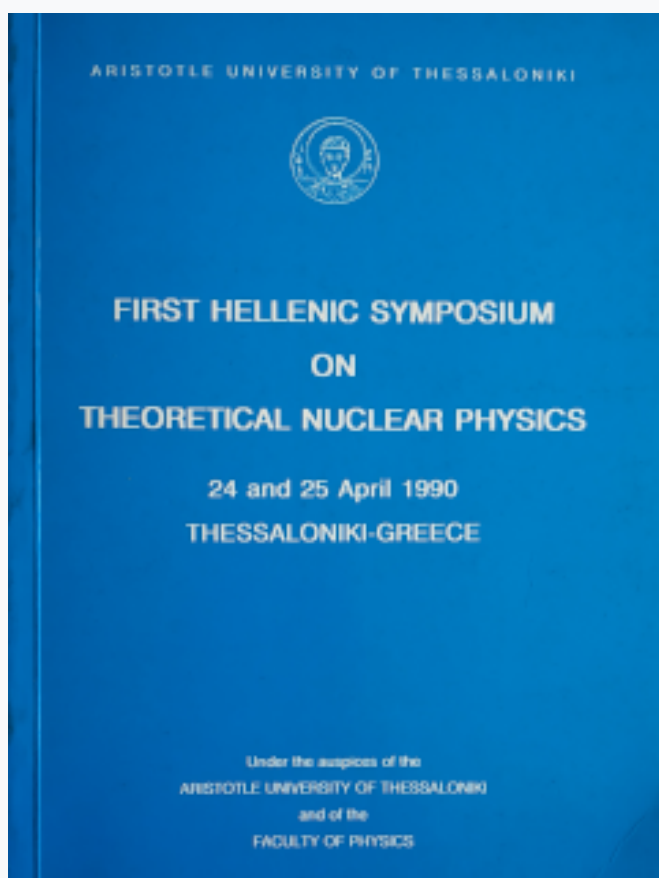


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*E. Mavrommatis, J. W. Clark*

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# Microscopic Study of the Response of Nuclear Matter <sup>1</sup>

E. Mavrommatis\* and J. W. Clark<sup>†</sup>

\*Physics Department, University of Athens, Panepistimioupoli, 15771 Athens, Greece

<sup>†</sup>Department of Physics and McDonnell Center for the Space Sciences,  
Washington University, St. Louis, Missouri 63130

**ABSTRACT:** The correlated random-phase approximation (CRPA<sub>1</sub>), which provides a description of the linear response and elementary excitations of nuclear matter, is summarized. The density-density response functions of symmetrical nuclear matter and pure neutron matter are calculated using a local version of CRPA<sub>1</sub> (LCRPA) based on the  $v_2$  model nucleon-nucleon interaction. Although simple, the calculation establishes some significant qualitative trends. It constitutes a prelude to calculations of response functions of realistic nucleon matter both with CRPA<sub>1</sub> and with theories that go beyond it.

## 1. INTRODUCTION

The dynamical response of nuclear systems carries vital information about their excitations, about the role of short-range dynamical correlations, and possibly about relativistic effects and nonnucleonic degrees of freedom. The aims of microscopic calculation of the response within conventional nuclear many-body theory are (a) to give a more realistic description beyond mean-field approximations and (b) to determine the extent to which nuclei can be described as systems containing nucleons alone, interacting via two-, three-, ... body interactions constrained by few-body data and meson-exchange theory. While many-body theories of ground-state properties of strongly interacting Fermi systems have reached a high degree of quantitative reliability, microscopic prediction of dynamic structure is at a relatively early stage.

<sup>1</sup> Presented by E. Mavrommatis

This work is part of a research program that deals microscopically with the excitations and response functions of symmetrical nuclear matter and pure neutron matter (with level degeneracy  $\nu = 4$  and  $\nu = 2$  respectively), using methods within the correlated-basis-functions theory (CBF) that are based on the correlated random-phase approximation CRPA<sub>I</sub> (Chen *et al.* 1982; Krotscheck 1982). CRPA<sub>I</sub> takes into account explicitly only 1p-1h excitations and performs the ring summation within CBF.

We will report here calculations of the density-density response function

$$\Pi(\mathbf{q}, \omega) = \frac{1}{A} \sum_{n \neq 0} \langle 0 | \rho^\dagger(-\mathbf{q}) | n \rangle \langle n | \rho(\mathbf{q}) | 0 \rangle \left[ \frac{1}{\hbar\omega - E_n + E_0 + i\epsilon} - \frac{1}{\hbar\omega + E_n - E_0 - i\epsilon} \right] \quad (1)$$

and dynamic structure function

$$S(\mathbf{q}, \omega) = -\frac{1}{\pi} \text{Im} \Pi(\mathbf{q}, \omega) = \frac{1}{A} \sum_{n \neq 0} |\langle 0 | \rho(\mathbf{q}) | n \rangle|^2 \delta(\hbar\omega - E_n + E_0) \quad (2)$$

In the above,  $\mathbf{q}$  and  $\omega$  are the momentum and energy transferred by the probe,  $|0\rangle$  and  $|n\rangle$  are the ground and excited eigenstates that correspond to  $E_0$  and  $E_n$  energy eigenvalues and  $\rho(\mathbf{q})$  is the density fluctuation operator ( $= \sum_i e^{i\mathbf{q} \cdot \mathbf{r}_i}$ ).

Impetus for microscopic calculation of  $\Pi(\mathbf{q}, \omega)$  and  $S(\mathbf{q}, \omega)$  for nuclear matter comes primarily from the current generation of electron-scattering experiments on medium and heavy nuclei in the quasielastic energy regime (Meziani *et al.* 1984; Blatchley *et al.* 1986; Deady *et al.* 1986) and the inability of independent-particle models to explain the available data on the longitudinal response function  $S_L(\mathbf{q}, \omega)$ . If one attempts to understand these experiments within the conventional nuclear picture, it appears that contributions from many-body effects of increasing complexity must be investigated (Meziani 1985). Microscopic calculation of  $S(\mathbf{q}, \omega)$  for infinitely extended nuclear matter is expected to yield valuable insights into the nature and importance of these complicated many-body processes. This expectation rests on an assumed proportionality of the measured  $S_L(\mathbf{q}, \omega)$  and the infinite-matter  $S(\mathbf{q}, \omega)$  (Fantoni and Pandharipande 1987, Fabrocini and Fantoni 1989), as well as on the relationship of nuclear matter results with those of finite nuclei that has been established recently with

the extraction of a nuclear-matter response function from the response of finite nuclei (Day *et al.* 1989).

A second motivation is that a microscopic evaluation of  $\Pi(\mathbf{q},\omega)$  for nuclear matter, together with consistent evaluation of the self-energy  $\Sigma(\mathbf{k},E)$ , contain fundamental information about the elementary excitations of the system. The properties of collective modes, typified by the zero-sound dispersion relation, may be extracted from  $\Pi(\mathbf{q},\omega)$ , while the nature of single-particle excitations is revealed by  $\Sigma(\mathbf{k},E)$ , from which one may derive an energy-dependent effective mass. These properties have obvious importance for a deeper understanding of nuclei. They are likewise basic to a description of the structure, dynamics and thermal history of neutron stars, being essential to the evaluation of such quantities as the specific heat, viscosity, superfluid gap, etc. (Maxwell 1979; Flowers and Itoh 1979; Chen *et al.* 1986). Since empirical constraints on the properties of neutron-star material are limited in the extreme, such astrophysical applications make it doubly important to develop our many-body calculations.

Previous microscopic treatments of  $S(\mathbf{q},\omega)$  include the calculation of Butler and Koonin (1988), based on the Brueckner-Goldstone theory and the Reid and Paris interactions, and the calculation of Fantoni and Pandharipande (1987) which is performed essentially at the Tamm-Dancoff level within a suitably developed correlated basis theory and which uses the  $v_{14}$ +TN1 interaction. (We should also call attention to the more phenomenological calculations of Alberico *et al.* 1980 and Pines *et al.* 1988).

In this paper we present an initial application of CRPA<sub>I</sub> theory to the calculation of  $S(\mathbf{q},\omega)$  in infinite nuclear systems (Mavrommatis *et al.* 1987; Mavrommatis and Clark 1990). We use primarily a simplified model of the nucleon-nucleon interaction, namely, the  $v_2$  potential (Pandharipande *et al.* 1975) and a simplified, local version of CRPA<sub>I</sub> (LCRPA) that has proven successful in applications to spin-polarized liquid  $^3\text{He}$  (Krotscheck *et al.* 1983), the electron gas (Krotscheck 1984) and spin-polarized deuterium (Davé *et al.* 1990). It will emerge that this initial application already leads to results of qualitative or semi-quantitative significance. Improvements of various aspects of the LCRPA calculation are currently being implemented.

We begin with a review of the theoretical basis of the correlated random-phase approximation CRPA<sub>I</sub> and its local version LCRPA.

## 2. CORRELATED RANDOM-PHASE APPROXIMATION

The correlated random-phase approximation (CRPA<sub>I</sub>) uses correlated-basis-functions (CBF) theory (Clark 1981) to extend the ordinary (first-order) random-phase approximation (RPA<sub>I</sub>) to the case of strongly interacting systems like liquid <sup>3</sup>He, nuclear matter, and nuclei. The ordinary RPA<sub>I</sub> may be extracted as the small-amplitude limit of time-dependent Hartree-Fock theory. To adapt this derivation to strongly-coupled systems (Chen *et al.* 1982) requires a replacement of all energy eigenstates  $|\phi_m\rangle$  of the noninteracting Fermi system by the corresponding correlated basis states

$$|\psi_m\rangle = F|\phi_m\rangle I_{mm}^{-1/2} \quad , \quad I_{mm} = \langle \phi_m | F^\dagger F | \phi_m \rangle \quad , \quad (3)$$

where F is a suitable static correlation operator, *e.g.*, of Feenberg or Jastrow form. One obtains the following set of supermatrix equations in place of the usual RPA<sub>I</sub> equations:

$$\begin{bmatrix} A & B \\ B^* & A^* \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \hbar \omega \begin{bmatrix} M & 0 \\ 0 & -M^* \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \quad . \quad (4)$$

Here, x and y are column matrices and A, B, and M are square matrices whose elements carry particle-hole (p-h) labels, *e.g.*,  $x = (x_{ph})$  and  $A = (A_{ph;p'h'})$ . The solutions of these equations yield approximate excitation energies  $\hbar\omega$  and amplitudes  $x_{ph}^*, y_{ph}^*$  for finding a given p-h pair present in or absent from the corresponding excited states. The matrices A and B (respectively Hermitian and symmetric) are now constructed in terms of the CBF effective interaction vertex V(12) and the CBF single-particle energies  $e(p)$  and  $e(h)$  assigned to particles and holes, while the matrix M is constructed in terms of the CBF nonorthogonality vertex N(12). Explicitly,

$$\begin{aligned} A_{ph;p'h'} &= [e(p) - e(h)] \delta_{pp'} \delta_{hh'} + \langle ph' | V(12) | hp' \rangle_a \quad , \\ B_{ph;p'h'} &= \langle pp' | V(12) | hh' \rangle_a \quad , \\ M_{ph;p'h'} &= \delta_{pp'} \delta_{hh'} + \langle ph' | N(12) | hp' \rangle_a \quad , \end{aligned} \quad (5)$$

wherein  $V(12)$  is in turn determined by  $W(12)$  (the CBF interaction vertex),  $N(12)$ , and  $e(k)$ . At nuclear densities, these CBF ingredients may be evaluated rather accurately by Fermi-hypennetted-chain (FHNC) procedures in the case of the Jastrow correlation factor  $F_J = \prod_{i < j} f(r_{ij})$ , which is adopted here in specific calculations.

The CRPA<sub>I</sub> equations (4) can be solved, with considerable effort, by standard diagonalization techniques on a suitable mesh (Kwong 1982). However, the nonorthogonality of the correlated basis, which is responsible for the appearance of the nontrivial metric matrix  $M$ , introduces an awkward energy dependence which is not present in ordinary RPA<sub>I</sub>. Fortunately, most of this energy dependence can be transformed away by rewriting the theory in terms of a p-h irreducible effective p-h interaction (Krotscheck 1982). The reformulation is accomplished as follows. First, one defines a correlation supermatrix

$$C = \begin{bmatrix} (C_{ph;p'h'})(C_{ph;h'p'}) \\ (C_{hp;p'h'})(C_{hp;h'p'}) \end{bmatrix} = \begin{bmatrix} \langle ph' | N(12) | hp' \rangle_a & \langle pp' | N(12) | hh' \rangle_a \\ \langle hh' | N(12) | pp' \rangle_a & \langle hp' | N(12) | ph' \rangle_a \end{bmatrix} \quad (6)$$

and a corresponding interaction supermatrix  $W$  in which the vertex  $N(12)$  is replaced by  $W(12)$ . The p-h irreducible components of these matrices, denoted respectively by  $X$  and  $X'$ , are then extracted via the relations

$$C = X + \frac{1}{2} CX \quad , \quad W = (1 + \frac{1}{2}C)X'(1 + \frac{1}{2}C) \quad . \quad (7)$$

It can be checked that neither  $X$  or  $X'$  so determined contains any diagrams which can be visually identified as p-h reducible (*i.e.*, separable into two disjoint parts by cutting a single pair of p-h lines). In particular, no chain diagrams appear in  $X$ . Setting

$$\Omega = \begin{bmatrix} [e(p) - e(h) - \hbar\omega - i\eta]\delta_{pp'}\delta_{hh'} & 0 \\ 0 & [e(p) - e(h) + \hbar\omega + i\eta]\delta_{pp'}\delta_{hh'} \end{bmatrix} \quad (8)$$

(where  $\eta$  is positive infinitesimal), we may then form a supermatrix

$$U(\omega) = X' - \frac{1}{4} X\Omega X \quad , \quad (9)$$

which plays the role of a p-h irreducible p-h interaction. In terms of  $\Omega$  and  $U(\omega)$ , the CRPA<sub>I</sub> equations (4) may be rewritten as

$$[\Omega + U(\omega)] \begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix} = 0 \quad , \quad (10)$$

where

$$\begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix} = \left( 1 + \frac{1}{2} C \right) \begin{bmatrix} x \\ y \end{bmatrix} .$$

Apart from a minor residual energy dependence of  $U$  (henceforth ignored), these equations are formally identical with those of ordinary RPA<sub>I</sub> and can be studied with quite conventional procedures. Their detailed solution is a computationally demanding task, which we have just started to pursue. Here we report on the results of the application of a simple local approximation scheme, called local CRPA<sub>I</sub> (LCRPA), which allows ready comparison with semiphenological approaches, notably the polarization-potential description (Pines *et al.* 1988).

### 3. LOCAL CORRELATED RANDOM-PHASE APPROXIMATION

The local approximation that we have used was proposed by Krotscheck (1982). Here “local” implies that the pertinent matrix elements of the  $p$ - $h$  channel of  $N(12)$  and  $W(12)$ , and indeed the composite quantity  $U$  of Eq. (9), become functions only of the momentum transfer  $q = |\mathbf{p} - \mathbf{h}|$  in the direct  $p$ - $h$  channel (apart from momentum-conserving delta functions). In particular,

$$U_{ph;p'h'} = A^{-1} U(q) \delta(\mathbf{p} + \mathbf{p}' - \mathbf{h} - \mathbf{h}') \quad , \quad (11)$$

where  $A$  is the particle number and  $U(q)$  is the local  $p$ - $h$  force. The latter is constructed assuming a Jastrow correlation factor  $F_J$ , and requiring (a) that the approximation to the  $N(12)$  matrix elements preserves the relation of these matrix elements to the static structure factor  $S(q)$  of the Jastrow ground-state trial function  $F_J \phi_0$  and (b) that the approximation to the  $W(12)$  matrix elements preserves their analogous role in the optimization condition for the Jastrow two-body correlation function  $f(r)$ ,

$$\frac{\delta \langle H \rangle_0}{\delta \ln f^2(r)} \equiv \Delta(r) = 0 \quad , \quad (12)$$

where  $\langle H \rangle_0$  is the energy expectation value in the Jastrow trial ground state  $F_J \phi_0$ . The proposed  $U(q)$  is simply

$$U(q) = \Delta(q)S^{-2}(q) + \frac{\hbar^2 q^2}{4m} [S^{-2}(q) - S_F^{-2}(q)] \quad , \quad (13)$$

where  $S_F(q)$  is the static structure factor of the noninteracting Fermi system and

$$\Delta(q) = \rho \int dr \exp(i\mathbf{q} \cdot \mathbf{r}) \Delta(r) = \frac{\hbar^2 q^2}{4m} [S(q) - 1] + S'(q) \quad . \quad (14)$$

The Jastrow  $S(q)$  entering (13)-(14) may be evaluated with good accuracy (near nuclear densities) by solving the FHNC/C equations, while its graphical derivative  $S'(q)$  appearing in (14) may be obtained by solving the FHNC/C' equations (Krotscheck *et al.* 1981). The vanishing of  $\Delta(q)$  is equivalent to the optimization condition. Hence, for optimal Jastrow correlations the p-h force  $U(q)$  depends only on  $S(q)$  and properties of the noninteracting system.

With a local p-h force, one has quite standard algebraic RPA formulas (see, *e.g.*, Brown 1972), which lead to the familiar  $RPA_I$  expression of the density-density response function

$$\Pi(q, \omega) = \frac{\Pi_0(q, \omega)}{1 - U(q)\Pi_0(q, \omega)} \quad . \quad (15)$$

The response function  $\Pi_0(q, \omega)$  is the p-h propagator of the free Fermi system, *i.e.*, the Lindhard function.

The dynamic structure factor and the properties of zero sound (if present) are derived from the relation (15) in the usual manner. Thus

$$S(q, \omega) = -\frac{1}{\pi} \text{Im} \Pi(q, \omega) \quad , \quad (16)$$

while the zero-sound dispersion relation  $\omega = \omega_{zs}(q)$  is determined by the roots of the denominator of (15), *i.e.*, the roots of

$$1 - U(q)\text{Re} \Pi_0(q, \omega) = 0 \quad (17)$$



in the region where  $\text{Im } \Pi_o(q, \omega) = 0$ . The strength  $Z_{zs}$  of the zero-sound mode is given by

$$Z_{zs}^{-1}(q) = U^2(q) \left. \frac{d}{d\omega} \text{Re } \Pi_o(q, \omega) \right|_{\omega=\omega_{zs}}. \quad (18)$$

With  $Z_{zs}$  taken into account, LCRPA satisfies the  $\omega^0$  and  $\omega^1$  sum rules.

A qualitative defect of LCRPA, evident in the form (15), is that the  $(q, \omega)$  domain corresponding to individual 1p-1h excitations is the same in LCRPA as in the free system. However, the  $\text{RPA}_I$  denominator in expression (15) introduces nontrivial correlation effects in that region. Moreover, outside that region and in the region where  $\text{Im } \Pi_o(q, \omega) = 0$ , zero sound may emerge as a distinct collective mode, corresponding to vanishing of the denominator.

LCRPA will also suffer, at a quantitative level, from the static nature of the effective interaction  $U(q)$  entering (15). For example, one does not expect the momentum dependence of the self-energy to be adequately reproduced within this scheme, particularly in the very delicate case of unpolarized liquid  $^3\text{He}$  (Friman and Krotscheck 1982). In spite of its deficiencies, LCRPA offers a simple and straightforward underpinning for phenomenological theories of comparable structure, such as the polarization potential model (Pines *et al.* 1988).

#### 4. RESULTS FOR MODEL NUCLEAR INTERACTIONS

Based on the LCRPA scheme outlined in Sec. 3, we have studied the dynamical response of a simple model of nucleon matter in which the bare interaction between nucleons is taken as the  $v_2$  "homework-model" potential shown in Figure 1 (Pandharipande 1975). This potential consists of the central part of the  $^3S_1$ - $^3D_1$  component of the Reid soft-core interaction (Reid 1968), assumed to act in all partial waves. It has been widely used in tests of many-body methods (see, for example, Clark 1979, Ramos *et al.* 1989). Explicitly,

$$v_2(r) = [9924.3 \exp(-4.2r) - 3187 \exp(-2.8r) + 105.468 \exp(-1.4r) - 10.463 \exp(-0.7r)]/(0.7r) \quad (19)$$

Our calculations are based mainly on the parametrized Jastrow correlation factor [denoted (C)]

$$f(r) = \exp[-Ae^{-Br}(1 - e^{-r/D})/r] \quad (20)$$

Specific values of the parameters in (20) have been determined by Cepeley *et al.* (1977), by minimizing the Jastrow ground-state energy  $E_J$ , using the Metropolis Monte Carlo algorithm. Additionally, for  $\nu = 4$  we have considered two versions of the correlation function

$$f(r) = (1 - e^{-r^2/b^2})^n + gr^m e^{-r^2/\gamma^2} \quad (21)$$

studied by Benhar *et al.* (1976a,b). In this case the parameters have been determined by minimization of the energy expectation value truncated at three-body cluster order. In the simpler version of (21) [denoted (B1)],  $g$  has been set equal to zero, while in the other [denoted (B2)], it has been fixed with the help of the normalization condition (see, for example, Clark 1979).

For the sake of comparison, we also present nuclear matter ( $\nu = 4$ ) results for the case that the nucleon-nucleon interaction is replaced by a pure hard-core potential (HC) of core radius  $c = 0.5$  or  $0.4$  fm. For this potential a simple choice of  $f(r)$  is used,

$$f(r) = \begin{cases} 0 & r \leq c \\ 1 - \exp[-\mu(r-c)] & r > c \end{cases} \quad (22)$$

with  $\mu$  determined by minimizing the FHNC/C approximation to the ground-state energy (Flynn 1984). Plots of some of the above correlation functions for nuclear matter ( $\nu = 4$ ) are given in Figure 1.

None of these correlation functions is optimal in the sense of being a solution of the Euler equation, but it is expected that the C choice will not depart substantially from the true optimal function except at large  $r$ . The discrepancy at large  $r$  will produce a significant departure of  $\Delta(q)$  from zero at low  $q$  values and will accordingly affect the behavior of  $U(q)$  at small  $q$ . We have chosen to set  $\Delta(q)$  equal to zero, since the behavior of  $U(q)$  at small  $q$  is already suspect because of the local approximation itself.

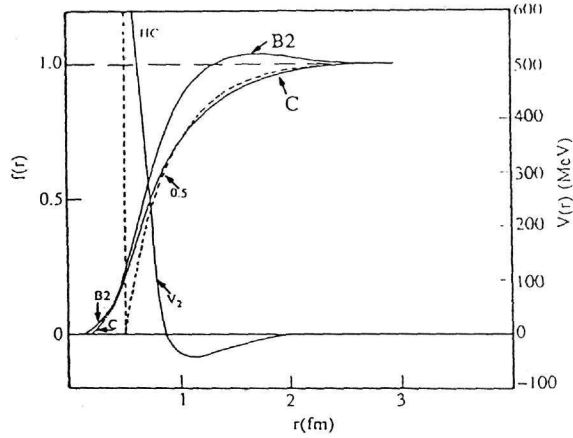


Figure 1. Nuclear matter (a) solid curves: the potential  $v_2$  and the corresponding correlation functions  $C$  and  $B2$  at  $k_F = 1.39 \text{ fm}^{-1}$  and  $1.4 \text{ fm}^{-1}$  respectively, (b) dashed curves: the hard-core potential of radius  $c = 0.5 \text{ fm}$  and the optimized correlation function of Eq. (22) at  $k_F = 1.4 \text{ fm}^{-1}$ .

Numerical results based on (20) have been obtained for symmetrical nuclear matter, *i.e.*,  $\nu = 4$ , at  $k_F = 1.39 \text{ fm}^{-1}$ , corresponding to a density  $\rho = 0.182 \text{ fm}^{-3}$  near nuclear saturation, and for pure neutron matter, *i.e.*,  $\nu = 2$ , at  $k_F = 1.75, 2.25, 2.90 \text{ fm}^{-1}$ , corresponding to three densities  $\rho = 0.182, 0.386, 0.822 \text{ fm}^{-3}$  of relevance in the study of neutron stars. For both level degeneracies, we have examined a range of wave-number transfers  $q$  from 0 to about  $4 \text{ fm}^{-1}$ . Results based on (21) are available only for symmetrical nuclear matter at Fermi wave number  $k_F = 1.4 \text{ fm}^{-1}$ , and hard-core results have also been obtained for this case using (22). (We should note hard-core results based on (22) with  $c = 0.4 \text{ fm}$ , for both  $\nu = 4$  and  $\nu = 2$  systems, may be found in Mavrommatis *et al.* 1987.)

Let us first discuss the results for symmetrical nuclear matter. The CBF p-h interaction appropriate to the  $v_2$  potential at  $k_F = 1.39 \text{ fm}^{-1}$ , for the  $C$  choice of  $f(r)$ , is shown in Figure 2(a), along with the  $c = 0.5 \text{ fm}$  hard-sphere result at the same density. Both potentials sustain a collective mode corresponding to zero sound. In both cases, this mode emerges from the p-h continuum around  $0.3\text{--}0.4 \text{ fm}^{-1}$  and sinks back into it at about  $1.53 \text{ fm}^{-1}$ . The associated zero-sound dispersion relation and the strength of zero sound for the  $v_2$  potential are plotted against  $q$  in Figures 3(a) and 3(b),

respectively. The corresponding curves for the hard-core potential show a remarkable agreement with those for  $v_2$

We see in Figure 2 that the effective interaction derived from the soft-core  $v_2$  potential is very close to that derived from the HC potential. This concurrence indicates that the two potentials are similarly effective in lifting particles from below to above the Fermi surface, at least when the momentum transferred to the p-h pair is not excessive and the density is near the saturation value for nuclear matter. In turn this observation suggests that a useful measure of the strength of the p-h force may be provided by the wound parameter  $\kappa$  (Clark 1979). Indeed, referring to Figure 2, we have  $\kappa = 0.232$  for the C choice of  $f(r)$  and  $\kappa = 0.263$  for the  $c = 0.5$  fm HC choice of Eq. (22). One comes to the same conclusion upon comparing the p-h interaction derived from C and from the B1 and B2 choices of  $f(r)$ . The substantial differences may be attributed to the different values of the wound parameters (which have the values  $\kappa = 0.146$  and  $0.143$  for B1 and B2 respectively). We also realize that the behavior of  $U(q)$  and  $\omega_{zs}(q)$  for small  $q$  is not correct. As previously mentioned, this is due to the non-optimality of the correlation functions.

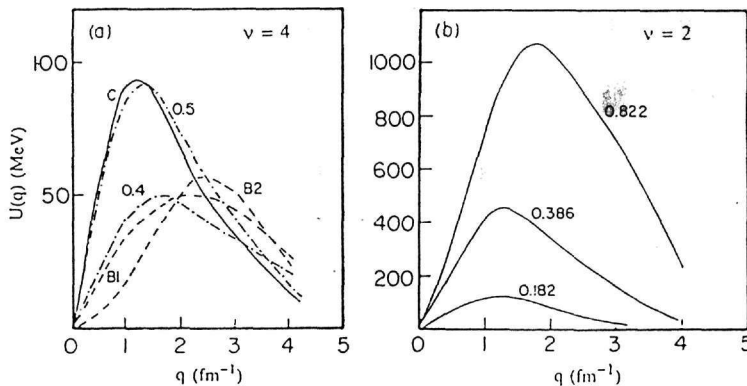


Figure 2. (a) Wave-number dependence of local p-h interaction  $U(q)$  for  $v_2$  model of nuclear matter at  $k_F \approx 1.4 \text{ fm}^{-1}$ , based on correlation functions C, B1 and B2 and for hard-sphere nucleons with  $c = 0.4$  and  $0.5$  fm based on Eq. (22). (b)  $U(q)$  for  $v_2$  model of pure neutron matter, correlation function C and different densities (as labeled, in  $\text{fm}^{-3}$ ).

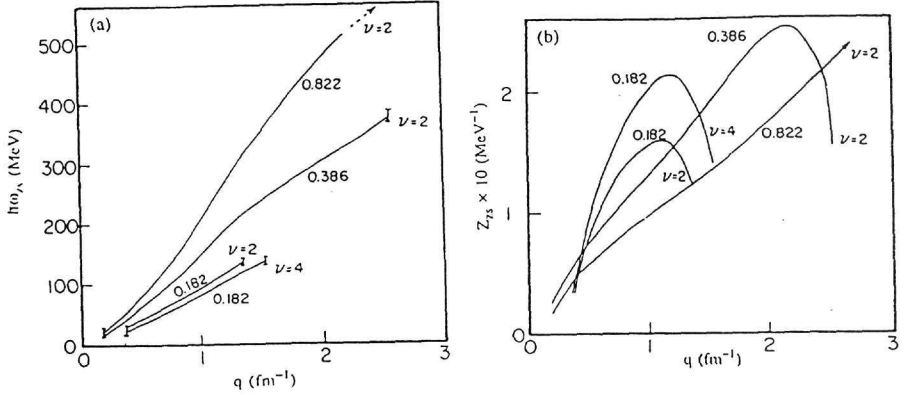


Figure 3. (a) Zero-sound dispersion relation  $\omega_{zs}(q)$  and (b) zero-sound strength  $Z_{zs}(q)$ , for  $v_2$  model of nuclear matter at  $\rho = 0.182$  fm $^{-3}$  and of neutron matter at  $\rho = 0.182, 0.386, 0.822$  fm $^{-3}$ , based on correlation functions of C type.

The remainder of the discussion pertains to the C choice of correlations. We have calculated  $S(q, \omega)$  as a function of energy transfer  $\hbar\omega$  for several values of  $q$ , including values for which experimental data are available. Results are presented here only for  $q = 1.47$  fm $^{-1}$  (Figure 4) and  $q = 2.76$  fm $^{-1}$  (Figure 5), at  $k_F = 1.39$  fm $^{-1}$ . The figures include, for comparison, the Fermi gas structure function  $S_F(q, \omega)$  as well as the LCRPA result for the  $c = 0.5$  fm hard-core potential at the same density. We observe the well-known quenching of the response at low energies compared to the independent-particle-model result (Meziani 1985), but this effect is too emphatic in our results at the lower  $q$  values. For a given  $q$ , the strength is shifted to values of  $\hbar\omega$  higher than the experimental peak energy. This effect, as well as the excessive quenching at low  $\omega$ , can presumably be attributed to the excessively repulsive character of the  $v_2$  and HC potentials, which act equally in all partial waves, in contrast to the strong partial-wave dependence of realistic nucleon-nucleon interactions. Similar trends seen in the results of Fantoni and Pandharipande (1987) and of Pines *et al.* (1988) are much milder. The rather unphysical piling up of the strength at the high- $\omega$  boundary that is observed for the lower value of  $q$  can be attributed to LCRPA, which does not give response outside the  $(q, \omega)$  domain implicated for the free system. As we go to

high  $q$ , the departure of the LCRPA results from those of the free Fermi system and from those of the other theoretical predictions become less noticeable, as expected from the decreasing importance of  $U(q)$  at large  $q$ .

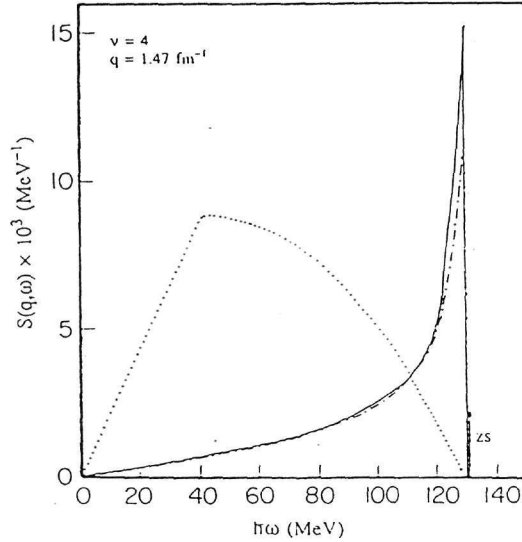


Figure 4. Dynamic structure function  $S(q, \omega)$  versus energy transfer  $\hbar\omega$  at fixed wave number transfer  $q = 1.47 \text{ fm}^{-1}$  and  $k_F = 1.39 \text{ fm}^{-1}$ . Solid curve: for  $v_2$  model of nuclear matter based on C correlation function of Figure 1. Dot-dashed curves: for hard-sphere nucleons with  $c = 0.5 \text{ fm}$  and the dashed correlation function of Figure 1. Dotted curves: for free nucleons. Zero-sound contribution in  $v_2$  case is indicated by vertical spike.

Generally speaking, the LCRPA results for neutron matter are qualitatively similar to those of symmetrical nuclear matter. The corresponding p-h forces  $U(q)$ , zero-sound dispersion relations  $\omega_{zs}(q)$ , and strengths  $Z_{zs}(q)$  are plotted in Figures 2 and 3 respectively, for the three densities mentioned above. The features remarked previously for  $S(q, \omega)$  at level degeneracy  $\nu = 4$  are also seen at  $\nu = 2$ , and density dependences for all relevant quantities are as expected. We should point out that although the Jastrow description should be more accurate for neutron matter than symmetrical nuclear matter, due to the absence of interaction in triplet-even states, the LCRPA must be less reliable due to the increased importance of nonlocalities implied by the lower level degeneracy.

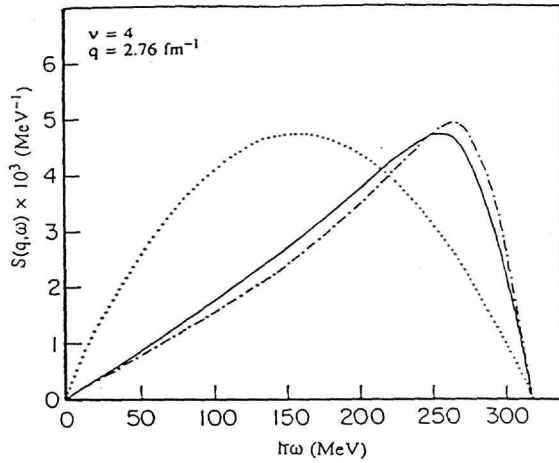


Figure 5. Same as Figure 4, except  $q = 2.76 \text{ fm}^{-1}$  and there is no zero sound.

## 5. CONCLUDING REMARKS

The density-density response functions of symmetrical nuclear matter and pure neutron matter have been studied with a simple theory, the local correlated random-phase approximation LCRPA, and a simple model interaction,  $v_2$ . Relative to an ordinary local RPA treatment, LCRPA takes account, approximately, of important dispersive, polarization, and geometrical effects arising from the strong interactions. It has the virtue of easy application and it establishes correctly some qualitative trends, but it has shortcomings, mainly at low  $q$ . The latter deficiency may be due in part to the omission of the  $\Delta(q)$  term, which is strictly not justified because non-optimal correlation functions have been used. (A quantitative study of this point is currently in progress.) However, it is likely that the shortcomings at low  $q$  are mainly due to the omission of nonlocalities, as has been pointed out recently in a comparison of LCRPA and the self-consistent Green's function theory (Dickhoff 1988) in the case of spin polarized  $^3\text{He}$ . We are therefore beginning calculations with CRPA<sub>I</sub> itself (in collaboration with N. H. Kwong), solving numerically the full CRPA<sub>I</sub> equations on a suitable grid in momentum space, with techniques similar to those used earlier for OMY models of nucleon matter (Sandler and Kwong 1984). Besides the solution of the full CRPA<sub>I</sub>, a satisfactory treatment of the response of realistic nucleon matter obviously requires the use of

realistic interactions and state-dependent correlation factors  $F$ . Initially, the  $v_4$  and  $v_6$  models of nucleon matter will be considered. Since our goal is a truly quantitative description of the response and elementary excitations in nuclear systems, theories are required that go beyond CRPA<sub>I</sub> and include extra correlated multipair effects. Two such extensions are currently under investigation. The first (CFRPA) treats  $F$  as a dynamical quantity, and the second (CRPA<sub>II</sub>) is the extension within CBF of the second-order RPA (see, for example, Yannouleas *et al.* 1983). Finally, we should remark that our calculations will be extended to spin and isospin-density response functions.

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