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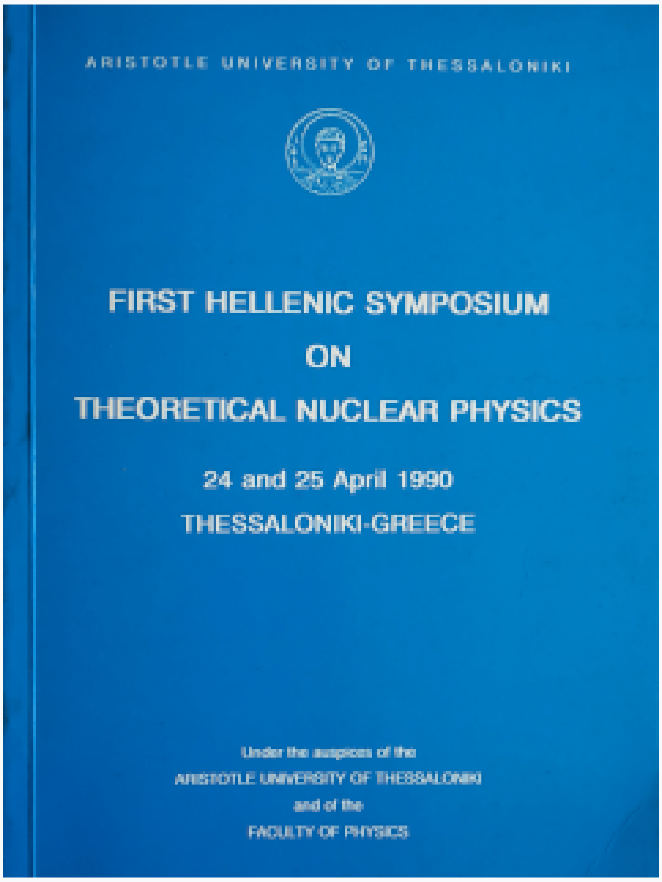

Double beta decay without invoking closure

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Double beta decay without invoking closure^{+}

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ABSTRACT : The nuclear matrix elements of all operators entering in the 0ν $\beta\beta$-decay of $^{76}Ge \rightarrow {}^{76}Se$ have been calculated explicitly in the context of QRPA within the model space 0f $f^{7/2}$ - 0h $^{11/2}$. The validity of the closure approximation has been tested and seems to be quite satisfactory for those matrix elements which are not usually suppressed. Our results indicate that they are dominated by multipoles other than 0^{+} and 1^{+} and that the matrix elements are comparable to those of shell model calculations.

1.INTRODUCTION

Neutrinoless double β-decay (0ν $\beta\beta$-decay), if it occurs in nature, is expected to play a crucial role in discriminating between the various gauge models which go beyond the standard model. The extraction of useful constraints on the parameters of these models hinges upon our ability to reliably calculate the relevant nuclear matrix elements. It is interesting therefore to obtain results for all types of nuclear matrix elements entering the $\beta\beta$-decay.

The evaluation of double β-decay nuclear matrix elements is not trivial for two reasons: First the nuclei which can undergo double β-decay have complicated structure. Second the relevant decay rates to the ground state or in some instances the first few excited states of the final nucleus seem to exhaust a small fraction of the available strength. Thus, effects which are normally small cannot be ignored here. Such are, e.g., the role of the two nucleon short range correlations and the validity of the closure approximation.

We have shown recently (1) that if one works in momentum space it is fairly easy to obtain a multipole expansion of the transition operator. As a result one needs not involve closure once the structure of the intermediate states is known. It was in fact possible to give fairly simple compact formulae which express the matrix elements in terms of the relevant one-body reduced matrix elements $|0_i^+\rangle \rightarrow |mJm)$, $|mJm) \rightarrow |0_f^+\rangle$ and a small number of energy dependent

$^{+}$Presented briefly by T. Kosmas

radial integrals. The analogous expressions in coordinate space are much more complicated and calculations of this nature are time consuming.

In momentum space however, these calculations are simpler. In a recent paper (2) we have reported calculations on ^{48}Ca for which the intermediate nuclear states can easily be constructed. This is done in the context of the Quasi Particle Random Phase Approximation (QRPA). In this work we shall report our calculations on ^{76}Ge.

2. RESULTS AND DISCUSION

It has been shown previously (1) that all nuclear matrix elements which are relevant in neutrinoless double β-decay can be expressed in terms of 16 operators which are functions of the initial and final momenta of the interacting nucleons. i.e.

$$\vec{q}_i = \frac{1}{\sqrt{2}}(\vec{p}'_i - \vec{p}_i) \text{ and } \vec{Q}_i = \frac{1}{\sqrt{2}}(\vec{p}'_i + \vec{p}_i)$$

The recoil terms which are important for the contributions coming from right handed currents are associated with the operators which only depend on Q_i. Since their dependence is at most linear on Q_i, their evaluation amounts to trivial integration. Other operators depend on integrals $I_{nl,m,p}$ which are functions of the intermediate state energies. (label m). These integrals are reduced to expressions involving only one single harmonic oscillator wave function. The allowed values of l are l=0 (scalar) l=2 (tensor) and l=1 (vector), for operators 14-16 which are associated with the recoil contribution. The value of n is also restricted to be $2n+l \leq \max(\bar{N}_\alpha + \bar{N}_{\alpha'}, +\bar{N}_\beta + \bar{N}_{\beta'})$. Thus, the same integrals may be involved in many single particle orbitals. The interaction- matrix elements which are employed in our QRPA calculation were obtained from the Bethe-Goldstone equation for the nuclear-matter G-matrix with a realistic one-boson- exchange potential (3). The bare G-matrix has been slightly renormalized in order to take into account the finite model space and the starting energy W in the Bethe-Goldstone equation is chosen to be -25 MeV. No corrections have been made to the particle-particle and particle-hole elements by some factors g_{pp} and g_{ph} and for the excitations relevant to QRPA we use two quasiparticle states of a model space consisting of the orbitals $0f^{7/2}$ - $0h^{11/2}$. The single particle energies were calculated with a Coulomb-corrected Woods-Saxon potential.

The nuclear matrix elements for all operators entering 0ν $\beta\beta$-decay are presented in tables Ia, Ib and Ic. In table Ia we present the results obtained in the exact treatment i.e. including the energy dependent operator. The short range two-nucleon correlation function and the nucleon form factor, however, were ignored in this case. In table Ib we present the same matrix elements obtained by using closure. Note that in this case $M'_F = M_{F\omega} = M_F$ and $M'_{GT} = M_{GT\omega} = M_{GT}$. The results shown in table Ic are analogous to those of table Ia except that in this case we included the short range two-nucleon correlation function. Furthermore for the short-range operators ($\Omega_2,\Omega_4,\Omega_9$) we included a nucleon form factor of a dipole shape. In table II we present the combinations of matrix elements as they enter 0ν $\beta\beta$-decay, all in the notation of ref. (4) except for x'_R which is in the notation of Tomoda and Faessler (5)

By comparing tables Ia and Ib we see that closure works pretty well for the matrix elements which are not unusually suppressed. In fact these closure matrix elements do no defer from those of the exact treatment by more than 15%. The greatest discrepancy appears in the case of the scalar recoil term. The discrepancy in the small Fermi like matrix elements is larger but their contribution to the 0ν $\beta\beta$-decay rate is negligible.

Our results indicate that the various multipoles do not contribute additively. We find cancellations between the multipoles even in the case of the larger matrix elements. The contribution of the 1^+ states to the Gamow Teller matrix elements is negligible which is in agreement with the results previously obtained near $g_{pp} \approx 1.0$. The largest contribution comes from the members of the 2^- multipoles. We should mention that the matrix element M_P, which is associated with the spin antisymmetric operator, is quite large and dominated by high negative parity multipoles ($3^-,5^-,7^-$). We also notice that, unlike the case of ^{48}Ca (within $0f_{7/2}$ shell), the matrix element M'_{GT} and M'_T are cancelling each other. Thus, the matrix elements $x_1^{\pm}$ are very small. Note, however, that again unlike ^{48}Ca the scalar and the tensor recoil contributions are additive which leads to large value of x'_R. By comparing tables Ia and Ic we see that the effect of the short range two-nucleon correlation function on the Gamow-Teler matrix element is small. Its effect on the relatively short range operators Ω_S^R, Ω_T^R is large as expected. As expected, the inclusion of the short range two-nucleon correlation function radically affects the heavy majorana neutrino matrix elements M_F^H and M_{GT}^H. The reduction of these matrix elements by only a factor of 2 can be traced back to the nucleon form factor which makes the operator have a finite range r=0.25 fm. We do not understand why the matrix element M_p is reduced by a factor of 2 once the correlation is turned on.

We also find the recoil contribution is dominant reconfirming the results of Tomoda and Faessler (5). For this reason the precise value of $x_1^{\pm}$ and x'_p is not relevant. As expected, the contribution of the recoil term is reduced by the presence of short range correlations by about 40% but is still quite high ($x'_p \approx 96$).

The matrix elements relevant for 2ν ββ-decay, properly weighted by energy denominators in units of $M_e c^2$ are very small i.e. $M_F^{2\nu} = 0.041$, $M_{GT}^{2\nu} = 0.020$. The smallness of these matrix elements is consistent with the previously obtained results (5).

Finally we would like to compare our present results with those of the existing shell model calculations.Our value for the matrix element $M_\nu=(1-x_f)M_{GT}$ is almost the same with the result of ref. (6) with monopole interaction, about 35% smaller than that of Ref. (7) and about a factor of two smaller than that of Ref.(8). As we have mentioned, our values of $x_1^{\pm}$ are tiny compared to those of the shell model calculations. These cancel the Fermi (M'_F) and Gamow Teller (M'_{GT}) ME. We have no explanation for this discrepancy. Also the value $M_p = 1.74$, obtained in the present calculation is somewhere between the values of the two above mentioned shell model calculations. The heavy neutrino matrix elements are a factor of two smaller than those of Ref.(8).

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	M_F	M_{GT}	M'_F	M'_{GT}	M'_T	$M_{F\omega}$	$M_{GT\omega}$	M_P	Recoil R_V	Recoil R_S	Recoil R_T	M''_F	M''_{GT}
0^+	0.4137		0.4875			0.3503						7.3	
1^+		0.5321		-0.5246	0.3287		-0.5448		0.0202	-6.2555	-2.3021		-39.2
0^-		0.1010		0.1115	-0.1276		0.0904			0.7086	0.7125		4.6
1^-	-0.6365	0.6817	-0.7417	0.7744	0.4220	-0.5314	0.5891	0.0116	-0.1248	3.7480	-1.8574	-22.5	24.6
2^-		2.9309		3.1148	-0.4173		2.7477		-0.0044	15.1570	2.3400		96.8
2^+	0.6948	-0.4848	0.7307	-0.5103	-0.3147	0.6588	-0.4592	-0.1339	-0.1362	-5.3813	2.6912	44.7	-33.6
3^+		1.2037		-1.2679	0.3603		-1.1392			-15.1954	-3.9094		-95.2
3^-	-0.5355	0.5417	-0.5586	0.5602	0.3550	-0.5123	0.5233	-1.1506	-0.1040	8.1479	-4.0741	-52.1	53.7
4^-		1.1089		1.1450	-0.2269		1.0726			17.9484	3.0571		121.4
4^+	0.3217	-0.2682	0.3337	-0.2792	-0.1752	0.3097	-0.2572	0.0326	-0.0804	-5.7287	2.8644	41.0	-34.1
5^+		-0.5287		-0.5478	0.1807		-0.5094			-13.3302	-3.7698		-84.4
5^-	-0.3265	0.3380	-0.3345	0.3543	0.2228	-0.3184	0.3305	-1.3388	-0.1220	9.2581	-4.6291	-55.1	58.1
6^-		0.5557		0.5694	-0.1303		0.5418			16.4846	2.9315		102.0
6^+	0.1260	-0.1129	0.1302	-0.1169	-0.0739	0.1216	-0.1087	0.0904	-0.0336	-4.1296	2.0648	28.1	-25.8
J>6	-0.1270	0.0579	-0.1279	-0.0603	0.1447	-0.1260	-0.0554	-1.3561	-0.0474	-1.1580	-4.5833	-34.0	-7.4
SUM	-0.0693	3.0696	-0.0803	3.3226	0.5514	-0.0477	2.8225	-3.8448	-0.6526	20.2734	-8.4637	-42.6	140.0

Table Ia: The various nuclear matrix elements entering the $0\nu\ \beta\beta$-decay of ^{76}Ge (For notation see ref 1). The exact energy dependent transition operator was used without short range two nucleon correlation function.

	M'_F, $M_F/M_{F\omega}$	M'_{GT}, $M_{GT}/M_{GT\omega}$	$M_{T'}$	M_P	Recoil			M^H_F	M^H_{GT}
					R_V	R_S	R_T		
0^+	0.5305							7.3	
1^+		-0.4595	0.3706		0.0321	-6.4591	-2.4240		-39.1
0^-		0.1134	-0.1512			1.7551	0.7568		4.6
1^-	-0.7717	0.7958	0.5283	0.0196	-0.1385	4.0539	-2.0271	-22.5	24.5
2^-		3.1389	-0.4289		-0.0046	15.8448	2.4048		96.8
2^+	0.7376	-0.5124	-0.3416	-0.1333	0.0213	-5.5717	2.7858	44.7	-33.7
3^+		-1.2731	0.3844		0.2144	-15.7483	-4.0270		-95.2
3^-	-0.5773	0.5895	0.3931	-1.1350	-0.0008	8.8027	-4.4013	-52.0	53.2
4^-		1.2193	-0.2597		-0.0276	20.0295	3.3969		121.1
4^+	0.3261	-0.2712	-0.1809	-0.0343	0.0015	-5.7280	2.8640	41.2	-34.6
5^+		-0.5604	0.1900		0.0465	-13.9592	-3.8512		-84.4
5^-	-0.3487	0.3530	0.2354	-1.2861	-0.1374	9.6279	-4.8140	-55.2	58.2
6^-		0.5702	-0.1367		-0.0092	16.8812	3.0260		102.1
6^+	0.1305	-0.1172	-0.0782	0.0982	-0.0349	-4.2688	2.1344	28.1	-25.8
$J \geq 6$	-0.1279	-0.0601	0.1466	-1.3436	-0.0077	-1.2982	-4.6036	-34.0	-7.0
SUM	-0.1009	3.5262	0.6712	-3.8145	-0.0449	24.0418	-8.7795	-42.4	140.3

Table Ib: The same quantities as in table Ia, but using closure.

	M_F	M_{GT}	M'_F	M'_{GT}	M'_T	$M_{F\omega}$	$M_{GT\omega}$	M_P	R_V	R_S	R_T	M_F^{II}
0^+	0.3128		0.4650			.3301						3.3
1^+		-0.4279		-0.4006	0.3293		-0.4511		0.0220	-2.4703	-2.0816	
0^-		0.0879		0.0973	-0.1277		0.0779			0.2756	0.6341	
1^-	-0.5836	0.6231	-0.6476	0.7000	0.4223	-0.4897	0.5430	-0.0044	-0.1277	1.6600	-1.7089	-10.4
2^-		2.6585		2.8201	-0.3812		2.4836		-0.0176	7.0256	2.0458	
2^+	0.5675	-0.3478	0.5945	-0.3613	-0.2854	0.5365	-0.3297	-0.1316	0.0010	-1.8200	2.1796	18.1
3^+		-0.7690		-0.7947	0.3397		-0.7308		-0.2029	-4.9771	-3.2822	
3^-	-0.4001	0.4119	-0.4110	0.4192	0.3727	-0.3816	0.3967	-0.8511	0.0105	3.1966	-3.7804	-19.5
4^-		0.8225		0.8315	-0.2436		0.7957		-0.0261	6.9854	2.8212	
4^+	0.1923	-0.1586	0.1930	-0.1597	-0.1704	0.1857	-0.1524	-0.0034	-0.0048	-1.8422	2.3902	14.1
5^+		-0.2896		-0.2866	0.1821		-0.2803		0.0329	-4.0618	-3.1676	
5^-	-0.1674	0.1699	-0.1611	0.1632	0.2286	-0.1651	0.1680	-0.6153	-0.1024	2.8484	-4.0096	-17.3
6^-		0.2649		0.2502	-0.1371		0.2634		-0.0040	4.9349	2.6432	
6^+	0.0438	-0.0376	0.0394	-0.0336	-0.0740	0.0440	-0.0378	0.0407	-0.0249	-1.0072	1.6771	7.1
J>6	-0.0225	-0.0323	-0.0167	-0.0337	0.1437	-0.0229	-0.0311	-0.1745	-0.0277	-0.2209	-3.6127	-7.1
SUM	0.0228	2.9759	0.0555	3.2113	0.5990	0.0370	2.7151	-1.7396	0.4365	10.5270	-7.7023	-11.7

Table Ic: The same as in table Ia but with two nucleon short range correlations included. Note that for the matrix elements R_S, M_F^{II} and M_{GT}^{II} a dipole shape nucleon form factor was included.

	ENERGY DEPENDENT NO-CORRELATION	CLOSURE NO-CORRELATION	ENERGY DEPENDENT WITH CORRELATION
M_{GT}	3.07	3.53	2.97
$1 - \chi_F$	1.02	1.03	0.99
χ_1^+	-0.046	-0.198	-0.092
χ_1^-	0.056	-0.086	-0.165
χ_2^+	0.916	1.003	0.935
χ_2^-	0.923	1.028	0.925
χ_P'	-1.01	-0.87	-0.47
χ_R'	140	142	95.8
M_{GT}^H	140	140	67.7
$1 - \chi_f^H$	1.24	1.21	1.11

Table II: The various matrix elements entering the 0ν $\beta\beta$-decay in the notati Ref. 4. For the recoil term χ_R' our definition agrees with that of Tomoda and Faessler (Ref. 5). M_{GT}^H and χ_F^H are quantities associated with heavy neutrino