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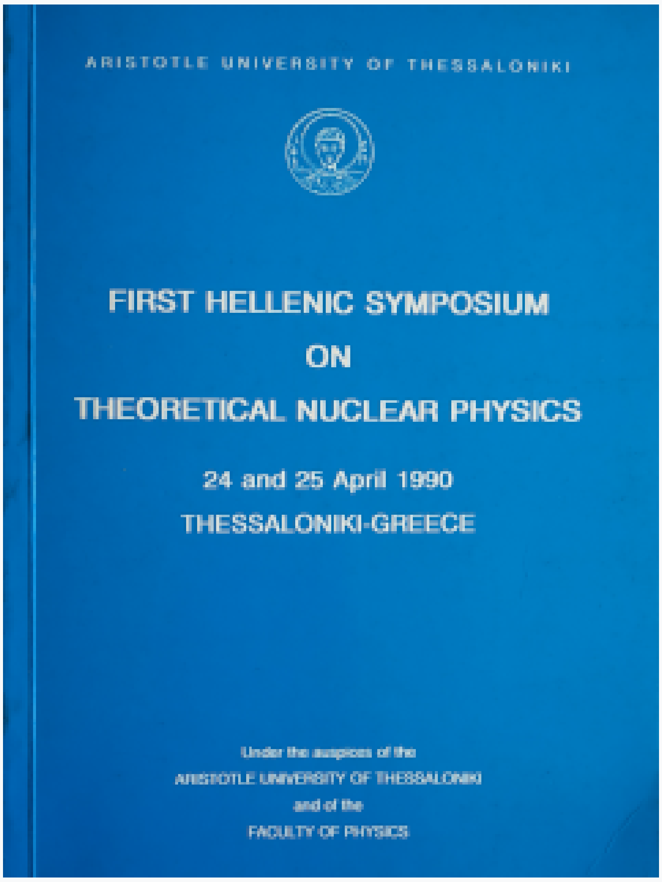

Relative probability of recoilless Λ-production in nuclei in the plane-wave impulse approximation

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Relative probability of recoilless Λ-production in nuclei in the plane-wave impulse approximation+

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Abstract The problem of estimating the relative probability P of recoilless Λ production in nuclei by means of the (K^-,π^-) reaction is investigated in the plane-wave impulse approximation, which leads to approximate expressions of the "Debye-Waller type". The values of the relevant average overlap integrals between Λ and neutron normalized harmonic oscillator wave functions are close to 1.

1. Introduction

It is well known that hypernuclei are mainly formed in strangeness exchange reactions like $^{A}Z(K^-,\pi^-)^{A}_{\Lambda}Z$. In light nuclei the dominant process is the recoilless Λ-production which is analogous[1] to the Mössbauer effect[2]. Thus, at first approximation, the relative probability P for the recoilless Λ production was estimated in ref.1 by using the following Debye-Waller factor:

$$P(n_i,n_i) = \exp\left[\frac{-(2n_i+1)\hbar^2 q^2}{2m_\Lambda \hbar\omega_\Lambda}\right] \qquad (1)$$

In this expression, n_i is the harmonic oscillator quantum number of the outer shell where the formation of the Λ is assumed to take place, q the momentum transfer, m_Λ the mass of the Λ and $\hbar\omega_\Lambda$ the harmonic oscillator spacing of the Λ which is a function of the mass number of the corresponding hypernucleus. Expression (1) may be extended to the case where the Λ is formed in inner shells as well.

+Presented by M.Grypeos

The probability P may also be calculated[3a], in a complicated way, using the shell model and the distorted wave impulse approximation. It should be noted[3a] that in the sum-rule approach[3b], which is much simpler than the shell model approach, one does not obtain[3a] the relative probability P.

In the present work we carry out a detailed calculation of P in the plane-wave impulse approximation (PWIA) which has some attractive features. More details can be found in ref. 4.

2. The analytic expression for P

Using the PWIA as basic initial approximation, the relative probability P of recoilless Λ-production is written[1]:

$$P(n_i,n_i) = |\langle n^{(\Lambda)}_{f=i}| e^{i\vec{q}\vec{r}_1} | n^{(n)}_i\rangle|^2 \qquad (2)$$

where $|n^{(n)}_i\rangle$ is the initial state (of the outer neutron shell) of the target nucleus while $|n^{(\Lambda)}_{f=i}\rangle$ is the final state (of this shell) of the hypernucleus in which one neutron has been transformed to a Λ particle with the same quantum numbers as those of the neutron. We assume that the initial state (of the outer neutron shell) of the nucleus, where the Λ is formed, can be approximated by a single Slater determinant of single-neutron states, while the final state (of this shell) of the hypernucleus $|n_{f=i}\rangle$ for recoilless Λ-production can be approximated by the same Slater determinant with one difference, namely that the single-neutron states in the column corresponding to one of the neutrons (say neutron 1) are substituted by the corresponding single Λ-particle states (with the same quantum numbers). Next, we consider that for the mechanism of recoilless Λ-production the condition: $qR << 1$ holds, where R is the nuclear radius. Our calculation leads to the following "Debye-Waller type" expression, if harmonic oscillator wave functions are used:

$$P(n_i,n_i) = \bar{I}^2_{n_i} \exp\left[\frac{-\hbar^2 q^2}{(\mu_\Lambda \hbar\omega_\Lambda + \mu_n \hbar\omega_n)}\left(\frac{2n_i}{3}+1\right)\right] \qquad (3)$$

where $\bar{I}_{n_i}$ is the average overlap integral

$$\bar{I}_{n_i} = \overline{\langle nl^{(\Lambda)} | nl^{(n)} \rangle}_{n_i} = \frac{1}{N_{n_i}} \sum_{\substack{nl \\ (2n+l=n_i)}} 2(2l+1)\langle nl^{(\Lambda)} | nl^{(n)} \rangle \qquad (4)$$

In expression (3) μ_Λ , μ_n are the reduced masses of the Λ-core and neutron-core systems and in expression (4) N_{n_i} is the number of neutrons of the outer shell which is assumed to be closed.

For the harmonic oscillator spacing of the neutron which is assumed to be the same with that of the proton we use the expression[5]:

$$\hbar\omega_n \approx 38.9\ A^{-1/3} - 23.2\ A^{-1} \qquad (5)$$

while for the corresponding spacing for the Λ the expression [6]:

$$\hbar\omega_\Lambda \approx \left(\frac{\hbar^2}{m_\Lambda} 2D\right)^{1/2} \left(\frac{2}{5}\right)^{1/3} \frac{1}{r_o} \left(1+ \frac{m_\Lambda}{m_n} A_c^{-1}\right)^{1/2} A_c^{-1/3} \qquad (6)$$

is used. D is the "potential depth" and r_o the radius parameter in expression (5) of ref. 7a for the "half-depth radius" c of the Woods-Saxon Λ-nucleus potential and A_c = A-1. This expression for c goes over to the well known expression[7b] $c=r_oA^{1/3}$ for $A_c \to \infty$.The values of the parameters D and r_o of the Woods-Saxon potential may be determined by fitting to known values of the (ground-state) energy of the Λ. The values used here are[4] D=28.3 MeV, r_o = 1.205 fm (for which the difuseness parameter a is 0.35fm).

Finally we note that if the transformation of a neutron to a Λ were not restricted to the neutrons of the outer shell but every neutron among the N neutrons of a closed-shell nucleus was allowed of being transformed to a Λ, the expression for P would be:

$$P(\{n_s\},\{n_s\}) = \bar{\bar{I}}^2 \exp\left[\frac{-\hbar^2 q^2}{(\mu_\Lambda \hbar\omega_\Lambda + \mu_n \hbar\omega_n)} \sum_{n_s} \left(\frac{2n_s}{3}+1\right) \frac{N_{n_s} \bar{I}_{n_s}}{N\bar{\bar{I}}}\right] \qquad (7)$$

where the average overlap integral over all the occupied neutron states $\bar{\bar{I}}$ is:

$$\bar{\bar{I}} = \bar{\bar{I}}\{n_s\} = \frac{1}{N} \sum_{n_s} N_{n_s} \bar{I}_{n_s} \qquad (8)$$

The $\bar{I}_{n_i}$ and $\bar{\bar{I}}$ may be given analytically for the various hypernuclei[4]. Also, the above expressions for P may be easily extended[4] to the case of nuclei with the outer shell open, by assuming that the neutrons in the open shell contribute on the average the same amount as if this shell were completely filled.

The relevant expressions are

$$P(\{n_s\},\{n_s\}) = \bar{\bar{I}}^2 \exp\frac{-\hbar^2 q^2}{(\mu_\Lambda \hbar\omega_\Lambda + \mu_n \hbar\omega_n)} \sum_{n_s=0}^{n_i-1}\left[\left(\frac{2n_s}{3}+1\right)\frac{N_{n_s}\bar{I}_{n_s}}{N\bar{\bar{I}}} + \left(\frac{2n_i}{3}+1\right)\frac{U_{n_i}\bar{I}_{n_i}}{N\bar{\bar{I}}}\right] \qquad (9)$$

and

$$\bar{\bar{I}} = \frac{1}{N}\left[\sum_{n_s=0}^{n_i-1}\sum_{\substack{nl\\(2n+l=n_s)}} 2(2l+1)\langle nl^{(\Lambda)}|nl^{(n)}\rangle + \frac{U_{n_i}}{N_{n_i}}\sum_{\substack{nl\\(2n+l=n_i)}} 2(2l+1)\langle nl^{(\Lambda)}|nl^{(n)}\rangle\right] \qquad (10)$$

where U_{n_i} is the number of valence neutrons.

We may note that in the various expressions for the relative probability instead of $2n_i+1$ or $(2n_s+1)$ which appear in the exponent of the Mössbauer effect expression, one has now $(2n_i/3)+1$ or $(2n_s/3)+1$ respectively. This leads to larger values of P. It might be instructive to consider explicitly the relevant expressions for P and notice the similarity and the differences in their structure. We consider the case of ${}^{12}_{\Lambda}C$ hypernucleus and write the various expressions for P:

i) Expression (1)

$$P(1,1) = \exp\left[\frac{-3\hbar^2 q^2}{2m_\Lambda \hbar\omega_\Lambda}\right] \qquad (11)$$

ii) Expression (3) obtained by approximating the expansion of $\exp(i\vec{q}\vec{r})$:

$$P(1,1)= A_{\Lambda n}^{5} \exp\left[-\frac{5}{3}\frac{\hbar^2 q^2}{\mu_\Lambda \hbar\omega_\Lambda+\mu_n \hbar\omega_n}\right] \tag{12}$$

$$= A_{\Lambda n}^{5} \left[1-\frac{1}{3}(aq)^2+\frac{1}{18}(aq)^4+\ldots\right] \exp(-a^2 q^2/2) \tag{13}$$

where

$$A_{\Lambda n} = \frac{2a_\Lambda a_n}{a_\Lambda^2+a_n^2} \quad \text{and} \quad a^2 = \frac{2a_\Lambda^2 a_n^2}{a_\Lambda^2+a_n^2} = \frac{2\hbar^2}{\mu_\Lambda \hbar\omega_\Lambda+\mu_n \hbar\omega_n} \tag{14}$$

iii) Expression derived from (2) without approximate treatment of the expansion of $\exp(i\vec{q}\cdot\vec{r})$ ("exact expression")

$$P(1,1) = A_{\Lambda n}^{5} \left[1-\frac{1}{3}(aq)^2+\frac{1}{36}(aq)^4\right]\exp(-a^2q^2/2) \tag{15}$$

It is seen that as long as (aq) is quite small, as it happens for the recoilless Λ-production, cases ii) and iii) give very close results, while in case i) the values of P are underestimated. In addition, it is interesting to note that the two first terms in expressions (13) and (15) which are the same, coincide essentially with the expression of the inelastic form factor squared derived by Esch[8] in his shell model analysis for the hypernuclear state $(1p_\Lambda)(1p_n)^{-1}$ (apart from the factor 1/2 introduced there and by taking the initial and final masses of the nuclear system to be approximately the same, as is the case).

The corresponding expressions for $P(\{n_s\},\{n_s\})$ in which the formation of the Λ in the inner shells is also allowed are:

a) $$P(\{0,1\}\{0,1\}) = \exp\left[\frac{-4\hbar^2 q^2}{2m_\Lambda \hbar\omega_\Lambda}\right] \tag{16}$$

β) P({0,1},{0,1})=

$$= \bar{\bar{I}}_{\{n_s=0,1\}} \left[1 - \frac{2}{3}(aq)^2B + \frac{2}{9}(aq)^4B^2 + \dots\right] \exp(-a^2q^2/2) \qquad (17)$$

γ) P({0,1},{0,1}) =

$$\left[\frac{A_{\Lambda n}^{3/2}(1+2A_{\Lambda n})}{3}\right]^2 \left[1 - \frac{2}{3}(aq)^2B + \frac{1}{9}(aq)^4B^2\right] \exp(-a^2q^2/2) \qquad (18)$$

where

$$B = A_{\Lambda n}/(1+2A_{\Lambda n}) \qquad (19)$$

The corresponding expressions, for other hypernuclei may be compared too. For example, in the case of $^{40}_{\Lambda}$Ca the expression in cases (i), (ii), α) and β) follow directly from the relevant general formulae and the values of $\bar{I}_{n_\ell=2}$ and $\bar{\bar{I}}(\{n_s\})$ given in Appendix of ref. 4. The expressions in cases γ) and (iii) (in which no approximate treatment of the expansion of $\exp(i\vec{q}\vec{r})$ is made) are

$$P(2,2) = \left[\frac{A_{\Lambda n}^{3/2}(1-5A_{n1}^2)}{4}\right]^2 \left[1 - \frac{1}{3}(aq)^2 - \frac{1}{12}(aq)^4C\right]^2 \exp(-a^2q^2/2) \qquad (20)$$

where

$$C = A_{\Lambda n}^2/(1-5A_{\Lambda n}^2) \qquad (21)$$

P({0,1,2},{0,1,2}) =

$$= \left[\frac{A_{\Lambda n}^{3/2}(15A_{\Lambda n}^2+6A_{\Lambda n}-1)}{20}\right]^2 \left[1-(aq)^2F_1 - \frac{1}{4}(aq)^4F_2\right]^2 \exp(-a^2q^2/2) \qquad (22)$$

where

$$F_1 = \frac{5A_{\Lambda n}^2 + A_{\Lambda n} - 1}{15A_{\Lambda n}^2 + 6A_{\Lambda n} - 1} \quad \text{and} \quad F2 = -\frac{A_{\Lambda n}^2}{15A_{\Lambda n}^2 + 6A_{\Lambda n} - 1} \qquad (23)$$

Again we may observe that the expressions in cases ii) and iii) as well as β) and γ) will give close values of P for small values of aq. In addition it is clear that the expression for P in case iii) ("exact expression") becomes increasingly more complicated with increasing mass number. On the contrary, the expression with the Debye-Waller factor (case ii) remains rather simple.

3. Numerical results and discussion

Expression (3) is simple enough and suitable for numerical estimates of P. For the sake of comparison we used expression (7) (and (9)) as well. The results of our calculation are displayed in table 1 for a number of nuclei, using for $\hbar\omega_\Lambda$ and $\hbar\omega_n$ expressions (5) and (6), respectively. In the same table we also give the values of the average overlap integrals $\bar{I}_{n_i}$ and $\bar{\bar{I}}\{n_s\}$ and the values of the momentum P_K of K^- in the corresponding experiments (K^-,π^-). A reasonable choice of q is made[4].

Table 1. Values of the relative probability for recoilless Λ-production estimated with expressions (3) for $P(n_i,n_i)$ and (7) (or (9)) for $P(\{n_s\},\{n_s\})$ using the values of the oscillator spacings $\hbar\omega_n$ and $\hbar\omega_\Lambda$ resulting from expressions (5) and (6), respectively. The values of the average overlap integrals $\bar{I}_{n_i}$ and $\bar{\bar{I}}\{n_s\}$ and the values of the incident K^- momentum P_K in the corresponding (K^-,π^-) experiments are also given.

Hyper-nucleus	A_c	P_K MeV/c	$\hbar\omega_n$ expr. (5)	$\hbar\omega_\Lambda$ expr. (6)	$\bar{I}_{n_i}$	$P(n_i,n_i)$ %	$\bar{\bar{I}}\{n_s\}$	$P(\{n_s\},\{n_s\})$ %
$^{12}_{\Lambda}C$	11	720	15.0	12.9	1.000	81	1.000	83
$^{16}_{\Lambda}O$	15	720	14.0	11.4	1.000	80	1.000	82
$^{27}_{\Lambda}Al$	26	720	12.1	9.4	0.996	69	0.997	75
$^{32}_{\Lambda}S$	31	720	11.5	8.8	0.995	68	0.996	73
$^{40}_{\Lambda}Ca$	39	790	10.8	8.1	0.993	55	0.995	60
$^{51}_{\Lambda}V$	50	720	10.0	7.5	0.988	56	0.992	64
$^{89}_{\Lambda}Y$	88	720	8.4	6.2	0.977	42	0.986	53
$^{209}_{\Lambda}Bi$	208	640	6.4	4.6	0.950	38	0.970	50

Table 2. Comparison of the values of the relative probabilities P obtained with "Debye-Waller" type expressions (3) and (7) (or(9)) with those calculated with the "exact" expression,for a number of hypernuclei. The experimental values of $\hbar\omega_n$ and $\hbar\omega_\Lambda$ and those obtained with expressions (5) and (6) are given.The values of the incident K^- momentum P_k in the corresponding (K^-,π^-) experiments are also given.

Hyper-nucleus	A_c	P_K	experimental $\hbar\omega_n$	experimental $\hbar\omega_\Lambda$	$P(n_i,n_i)$ %		$P(\{n_s\},\{n_s\})$ %		$\hbar\omega_n$	$\hbar\omega_\Lambda$	$P(n_i,n_i)$ %		$P(\{n_s\},\{n_s\})$ %	
					Debye-Waller type expression (3)	"Exact" expression	Debye-Waller type expression(7)or(9)	"Exact" expression	expres (5)	expres (6)	Debye-Waller type expression (3)	"Exact" expression	Debye-Waller type expression (7) or (9)	"Exact" expression
$^{12}_{\Lambda}C$	11	720	15.4	11.0	78	78	81	81	15.0	12.9	81	81	83	83
$^{16}_{\Lambda}O$	15	720	13.4 (12.3)	10.0	77 (77)	77 (77)	79 (79)	79 (79)	14.0	11.4	80	80	82	82
$^{40}_{\Lambda}Ca$	39	790	10.0	8.0	54	53	59	59	10.8	8.1	55	54	60	60

It is seen from the results displayed in this table that the values of the average overlap integrals are close to unity and may usually be omitted in the expressions where these integrals or their squares appear as factors. We also note that the values of P are large for light hypernuclei, as expected.

Finally in table 2 the results of numerical calculations for $^{12}_{\Lambda}C$, $^{16}_{\Lambda}O$ and $^{40}_{\Lambda}Ca$ are displayed with the aim of comparing the values of the relative probabilities $P(n_i,n_i)$ and $P(\{n_s\},\{n_s\})$ obtained with the Debye-Waller-type expressions (3) and (7) (or (9)) with the corresponding "exact" ones (those derived from expression (2) and the corresponding one for $P(\{n_s\}\{n_s\})$ without approximate treatment of the expansion of $\exp(i\vec{q}\vec{r})$). It is seen that for these nuclei the agreement of the results obtained with the two kinds of expressions is remarkably good. In the same table the values obtained using the "experimental values" of $\hbar\omega_n$ and $\hbar\omega_\Lambda$ are also displayed. These lead to slightly smaller values of P, in particular for the lighter elements.

In conclusion, expression (3) for P seems to be a suitable expression for a rough estimate of the relative probability of recoilless Λ-production in nuclei in the region where PWIA is valid. Its merit is its remarkable simplicity.

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