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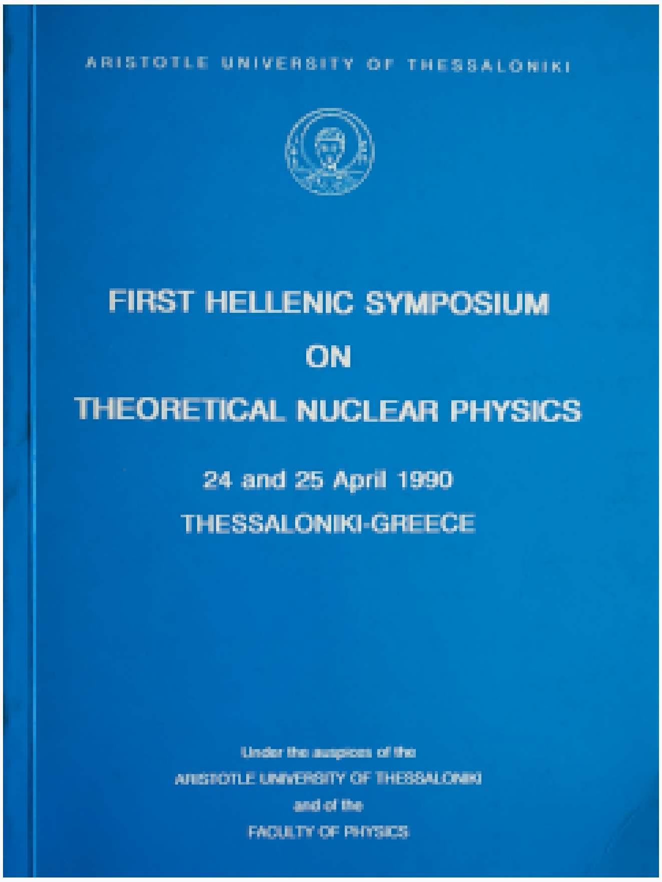

Correlated charge form factors and densities of the sd-shell nuclei

S. E. Massen

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S.E.Massen

Department of Theoretical Physics.University of Thessaloniki Thessaloniki 54006, Greece

ABSTRACT: The expression of the two body term in the factor cluster expansion of the charge form factor of ^{40}Ca is derived. It contains the harmonic oscillator (HO) parameter b_1 and the parameter λ which originates from the Jastrow correlation function. This expression together with the corresponding one of ^{16}O nucleus helps to find a mass dependence of λ and an approximate and fairly simple expression of the two body term of open shell nuclei in the region $16 \leq A \leq 40$ which contains one free parameter, the HO parameter b_1. The fitting to the corresponding experimental charge form factor is quite improved in comparison to the HO one without correlations.

1. INTRODUCTION

The factor cluster expansion of Ristig et al (1971) (Clark 1981) has been used by Nassena (1979,1981) and a generalized expression for the charge form factor, $F_{ch}(q)$, of light closed shell nuclei was derived. This formula was simplified (Massen et al 1988) using normalized correlated wave functions of the relative motion and was applied to the ^{16}O nucleus. Finally in a recent paper (Massen et al 1989) various approximations to the two-body term of the cluster expansion of the $F_{ch}(q)$ have been used and an approximate expression of it for the ^{4}He and ^{16}O nuclei has been derived. That formula was extended approximately to the other p shell nuclei.

The purpose of the present work is to extend the previous works to the ^{40}Ca nucleus and to the other s-d shell nuclei. This extension seems to be necessary for two reasons. First it is worth seeing if the correlation parameter $(b_1^2/\lambda)^{1/2}$ remains constant in the s-d shell nuclei as it was the case in the p shell nuclei. On the other hand the work of finding the two body term of the cluster expansion of $F_{ch}(q)$ for each nucleus in the s-d shell is a laborious one, so it is worth to find a simple treatment of the correlated charge form factor in this region of nuclei. For these reasons the "exact" formula of $F_{ch}(q)$ for ^{40}Ca nucleus, which is a sum of the one and two body-term in the cluster expansion of it, has been found. This formula which is more complicated than the corresponding one of ^{16}O nucleus has two free parameters, the HO parameter b_1 and the correlation parameter λ. These parameters have been determined by fitting to the experimental data of the charge form factor. In the next step an approximate formula of $F_{ch}(q)$ for ^{40}Ca has been found which is similar to the corresponding one of ^{16}O. This approximate formula has the advantage that it can be used in finding a mass dependence of the correlation parameter λ which is related to the dependence of the HO parameter b_1 on the mass number. Finally from the approximate expression of F_{ch} for ^{16}O which has been found in our previous work and the one of ^{40}Ca an approximate expression of $F_{ch}(q)$ for the s-d shell nuclei has been found by making some reasonable assumptions. This expression has one free parameter, the HO parameter b_1, which can be determined for each nucleus separately by fitting to the experimental $F_{ch}(q)$. Such a procedure has the advantage of simplifying the calculations very considerably. In section 2 the "exact" expression of F_{ch} for ^{40}Ca (which is a sum of one and two body terms) is derived while in section 3 an approximate expression of F_{ch} for this nucleus is derived and results are reported and discussed in both cases In section 4 the approximate expression is extended to other s-d shell nuclei and results for ^{20}Ne, ^{24}Mg, ^{28}Si, ^{31}P, ^{32}S and ^{39}K are also given and discussed. In section 5 the charge densities of these nuclei are given and compared with the experimental ones. Concluding remarks are made in section 6.

2. THE EXPRESSION OF THE CHARGE FORM FACTOR OF ^{40}Ca NUCLEUS

In a previous work (Massen et al 1988) a general expression of the charge form factor of light closed shell nuclei was derived by using the factor cluster expansion of Ristig et al (1971) by considering a normalized correlated wave function of the relative motion. This expression has the form

$$F_{ch}(q) = f_p(q)f_{CM}(q)[F_1(q)+F_2(q)] \quad (1)$$

where $f_p(q)$ and $f_{CM}(q)$ are the corrections due to the finite proton size and the center of mass motion (Massen et al 1988) and

$$F_1(q)= \frac{1}{A} <O>_1 = \frac{1}{A} 4 \sum_{n_i l_i} (2l_i +1)<n_i l_i | j_0(qr_1)| n_i l_i > \quad (2)$$

is the contribution of the one body term to $F_{ch}(q)$ while the contribution of the two body term to $F_{ch}(q)$ is

$$F_2(q)= \frac{1}{A} <O>_2 = \frac{1}{A} [<O>_2^{(1)} - (A-1)<O>_1] \quad (3)$$

where:

$$<O>_2^{(1)} = \sum_{\substack{n_i l_i \\ n_j l_j}} \sum_{\substack{nl \\ n'l'}} \sum_{\substack{NL \\ N'L'}} \sum_{\substack{\lambda \\ mM}} <lmLM|\lambda\mu><l'mL'M|\lambda\mu><nlNL\lambda| n_i l_i n_j l_j \lambda>$$

$$x<n'l'N'L'\lambda| n_i l_i n_j l_j \lambda><NLM| e^{i\bar{q}\cdot\bar{R}} | N'L'M>B(nlm,n'l'm) \quad (4)$$

The matrix element B(nlm,n'l'm) depends on the wave function of the relative motion and the operator which introduces the correlations. If the operator **F** is spin independent the matrix element B has the form

$$B(nlm,n'l'm) = [16-4(-1)^{l'}]<nlm| F_{12}^{+} e^{iq.r/2} F_{12} | n'l'm>$$

The application of the above formula to the ^4He is straight forward while for ^{16}O is more difficult but still it is easy to be handled (Massen et al 1988). For the case of ^{40}Ca it is extremely difficult to find the expression of the two body term $F_2(q)$ by hand because the possible combinations of the quantum numbers nl,NL,λ, mM are about 2000. For this reason a computer program which calculates $F_2(q)$ was made. In this way we have found that the two body term, $F_2(q)$, of the $F_{ch}(q)$ has the form

$$F_2(q) = \bar{F}_2(q) + \bar{\bar{F}}_2(q) \quad (5)$$

where

$\bar{F}_2(q)=$

$$\frac{1}{40}\Big[12\Big[(\frac{185}{8} -40y+\frac{83}{4}y^2 -4y^3 +\frac{1}{4}y^4)A_{00}(j_0)+(\frac{175}{8} - \frac{50}{3}y+\frac{31}{12}y^2)A_{02}(j_0)$$

$$+\frac{27}{8}A_{04}(j_0) + (\frac{35}{8} - \frac{10}{3}y+\frac{2}{3}y^2)A_{10}(j_0)+ \frac{15}{8}A_{12}(j_0)+ \frac{3}{8}A_{20}(j_0)$$

$$+(-\frac{10}{3}y+\frac{20}{21}y^2)A_{02}(j_2)+ \frac{9}{7}y^2 A_{02}(j_4)\Big] +20\Big[(30-35y+11y^2 -y^3)A_{01}(j_0)$$

$$+(\frac{21}{2} -\frac{7}{2}y)A_{03}(j_0)+(\frac{9}{2} - \frac{3}{2}y)A_{11}(j_0)+(-\frac{25}{2}y+8y^2 -y^3)A_{01}(j_2)$$

$$-\frac{9}{10}yA_{11}(j_2)-\frac{7}{5}yA_{03}(j_2)]]e^{-y} - 39(1 - 2y + \frac{4}{5}y^2)e^{-2y} \qquad (5a)$$

and

$\bar{\bar{F}}_2(q) =$

$$\frac{1}{40}[\sqrt{6}(-25y+16y^2-2y^3)A_{00}^{10}(j_0)+\frac{3}{5}\sqrt{30}y^2A_{00}^{20}(j_0)-5\sqrt{14}yA_{02}^{12}(j_0)$$

$$+4\sqrt{10}(-5y+y^2)A_{01}^{11}(j_0)-2\sqrt{5}yA_{10}^{20}(j_0)+\sqrt{15}(-50y+32y^2-4y^3)A_{00}^{02}(j_2)$$

$$+12\sqrt{\frac{15}{14}}y^2A_{00}^{12}(j_2)+\frac{40}{\sqrt{35}}(-21y+6y^2)A_{01}^{03}(j_2)+4\sqrt{10}(4y+y^2)A_{01}^{11}(j_2)$$

$$-\frac{108}{\sqrt{7}}yA_{02}^{04}(j_2)+\frac{20}{7}\sqrt{14}yA_{02}^{12}(j_2)-4\sqrt{2}yA_{02}^{20}(j_2)+2\sqrt{10}(8y-y^2)A_{02}^{10}(j_2)$$

$$+12\sqrt{14}yA_{03}^{11}(j_2)-2\sqrt{35}yA_{10}^{12}(j_2)+36\sqrt{\frac{3}{35}}y^2A_{00}^{04}(j_4)+\frac{240}{\sqrt{35}}y^2A_{01}^{03}(j_4)]e^{-y} \qquad (5b)$$

where

$$y = b_1^2q^2/8 \quad , \quad b_1=(\hbar/m\omega)^{1/2}$$

and

$$A_{nl}^{n'l'}(j_k) = \langle\psi_{nl}|j_k(qr/2)|\psi_{n'l'}\rangle \quad , \quad A_{nl}^{nl}(j_k) = A_{nl}(j_k) \qquad (6)$$

The one body term $F_1(q)$ has the form:

$$F_1(q) = (1-2y+\frac{4}{5}y^2)e^{-2y} \qquad (7)$$

If we approximate the correlated relative wave function by the normalized correlation functions

$$\psi_{nl}(r) = N_{nl}[1 - \exp(-\lambda r^2/b^2)]\phi_{nl}(r) \qquad (8)$$

the matrix elements $A_{nl}(j_k)$ and $A_{nl}^{n'l'}(j_k)$ can be found analytically. In expression (8) λ is the correlation parameter which is taken to be state independent, N_{nl} are the normalization factors $\phi_{nl}(r)$ is the HO radial wave function and $b=\sqrt{2}b_1$ is the HO parameter for the relative motion. The expressions for some of N_{nl}, $A_{nl}(j_k)$ and $A_{nl}^{n'l'}(j_k)$ are given in Massen and Panos 1989 while the others are similar.

Relation(1) can be used now for numerical calculations with the wave function (8), considering only two free parameters, the correlation parameter λ and the HO parameter b_1. The fitting to the experimental data of F_{ch} for ^{40}Ca (Sinha et all 1973) gives b_1=1.860fm, λ=13.915 and χ^2=19930 (case I). In the case

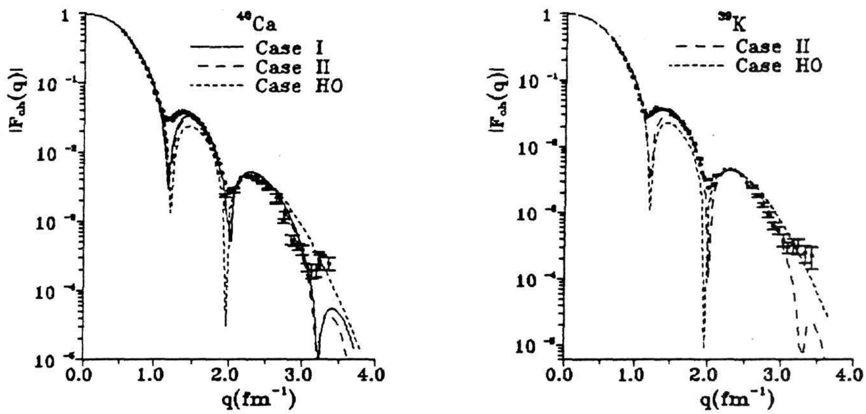

Figure 1. The charge form factors, $|F_{ch}(q)|$, of nuclei: **a)** ^{40}Ca and **b)** ^{39}K versus momentum transfer. For the cases I, II and HO see text. The experimental points and errors are from Sinha et al 1973.

of no correlations ($\lambda \rightarrow \infty$, case HO) the fitting gives $b_1=1.950$fm and $\chi^2=26847$. From these values of χ^2 we note that the introduction of correlations improves the overall fitting about 30% in comparison with the HO case, while from fig. 1a we can see that in case I the three diffraction minima are reproduced in the correct position while in case HO, only the position of the two diffraction minima are well reproduced and the overall fitting is worse. This is a general feature of wave functions with short-range correlations which reproduce theoretical F_{ch} at high momentum transfer better than those obtained with usual single particle potentials. A strong repulsion in the single particle potential may, however, improve the results considerably (Gibson et al 1968, Grypeos et al 1989). Also, form factors obtained with wave functions derived from usual Hartee-Fock calculations, are not expected to fit well the experimental $F_{ch}(q)$ for large values of q (Friedrich et al 1986).

3. DERIVATION OF AN APPROXIMATE EXPRESSION FOR THE CHARGE FORM FACTOR OF ^{40}Ca.

In our previous work (Massen and Panos (1989)) an approximate expression of the two body term, $F_2(q)$, of the charge form factor for ^{4}He and ^{16}O has been found which had the form

$$F_2(q) = \lambda^{-3/2}\left[A(y)e^{-y} + B(y)e^{-y_1} + C(y)e^{-y_2}\right]e^{-y} \qquad (9)$$

where

$$y=b_1^2 q^2/8 \ , \quad y_1 = y/(1+\lambda) \ , \quad y_2 = y/(1+2\lambda) \qquad (10)$$

and A(y), B(y), C(y) are polynomials of second order, with coefficients given in table 1.

Table 1. The values of the coefficients α_i, β_i, γ_i (i=0,...,4) which appear in the approximate expression of F_2 for nuclei ^{4}He, ^{16}O, ^{40}Ca.

	α_0	α_1	α_2	α_3	α_4
^{4}He	4.939	0.	0.	0.	0.
^{16}O	9.570	-8.644	0.	0.	0.
^{40}Ca	15.011	-27.475	10.434	0.	0.

	β_0	β_1	β_2	β_3	β_4
^{4}He	-6.	0.	0.	0.	0.
^{16}O	-11.625	10.5	-1.5	0.	0.
^{40}Ca	-18.234	31.125	-15.675	2.7	-0.15

	γ_0	γ_1	γ_2	γ_3	γ_4
^{4}He	1.061	0.	0.	0.	0.
^{16}O	2.055	-1.856	0.265	0.	0.
^{40}Ca	3.223	-5.502	2.789	-0.477	0.027

Following the same procedure for ^{40}Ca we found that the approximate expression $F_2(q)$ is given again by expression (9), the only difference is that A(y), B(y) and C(y) are now polynomials of fourth order, that is

$$A(y)=\alpha_0+\alpha_1 y+\alpha_2 y^2+\alpha_3 y^3+\alpha_4 y^4, \quad B(y)=\beta_0+\beta_1 y+\beta_2^2 y+\beta_3^3 y+\beta_4^4 y$$

$$C(y)=\gamma_0+\gamma_1 y+\gamma_2 y^2+\gamma_3 y^3+\gamma_4 y^4 \qquad (11)$$

where the coefficients α_i, β_i, γ_i (i=0,1,2,3,4) are given in table 1. From these values we can see that

$$F_2(q) = 0 \quad \text{for } q=0 \text{ or/and } \lambda \to \infty \qquad (12)$$

and

$$\alpha_0 + \beta_0 + \gamma_0 = 0 \qquad (13)$$

If instead of expression (5) we use expression (9) for F_2 in fitting the F_{ch} for ^{40}Ca to the experimental data we obtain, with b_1=1.860fm and λ=13.915, the value χ^2=21109 (case II). This value of χ^2 differs less than 6% from the corresponding value of χ^2 which has been found in case I. From figure 1a we can see that the fitting with the approximate expression of F_2 reproduces again the three diffraction minima in the correct position and the overall fitting is almost as good as in case I. Thus the above approximate expression of F_2 is reasonable and can be used instead of the "exact" one.

4. THE APPROXIMATE EXPRESSION OF $F_2(q)$ FOR s-d SHELL NUCLEI

Having found the approximate expression of F_2 for ^{40}Ca in section 3 and for ^{16}O in Massen et el (1989) it is worth seeing if we can make a reasonable estimate of the two body term in the cluster expansion of F_{ch} for the other s-d shell nuclei so that it is not necessary to make the same laborious work for these nuclei separately. For this reason we make the following assumptions:

i) The expression of the two body term, F_2, in the factor cluster expansion of F_{ch} for the open s-d shell nuclei has the same structure as in expression (9) as it should be expected.

ii) The values of the parameter $(b_1^2/\lambda)^{1/2}$ which were found in the fitting of F_{ch} with the "exact" expression of F_2 for ^4He, ^{16}O and ^{40}Ca are nearly equal as can be seen from table 2, that is

$$(b_1^2/\lambda)^{1/2} \approx \text{constant} \qquad (14)$$

This relation together with the fact that the leading term of the expansion of b_1 in powers of A is $A^{1/6}$ (Bertsch 1972, Daskaloyannis et al 1983) leads to $\lambda \approx A^{1/3}$. It should be noted that this relation indicates that the leading term of λ in an expansion of A is $A^{1/3}$. For the sake of simplicity we take the A dependence of λ to be

$$\lambda \approx \lambda_0 + \lambda_1 A^{1/3} \qquad (15)$$

where the values of λ_0 and λ_1 can be found from the known values of λ for ^{16}O and ^{40}Ca.

iii) For the coefficients α_i, β_i, γ_i (i=0,1,2,3,4) of the s-d shell nuclei we make a linear interpolation, between the corresponding values of ^{16}O and ^{40}Ca, of the form

$$\alpha_i = \alpha_i^{(0)} + \alpha_i^{(1)}(Z-8), \quad \beta_i = \beta_i^{(0)} + \beta_i^{(1)}(Z-8), \quad \gamma_i = \gamma_i^{(0)} + \gamma_i^{(1)}(Z-8) \qquad (16)$$

where Z-8 is the number of protons in the s-d shell.

In the case of s-d shell nuclei it is not easy to find an A dependence for the coefficients α_0, β_0, as we found in the case of p shell nuclei (Massen and Panos 1989) because the contribution of the two body term to the second moment of the density, $\langle r^2 \rangle_2$, for ^{40}Ca does not depend only on the parameters α_0 and β_0 but depends also on the parameters α_1, β_1, γ_1. The expression of $\langle r^2 \rangle_2$ is now

$$\langle r^2 \rangle_2 = \frac{3}{2}\left[\alpha_0 + \beta_0 \frac{1+\lambda/2}{1+\lambda} + \gamma_0 \frac{1+\lambda}{1+2\lambda} - \frac{1}{2}(\alpha_1 + \beta_1 + \gamma_1)\right] b_1^2 \lambda^{-3/2} \qquad (17)$$

This expression of $\langle r^2 \rangle_2$ remains the same (as in the case for p shell nuclei) when the corrections due to the center of mass motion and the finite proton size are included.

If the above assumptions are reasonable, we should obtain better results with the approximate formula (9) for F_2, compared to those obtained with harmonic oscillator wave functions without correlations. Indeed this is the case as we will see below.

The calculation of F_{ch} for each s-d shell nucleus using the approximate expression (9) for $F_2(q)$ is as follows: First the values of α_i, β_i, γ_i (i=0,...,4) and the value of the correlation parameter λ are found from equations (16) and (15) and the corresponding values for ^{16}O and ^{40}Ca from tables 1 and 2. Secondly the one body term of $F_{ch}(q)$ is calculated by the formula

$$F_1(q) = \left[1 - \frac{8(Z-5)}{3Z} y + \frac{4(Z-8)}{3Z} y^2\right] e^{-2y} \qquad (18)$$

Finally $F_{ch}(q)$ is found from expression (1). It should be noted that in this procedure the $F_{ch}(q)$ for each s-d shell nucleus is a function of q with only one free parameter, the harmonic oscillator parameter b_1.

We have used this procedure for the nuclei ^{20}Ne, ^{24}Mg, ^{28}Si, ^{31}P, ^{32}S and ^{39}K. The parameter b_1 for each of these nuclei has been determined by least squares fitting to the experimental $F_{ch}(q)$. The experimental values of F_{ch} for ^{20}Ne, ^{24}Mg and ^{28}Si

are from Horikawa 1971, for ^{31}P and ^{32}S are from Sinha et al 1972 and for ^{39}K are from Sinha et al 1973. The values of b_1, $(b^2/\lambda)^{1/2}$ and χ^2 for these nuclei are shown in table 2. From this table we can see that the correlation parameter $(b^2/\lambda)^{1/2}$ remains almost constant. In most cases the difference between the larger and the smaller values of this parameter is less than 6%, but there is an exception for ^{20}Ne where the difference is 8%. The values of b_1 and χ^2 mentioned previously can

Table 2. The values of the HO parameter b_1, the correlation parameters λ, $(b_1^2/\lambda)^{1/2}$, the χ^2, the charge RMS radius $\langle r_{ch}^2\rangle^{1/2}$, the contribution to the charge RMS radius from the two body terms $\langle r_{ch}^2\rangle_2^{1/2}$ and the experimental RMS radius for the ^4He, ^{16}O and s-d shell nuclei (distances in fm). For the various cases see text.

Case	Nucl	b_1	λ	$\sqrt{b_1^2/\lambda}$	χ^2	$\langle r_{ch}^2\rangle^{1/2}$	$\langle r_{ch}^2\rangle_2^{1/2}$	$\langle r_{ch}^2\rangle_{ex}^{1/2}$
I	^4He	1.215	5.967	0.497	152.4	1.578	0.514	1.630[a]
II	^4He	1.215	5.967	0.497	392.5	1.595	0.562	
HO	^4He	1.363	∞	0	1592.8	1.630	0.	
I	^{16}O	1.679	12.767	0.470	6226.	2.659	0.654	2.728[b]
II	^{16}O	1.679	12.767	0.470	7193.	2.655	0.639	
HO	^{16}O	1.786	∞	0	9013.	2.728	0.	
II	^{20}N	1.659	13.016	0.460	2.000	2.771	0.662	2.910[c]
HO	^{20}N	1.743		0	0.924	2.816	0.	
II	^{24}Mg	1.760	13.233	0.484	9.366	3.028	0.733	3.03[c]
HO	^{24}Mg	1.807	∞	0	17.695	3.011	0.	
II	^{28}Si	1.821	13.427	0.497	11.267	3.201	0.788	3.14[c]
HO	^{28}Si	1.891	∞	0	16.974	3.215	0.	
II	^{31}P	1.746	13.560	0.474	9274.	3.105	0.767	3.19[d]
HO	^{31}P	1.849		0	11902.	3.176	0.	
II	^{32}S	1.793	13.603	0.486	2940.	3.210	0.804	3.245[d]
HO	^{32}S	1.860	∞	0	4664.2	3.217	0.	
II	^{39}K	1.866	13.879	0.501	24848.	3.399	0.876	3.408[e]
HO	^{39}K	1.969	∞	0	26122.	3.456	0.	
I	^{40}Ca	1.860	13.915	0.499	19930.	3.419	0.936	3.482[e]
II	^{40}Ca	1.860	13.915	0.499	21109.	3.406	0.887	
HO	^{40}Ca	1.950	∞	0	26847.	3.439	0.	

a) DeJager et al 1974, b) Sick and McCarthy 1970
c) Horikava et al 1971, d) Sinha et al 1972, e) Sinha et al 1973

be compared with the corresponding values of b_1 and χ^2 when the fitting to the experimental values of F_{ch} is made with $\lambda=\infty$ (case HO). From these values of χ^2 which are also shown in table 2 we can see that for the above mentioned nuclei, except for ^{20}Ne, the values of χ^2 in most cases are more than 20% bigger compared with the corresponding values of χ^2 of case II. Finally it may be seen from figures 1b, 2 and 3, where F_{ch} (in cases II and HO) is compared with the corresponding experimental values of F_{ch}, that for all nuclei the diffraction minima are in the correct position while the overall fitting is better in case II than in case HO, except for ^{20}Ne. The above agreement in

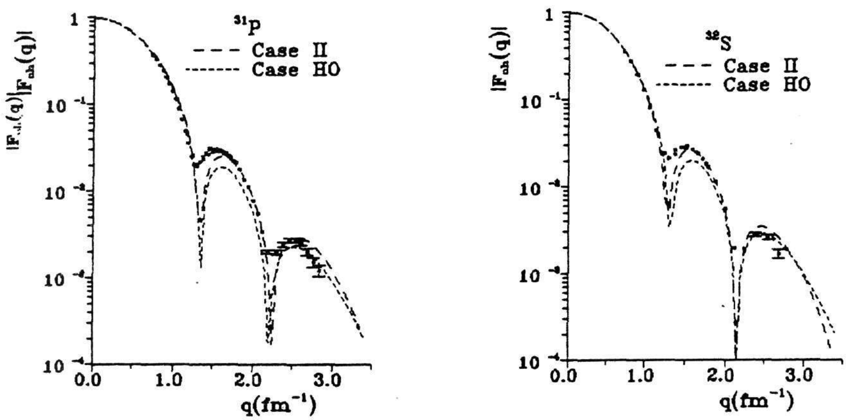

Figure 2. The charge form factors, $|F_{ch}(q)|$, of nuclei: a) ^{31}P and b) ^{32}S versus momentum transfer. For the cases II and HO see text. The experimental points and errors are from Sinha et al 1972.

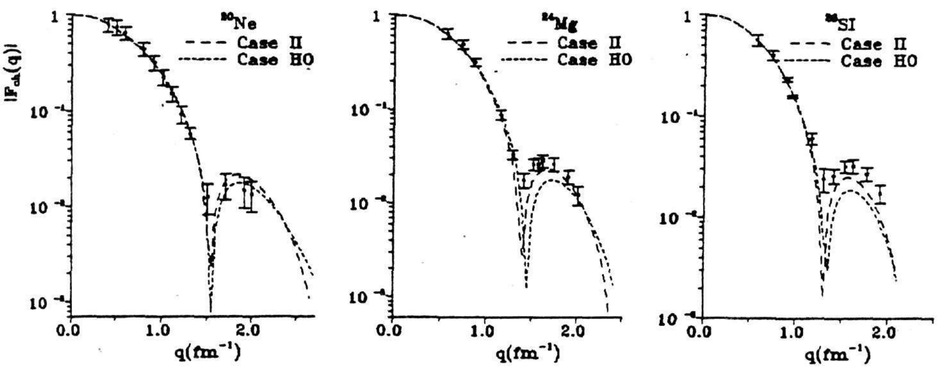

Figure 3. The charge form factors, $|F_{ch}(q)|$, of nuclei: a) ^{20}Ne, b) ^{24}Mg and c) ^{28}Si, versus momentum transfer. For the cases II and HO see text. The experimental points and errors are from Horikawa 1972.

case II with the experiment indicates that the assumptions made are reasonable and expression (9) of F_2 can be used as a reasonable approximation to include in a way short range correlations in the s-d shell nuclei. The disagreement in the form factor of the case of ^{20}Ne is not surprising because this nucleus is a peculiar one in many shell model analyses and it maybe that we need for its description othcr degrees of freedom such as rotational or/and a cluster model treatment (Abgrall et al 1974).

5. THE APPROXIMATE EXPRESSION OF THE CHARGE DENSITY DISTRIBUTION

The above described method has the advantage that it offers the possibility of finding the approximate correction to the uncorrelated charge densities analytically by the Fourier transform of $F_2(q)$ given by (9). That is:

$$\rho_2(r) = \frac{1}{2\pi^2}\int_0^\infty \frac{\sin(qr)}{qr} q^2 F_2(q) f_{CM}(q) f_p(q)\, dq \qquad (19)$$

If for $f_p(q)$ we use a sum of n gaussians, $\sum_{i=1}^{n} A_i e^{-a_i^2 q^2/4}$, $\sum_{i=1}^{n} A_i = 1$ $\rho_2(r)$ becomes:

$$\rho_2(r) = \sum_{i=1}^{n} A_i\, \rho_2(r,a_i) \quad , \qquad (20)$$

where $\rho_2(r,a_i)$ has the form:

$$\rho_2(r,a_i) = \frac{1}{2\pi^{3/2}}\lambda^{-3/2}\left[I(x)e^{-x} + J(x_1)e^{-x_1} + K(x_2)e^{-x_2}\right] \qquad (21)$$

In the above formula of $\rho_2(r,a_i)$, x, x_1 and x_2 are:

$$x = r^2/\bar{b}_1^2 , \qquad x_1 = r^2/\delta_1^2 \quad , \qquad x_2 = r^2/\delta_2^2 \qquad (22)$$

where

$$\bar{b}_1^2 = (1-\frac{1}{A})b_1^2 + a_i^2 \quad , \quad \delta_1^2 = (\frac{1+\lambda/2}{1+\lambda} - \frac{1}{A})b_1^2 + a_i^2 \quad , \quad \delta_2^2 = (\frac{1+\lambda}{1+2\lambda} - \frac{1}{A})b_1^2 + a_i^2 \qquad (23)$$

The function $I(x)$ is:

$$I(x) = \frac{1}{\bar{b}_1^3}\Big[2\alpha_0 + \frac{3}{2}\alpha_1 \frac{b_1^2}{\bar{b}_1^2}\, {}_1F_1(-1;\frac{3}{2};x) + \frac{15}{8}\alpha_2 \frac{b_1^4}{\bar{b}_1^4}\, {}_1F_1(-2;\frac{3}{2};x)$$

$$+ \frac{105}{32}\alpha_3 \frac{b_1^6}{\bar{b}_1^6}\, {}_1F_1(-3;\frac{3}{2};x) + \frac{945}{128}\alpha_4 \frac{b_1^8}{\bar{b}_1^8}\, {}_1F_1(-4;\frac{3}{2};x)\Big] \qquad (24)$$

while the function $J(x_1)$ or the function $K(x_2)$ can be derived from the function $I(x)$ if instead of x, b_1 and α_i $(i=0,1,2,3,4)$ we put x_1, δ_1 and β_i or x_2, δ_2 and γ_i respectively.
The contribution of the one body term to the density is

$$\rho_1(r)=\sum_{i=1}^{n} A_i\,\rho_1(r,a_i) \qquad (25)$$

where $\rho_1(r,a_i)$ has the form:

$$\rho_1(r,a_i)=\frac{1}{\pi^{3/2}\,b_1^{-3}}\left[1-\frac{2(Z-5)}{Z}\,\frac{b_1^{2}}{b_1^{-2}}\,{}_1F_1(-1;\tfrac{3}{2};x)+\frac{5(Z-8)}{4Z}\,\frac{b_1^{4}}{b_1^{-4}}\,{}_1F_1(-2;\tfrac{3}{2};x)\right]e^{-x} \qquad (26)$$

Expressions (20) and (25) have been used for calculations of the charge densities of ^{16}O, ^{40}Ca and for the other s-d shell nuclei mentioned in chapter 4 using the parameters of table 2. The calculated charge densities for ^{16}O, ^{24}Mg, ^{28}Si, ^{32}S, ^{39}K and ^{40}Ca are plotted and compared with the model independent charge distributions (Sick 1979) in figure 4. In the same figure

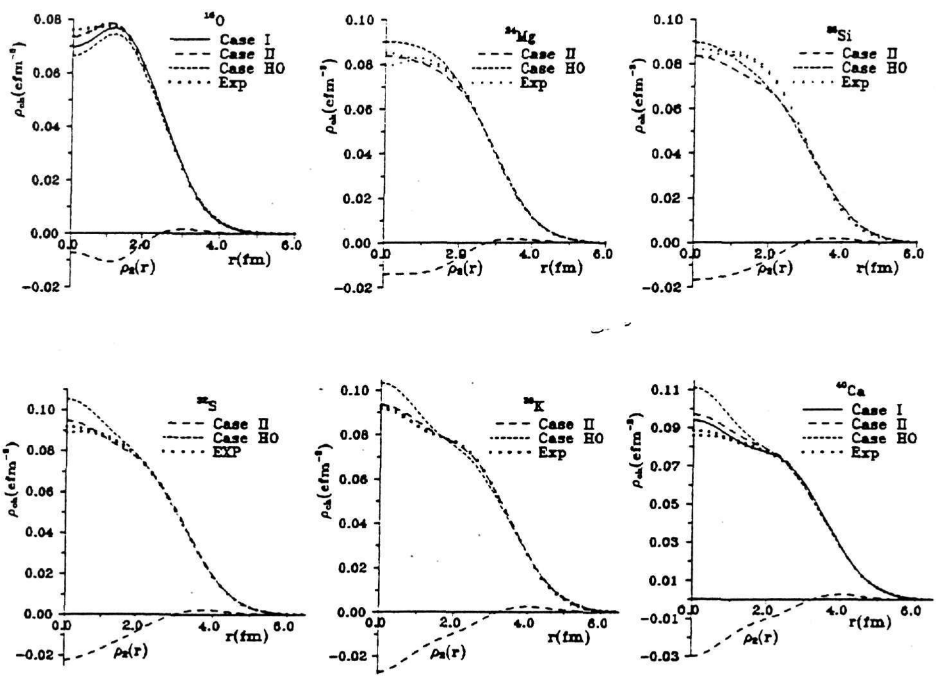

Figure 4. The charge distributions of nuclei: ^{16}O, ^{24}Mg, ^{28}Si, ^{32}S, ^{39}K and ^{40}Ca. For the cases I, II and HO see text. The experimental points are from Sick 1979 (See also Malaguti et al 1982) .

the approximate expression of $\rho_2(r)$ and the charge distribution of ^{16}O and ^{40}Ca calculated numerically from the "exact" expres sion of $F_{ch}(q)$ are also shown. From this figure we can see that the introduction of the correlations in the uncorrelated charge densities in the "exact" (case I) or the approximate form (case II) gives better charge distributions compared with the ones in the case HO, while in the case of nuclei ^{16}O,^{32}S,^{39}K and ^{40}Ca the agreement with the model independent charge densities is very good. We may also note that the introduction of the correlations leads to a decrease of the central part of the density and an increase of the surface part of it. This is an effect of the repulsion between the particles at small mutual distances and it seems that it contributes,to the charge densities in the right way Finally the RMS charge radii, $\langle r^2_{ch}\rangle^{1/2}$, and the contribution of the two body term to it, $\langle r^2_{ch}\rangle^{1/2}_2$, for the s-d shell nuclei are shown in table 2.

6. SUMMARY AND CONCLUDING REMARKS

In this paper an "exact" formula (in the two body approximation) and an approximate one for the correlated charge form factor of ^{40}Ca have been derived which reproduce quite well the experimental charge form factor. The two formulae give similar form factors for momentum transfers up to $q \approx 3.5 fm^{-1}$. Thus the assumption made for deriving the approximate formula is rea sonable. The correlation parameter $(b_1^2/\lambda)^{1/2}$, which characterises the "strength" of the correlations,has a value which is almost the same with the one which was found for nuclei ^4He and ^{16}O. On the basis of this we obtain a mass dependence for the correlation parameter λ. This feature for λ together with some other reasonable assumptions is useful in extending the approximate formulae of $F_{ch}(q)$ for ^{16}O and ^{40}Ca to other s-d shell nuclei so that we do not need to repeat the laborious work as in the case of ^{16}O and ^{40}Ca. The approximate formula of F_{ch} for the s-d shell nuclei derived using correlations (which has one free parameter, the HO parameter b_1) gives better χ^2 for almost all the nuclei we considered than in the case without correlations Thus, this method has the advantage that it offers the possibility of a simple treatment, in an approximate way,of the correlated charge form factor of open shell nuclei, not only in the region $4 \leq A \leq 16$ but also in the region $16 \leq A \leq 40$. The present work has the limitation of not taking properly into account the effect of long range correlations whose contribution is characterized by fluctuations with A. Because of this, it is natural to expect that there will be some deviations of the obtained values of the parameters from their actual values and also that this should affect their A dependence to some extent.

The correlated charge densities of these nuclei have been found analytically and compare quite well with the model independent charge densities. The introduction of the correlations has the feature of reducing the central part of the densities. We also note that the approximate expression of $F_2(q)$ was derived by expanding the matrix elements $A_{nl}^{n'l'}(j_k)$ and the normalization factors N_{nl} in powers of λ and keeping powers of λ up to $\lambda^{-3/2}$. Our results show that, as a first approximation, only the s states depend on λ. Thus, if the correlation parameter is taken to be state independent, as it was assumed in the present work, short-range correlations are mainly important in the s-states.

The question arises whether we can extend this method for A>40. In this case the degree of the polynomials A(y), B(y) and C(y) will be greater than four which means it will be very difficult to find the coefficients $\alpha_i, \beta_i, \gamma_i$ for heavy nuclei.

There is also the possibility of using this method in the case where the uncorrelated wave function is not a HO one but a wave function coming from more realistic single particle potentials, such as Woods-Saxon or Skyrme type interactions. If this is difficult then perhaps the approximate expression of the two body term, F_2 given by (9), could be used to include correlations in "a minimal way" when a more realistic single particle potential is used, in the same way as one uses the correction due to the center of mass motion.

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