals

$$\gamma = \frac{\bar{V}_{\Xi^- N}}{\bar{V}_{\Lambda N}} \tag{17}$$

was found to be $\gamma = 0.77$.We also found that similar conclusions are reached by using in the same way other potential models such as the Woods-Saxon one ($\gamma = 0.79$).For details see ref. 6.

Our analysis corroborates the finding of other similar studies [10,18] that the volume integral of the Ξ^- nucleon potential is smaller than the corresponding Λ nucleon one. The ratio of the two volume integrals γ according to the present estimates is indicated to be $\gamma \lesssim 0.8$.Its precise determination seems to be beyond our present capabilities.

4. Summary

A single particle hyperon-nucleus potential for which the Schrödinger eigenvalue problem can be solved analytically for states with l=0 is proposed for the study of various energy quantities of a Λ and a Ξ^- particle in hypernuclei. The potential is characterized by an extended surface region, which makes it suitable for relatively light hypernuclei but not (in general) for the heavy ones.

Using this potential model a "semi-empirical mass formula" for $B_\Lambda = B_\Lambda(A)$, the g.s. binding energy of a Λ in hypernuclei is derived. This formula for which the leading term (apart from the constant one) in its expansion in powers of A is proportional to $A^{-1/3}$ seems to be sufficiently accurate for quite a wide region of A ($12 \lesssim A \lesssim 140$).

The oscillator spacing $\hbar\omega_\Lambda$ in hypernuclei may be given by an approximate closed form analytic expression containing the dependence on the mass number A of the core nucleus to all orders of $A^{-1/3}$. This expression, or preferably its leading terms $c_1 A^{-1/3} - c_2 A^{-2/3}$, are suitable to estimate the lowest Λ excitation spacing for hypernuclei in the region: $12 \lesssim A \lesssim 40$.

An expression of the above form for $\hbar\omega_\Lambda$ is proposed for the first time on the basis of a potential model (with D and r_0 independent of A) and may be used for an estimate of the relative probability for the recoilless Λ production in the (K^-, π^-) reaction, in relatively light nuclei using the nuclear Debye-Waller factor.

The same potential model was applied to an analysis of the few available experimental values of the Ξ^- binding energy in the corresponding Ξ^- hypernuclei. A comparison of the volume integrals of the Ξ^- nucleon and Λ nucleon potentials has shown that the ratio $\gamma = \bar{V}_{\Xi^- N}/\bar{V}_{\Lambda N}$ is $\gamma \lesssim 0.8$ which is an indication that the $\Xi^- N$ interaction is weaker than the corresponding ΛN one, in accordance with findings in other similar studies.

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