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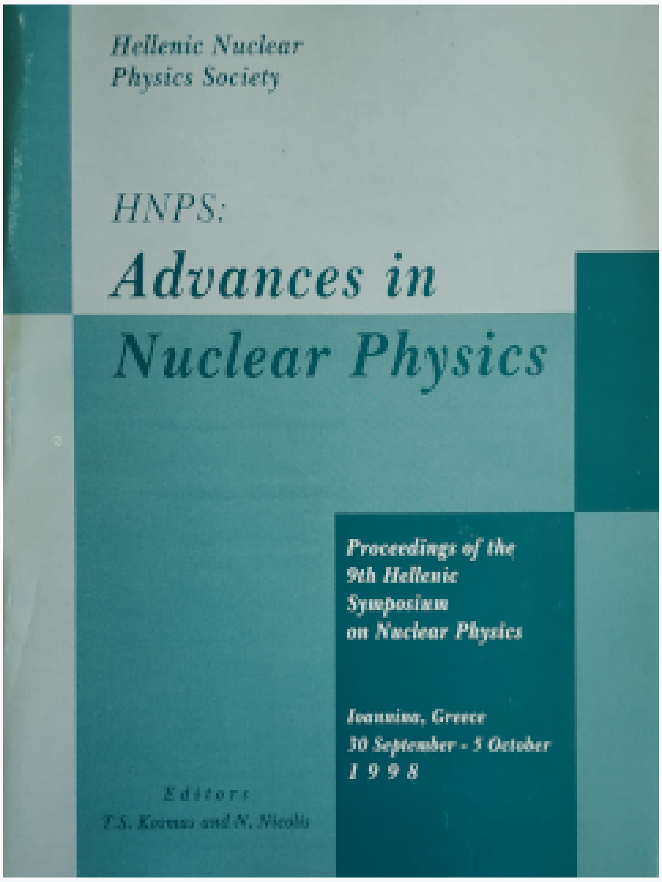

Liouville's theorem and quantum mechanics - time quantization and reality

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Liouville's theorem and quantum mechanics – time quantization and reality

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Abstract

The chrono-topology, as introduced axiomatically in a different context, is also supported by Liouville's theorem of statistical mechanics. It is shown that, if time is quantized, the distribution function (d.f.) becomes real. An elementary solution, g, of the classical Liouville equation has been found in phase-space and time, which can be used to construct any differentiable *d.f.*, $F(g)$, satisfying the same Liouville equation. The conditions imposed on $F(g)$ are reality and additivity. The reality requirement, ($Im\ F(g)=0$) quantizes: (i) $F(g)$ and makes it time-independent. (ii). The time variable. (iii) The energy. As a verification of chrono-topology, the Planck constant $\hbar$ has been calculated on the basis of the time quantization. The d.f. $F(g)$ becomes, after the time quantization, a real generalized Maxwell-Boltzmann *d.f.*, $F(g) = \exp[g(p, g; l_1, l_2, \ldots l_N)]$, depending on N quantum numbers. These facts are significant for quantum theory, because they uncover an intrinsic relationship between Liouville's theorem and quantum mechanics.

Keywords: Liouville theorem, chrono-topology, time quantization
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1 Introduction

It has already been observed [1] that Liouville's classical equation admits solutions with quantized time and energy. That first hint served as an incentive for the search of a rigorous definition of the time element, τ_λ, as in reference [2-7]. The conditions for the appearance of quantization in time have been shown to be:
(a) That the distribution function be real, and
(b) Satisfy a certain additivity condition similar to one of the basic requirements of *C*-Algebra* [8] in statistical mechanics.
If there were no observable changes in nature, then the time notion would be neither useful nor definable. The underlying principle of chrono-topology is that interactions are the exclusive agents of physical changes in the universe and that time has a part neither in the interactions nor in their results. Time does not

produce physical changes, but it is just useful to a human observer for the description of them[1].

This simple observation is used to give (mathematically) an operational time space definition. The time elements, τ_λ, are considered as a regular injective map of *an observed change of an observable* induced by means of a fundamental interaction process. These time elements are called *interaction proper time neighbourhoods* (IPN) and characterized in the following fundamental definition.

Definition 1.

An IPN, τ_λ^j, *is and injective map, f:* (a) *into observer's brain through one of his five senses or* (b) *into an appropriate device by means of a convenient detector, during observation of the* λ*-th change,* ΔO_λ^j, *of the j-th physical observable,* O^j*:*

$$f : \Delta O_\lambda^j \rightarrow f(\Delta O_\lambda^j) = \tau_\lambda^j \in \mathcal{T}_4^{(j)}.$$

Definition 1 enables us to characterize the microscopic system time by the following:

Definition 2.

The microscopic system time, $\mathcal{T}_4^{(j)}$, *of a particle system is defined as the union of a set of time elements,* $\{\tau_\lambda^j \mid \lambda \in \Lambda \subset Z^+\}$,

$$\mathcal{T}_4^{(j)} = \bigcup_{\lambda=1}^{\Lambda} \tau_\lambda^j, \Lambda \subset Z^+$$

for a finite $\Lambda \subset \mathcal{Z}^+$ *of interaction processes.*

The union of all $\{\tau_l^j\}$ corresponding to the j-th observable's change and belonging to a closed particle system is a disconnected topological space [9], satisfying the separation axioms of $\mathcal{T}_4^{(j)}$ (See Appendix). It can be used define the *Newtonian universal time,* N_t^1 -and this in fact is its physical origin. This in fact is the physical origin macroscopic time. This time, known from classical mechanics and from every-day life, comes about as the union of a set $\{\mathcal{T}_4^{(j)}\}$ which, although it is denumerable, many of its consecutive elements, τ_λ, are subjectively not discernible in all cases. This is due to human brain physiology (finite computing power resulting from finite speed, resulting to finite time resolution).

If the number of observable systems in the universe is *S*, the number of particles of the s-th system is *K(s)* the number of changing observables of the κ-th particle is *j* and the number of interactions of the s-th system of the κ-th particle changing the *j* observable is λ, then the Newtonian time in this universe is characterized by the following definition.

Definition 3.

Newtonian universal time space, N_t^1, *is the union of the maps* $\{\tau_{\lambda\kappa}^j\}$

[1] The clock hands do not show time. They present a varying angle due to the intersection of the spiral spring with the appropriate wheels. The successive maps of the varying angle in observer brain, or in any angle detector, are perceived (by him) as time. Similar is the case with all chronometers.

$$N_t^1 = \bigcup_{s\in S} \bigcup_{\kappa\in K(s)} \bigcup_{j\in J(\kappa,s)} \mathcal{T}_4^{(jks)\,2}$$

for S, $K(s)$, $J(\kappa, s)$, $\Lambda(j,\kappa, s)\subset Z^+$.

The time, $\mathcal{T}_4^{(j)}$, as defined in *chrono-topology*, satisfies the following axioms which are deduced from operational observation:

Axiom I.

All time measurements, classical or quantal, are based on an interaction process implementing a change of a physical observable that generates, if observed, a corresponding IPN. The generated IPN is a regular into map of just this change. The change map is stored either in observer's memory through one of his five senses or in the memory of an appropriate device.

Axiom II.

Every fundamental interaction process is associated with finite changes of the involved observables. Map sets, $\{\tau_l^j\}$ *of observables' changes have an intrinsically stochastic character, as regards their embedment in the Newtonian time space,* N_t^1. *Different ,* $\{\tau_l^j\}$ *start and end at irregular Newtonian times ,* $\{\tau_l^j\ \tau_l^j\}$, *and have, within limits, stochastically distributed durations,* $\{\delta(\tau_l^j)\}$. $\mathcal{T}_4$ *is (for living observers and for detectors incorporating a Newtonian universal timer) embedded in the time space,* N_t^1, *but it does not, in general, have the* N_t^1 *topology.*

Axiom III.

The elements of the empty set, Ø, of a class of map, $\{\tau_l^j\}$ *of* $\{\Delta O_\lambda^j\}$, *with* $j, \lambda\in Z^+$, *of physical observables,* $\{O^j\}$, *are not observable, and their values are identically equal to zero.*

The above definitions and axioms describe time quantization in a rigorous way. They are all basic, not yet deduced formally from any theory but they are the result of operational observation.

However, surprising though as it may appear, time as well as energy quantization follow formally from an elementary solution, *g(g, p, t)*, of the classical Liouville equation describing *N* interacting particles. The quantization in question follows only if two conditions are imposed which are naturally related to physical reality:

a) The $d.f., F_N(g(\mathcal{g},\mathcal{p},t))$, must be real (and therefore Im $F_N(g(\mathcal{g},\mathcal{p},t))=0)$.

b) The $d.f., F_N$, must be form invariant with respect to the number of the system particles, *N*.

The condition b) corresponds to the experimental fact that by adding two systems of identical particles one gets a system identical to the initial ones independently of the particle numbers:

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[2] If S,K,J,Λ are ale large numbers, then some or all of $\{\tau^{\kappa js}{}_\lambda\}$ may overlap partially or totally and give a time set compact in itself.

$F(g_{N1}(\mathscr{g}, \mathscr{p},t) x F(g_{N2}(\mathscr{g}, \mathscr{p},t)) = F(g_{N1}(\mathscr{g}, \mathscr{p},t) + g_{N2}(\mathscr{g}, \mathscr{p},t))$.

This expresses one of the main requirement of the *C*-Algebra in statistical mechanics.*

The *d.f.* *F(g)* may be any non-constant, differentiable function of *g* satisfying condition (b) above. Also, *g(g,p,t)* possesses first-order partial derivatives with respect to the components of $\mathscr{g}^{(n)} \in Q$, of $\mathscr{p}^{(n)} \in P$, for n= 1,2,... $N \in Z^{+}$ and of t.

Empirically, all interactions induce finite changes. Consequently, the interaction proper time neighbourhoods, $\{\tau_{\lambda}\}$, as continuous and regular, injective maps of such changes have in all cases finite diameters, $\delta(\tau_{\lambda})$.

Also, considering that all interactions come about by means of quanta exchange between interacting particles, the interaction time is necessarily finite, because every quantum change is finite. It becomes evident from this fact that the time variable for atomic and sub-atomic systems can only be a sectionally continuous variable.

The present work consists of 7 sections and one appendix: Since the assertion according to which time does not flow may appear to some readers as rather strange, two demonstrations based on chrono-topology are given in section 2. In section 3 an elementary solution, g(g,p,t), of Liouville equation is presented. It is shown that every differentiable function, $F_N(g)$, satisfies also the Liouville equation, provided that *g(g,p,t)* is a solution of it. The fact that by requiring $\mathrm{Im} F_N(g) = 0$ the *d.f.* becomes time-independent and quantized, is considered as a result of major physical significance. In section 4 it is demonstrated that the time and the energy of the system particles are quantized.

In view of these quantizations it would be not too much of an exaggeration to say that quantum theory might very well have been discovered long before Planck, by Liouville himself or, e.g., by Boltzmann.

In section 5 a new type of Maxwell-Boltzmann *d.f.* is presented. Its most striking property is that it depends on a set $\{l_1,....l_N\}$ of quantum numbers resulting from the classical Liouville's theory. It is also remarkable that its observability is a consequence of the time and energy quantizations which make $F_N(g)$ time-independent and, hence, observable[3].

In section 4 the Planck constant is given, based on energy and time quantization. Its theoretical value agrees with the experimental value 1.0544 x 10^{-34} *Js*. Finally, in sect. 7 some conclusions are given, together with the discussion of the results, while in Appendix useful topological definitions are given.

2 Time is discontinuous and does not flow

As stated in the introduction, the time space $\mathcal{T}_4$ may, but must not necessarily, have the topology of the *universal Newtonian time* N_t^1. This is because it is composed of time elements $\{\tau_{\lambda}\}$, which are, in general, disconnected, small subsets of an interval in R^1.

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[3] Experimentally observable are only eigenvalues, and they are time-independent.

A time element, τ_λ^j, precedes another time element $\tau_{\lambda'}^j, \tau_\lambda^j \prec \tau_{\lambda'}^j$, if for every $t \in \tau_\lambda^j$ and for every $t' \in \tau_{\lambda'}^j$ there holds $t' \rangle t$.

Remark 1

The successive maps, $\{\tau_\lambda^j\}$, implemented by means of a sensory organ of the observer create, as a matter of fact, the impression of a flow which is not the flow of any physical fluid.

In an electronic or any other kind of device, used for the observation and storage in succession of the observed observables' changes $\{\Delta O_\lambda^j \mid j, \lambda \in Z^+\}$ as well as of their maps of any regular type $\{\tau_\lambda^j \mid \lambda \in Z^+\}\}$, that is appropriate for the observation and measurement of the flow process of real physical fluids, no flow characteristics have ever been measured.

The basic principle in chrono-topology [6-8] is that interactions are the only causes of any change in the universe. Changes not associated with an interaction process do not exist. On the other hand, there may exist changes, whose interactions processes are unknown to the observer.

With the advent of the special relativity, it should have become clear that nothing in nature supports the metaphor of flowing time. This can be demonstrated either by means of a reality-based *gedankenexperiment* or directly, by using the Lorentz transformation:

$$x' = \gamma(x - vt), \tag{1}$$

$$t' = \gamma(t - \frac{\beta}{c}x), \tag{2}$$

where $\beta = \frac{v}{c}, \gamma = \sqrt{1-\beta^2}$, v is the relative velocity of two frames of reference and c the velocity of light.

According to the conventional view, time is *mathematically*, for every observer a continuous parameter. *Physically*, time is a sequence of excitations in observer's brain which are implemented by means of observer's brain nor outside it. What really does flow in not time. Here is the first proposition to prove by means of a 'gedankenexperimental'.

Proposition1

Given are a set of interaction proper time neighbourhoods, $\{\tau_\lambda, \tau_{\lambda+1}\}$ *such that* $\tau_\lambda \cap \tau_{\lambda+1} = \varnothing$.

Then the impression of a flow is implied by the properties $\tau_\lambda \cap \tau_{\lambda+1} = \varnothing$ *and* $\{\tau_\lambda \prec \tau_{\lambda+1}, \forall \lambda \in \Lambda \subset Z^+\}$.

Remark 2

We shall describe here some situations in which time cannot change continuously, if it is to generate in observer's mind the flow impression. The time discontinuity and the fact that time is embeddable in the Newtonian time-space, N_t^1, where observers live, produces psysiologically in observer's mind an application of Zermelo's *well-ordering theorem* on the generated time elements $\{\tau_\lambda, \tau_{\lambda+}\}$.

Proof

Let us consider the experiment of the laminar flow of a physical fluid, e.g., water. This flows with no air bubbles in a fully transparent glass tube. To make the flow observable solid pieces of any material (e.g., wood) are placed in it. The wood pieces $\{p_\lambda \mid \lambda = 1,2,...J \in Z^+\}$, upon entering the appropriately delimited optical field of the observer, create the maps $\{\tau_\lambda \mid \lambda = 1,2,...J \in Z^+\}$. Since the wood pieces pass sequentially and ordered, there hold the relations $\tau_\lambda \cap \tau_{\lambda+1} = \varnothing$ and $\{\tau_\lambda \prec \tau_{\lambda+1}, \forall\lambda \in Z^+\}$.

The maps into observer's brain $\{\tau_\lambda \mid \lambda = 1,2,...J \in Z^+\}$ are structured in the same way as the wood pieces are structured, $\{p_\lambda \mid \lambda = 1,2,...J \in Z^+\}$, in the flowing water. More precisely, this means that the lengths $\{l_\lambda \mid \lambda = 1,2,...J \in Z^+\}$ and the relative distances, $\{d_\lambda \mid \lambda = 1,2,...J \in Z^+\}$, of the wood pieces (or, equivalently, the diameters, $\{\delta(\tau_\lambda), \delta(\tau_{\lambda+1}) \mid \forall\lambda \in Z^+\}$, and the distances between the sets $\{\tau_\lambda \mid \forall\kappa \in Z^+\}$) are sets of proportional numbers, provided the passages of all $\{p_\lambda \mid \lambda = 1,2,...J \in Z^+\}$ have been mapped onto $\{\tau_\lambda \mid \forall\lambda \in Z^+\}$ by the same *f*.

Since the observer observes the flow of a physical fluid, the flow impression is created naturally in his mind by means of well-defined facts of brain physiology which, being well known, are not repeated here.

Let us isolate and keep in mind the *essential observational facts* from this physical flow experiment:

The physical observable whose changes are observed by the observer during the experiment is luminosity. The observer is affected by the differences of:

(i) Luminosity of each observed wood piece.

(ii) Luminosity of the transparent water between two successive wood pieces as these are seen passing.

The passing of the wood pieces and the water in the between result in changes in luminosity and create in observer's brain the corresponding maps, the interaction proper time neighbourhoods, $[\tau_\lambda \mid \forall\lambda \in Z^+\}$, of definite durations, $\{\delta(\tau_\lambda) \mid \forall\lambda \in Z^+\}$.

Between the end of the λ-th and the beginning of the $(\lambda+1)$-th passages there is also a definite duration which makes the maps, $\{\tau_\lambda \mid \forall\lambda \in Z^+\}$, of the passages disconnected. Two successive passages correspond to two successive neighbourhoods, $\tau_{\lambda'} \prec \tau_{\lambda''}$. This makes the maps ordered.

The observation of the water flow supplies, of course, the observer's brain with additional (irrelevant) information concerning the environment of the flow experiment. It causes adjacent excitations in observer's brain not essential to the flow information *per se*. They need not be considered in our case, because they do not contribute to the creation of the flow impression. So much for the real water flow impression.

Let us see next, what happens in the case of time 'flow' during the observation of successive changes of any physical observable whatsoever, not related to a physical flow but structured identically to the wood pieces and water passages in observer's optical field of the water flow experiment.

The observables' changes now create their corresponding interaction proper time neighbourhoods, $\{\overline{\tau_{\lambda}} \cap \overline{\tau_{\lambda+1}} = \varnothing\}$.

Every time neighbourhood, $\overline{\tau_{\lambda''}}$ 'observed', follows any preceding 'observed' $\overline{\tau_{\lambda'}}$, such that $\overline{\tau_{\lambda''}} \succ \overline{\tau_{\lambda'}}, \lambda'' \rangle \lambda'$. Every new map is added sequentially to the already existing set $\{\overline{\tau_{\lambda}}, \lambda \langle \lambda'\}$ in the brain. The disconnected set $\{\overline{\tau_{\lambda}} | \lambda \in Z^{+}$ the wood pieces and the water between them in the water flow experiment.

The sequential creations of disconnected maps in the brain, corresponding to observed observables' changes (e.g., differences of luminosities or anything else may be created by many, different physical processes) result in physical changes in the brain physiology, which are identical to those resulting in the case of the perception of physical water flow.

Other, non-relevant information may also accompany the perception process of the observable's changes, not related to any physical flow.

Therefore, an experimental measurement of the corresponding polarization sets in the microtubles in observer's brain, would give identical results in the two cases of observation of identically-ordered changes of *different physical observables*, the water flow in the one case and the changes of any other observable in other cases.

Hence, two identical flow impressions are created through observation of any different observables' changes, which as maps in the brain, are identically structured and ordered.

It would contradict the brain physiology and would also be a paradoxical phenomenon, if, since the extraneous excitation sets are identical, different results were implied in the brain and placed in the memory (e.g., if in the second of our example no flow impression were created). Human brain excels in recognizing patterns. These patterns in the two cases described are identical. It is, therefore, only natural for the brain to conclude that the causes are identical too. This represents an excellent example for possible differences between the 'being' and the 'phenomenon'.

Hence, the flow in the second case is the "time flow" and the proof that tome flow is an *impression* is now complete.

Remark 3

The flow impression is, of course, not physical, nor does it exist outside the observer's mind. Time does not flow, which also in agreement with relativity as is shown presently. The same process of ***Proposition 1*** is considered from a different point of view in the framework of relativity. Here time is considered as a parameter $t \in I \subset R^{1}$. This is in agreement with chrono-topology, because every τ_{λ} is a subset of *I*.

The proof will be based on the empirical fact that space does not flow.

Proposition 2

Considered are two observers, one in S and one in S', with relative velocity v. Then the time t' in S' does not flow, if the space coordinate x in S does not flow.

Proof

Let t=const. Any change Δt', of the time, *t'*, in (2) is a linear map of the change Δx in S:

$$\Delta t' = -\gamma \frac{\beta}{c} \Delta x \qquad (3)$$

The converse is also true. It follows from (1) for x – *const.*. that $\Delta x;$ is a linear map of the change, Δt:

$$\Delta x' = -\gamma\, v \Delta t. \qquad (4)$$

Solving (4) for Δt gives

$$\Delta t = -\Delta\, x'/\gamma\, v \qquad (4')$$

It follows from these relations that Δt flows, if and only if $\Delta x'$ flows. Since Δt is an arbitrary part of the time *t, t* does flow, if Δt does. However, according to all evidence, space *x* does not flow and, hence, time does not flow too.
This is, obviously, true for both observers, in *S* and in *S'*, and the proof is complete.

Remark 4

The above result looks overly trivial indeed. Despite its trivial appearance, it seems that this is probably the reason that, until now, impeded us to recognize that time is a map of *observed physical observables'* changes and it does not flow.

3 Liouville's theorem and a distribution function

In the previous section, arguments were presented showing that time for atomic and sub-atomic systems is a map of observed observables' changes but not *a continuous mathematical parameter on* R^1. It is interesting to observe that the truth of this result can also be demonstrated in a formal way based on the Liouville theorem in statistical mechanics.
The fact that the Planck constant has been determined from an elementary solution of Liouville's equation is considered as a verification of chrono-topology.
A system of *N* particles will be considered, that interact via forces, which are independent of the phase space coordinates, within each interaction proper time neighbourhood.
This assumption appears –strictly speaking- as unphysical. However, the conclusion that the system would cease to show quantum properties, because the inter-particle forces depended on the phase space coordinates looks, in our view, much more unphysical.
Furthermore, physical situations are imaginable in which the interaction forces change during the motion smoothly and only slightly, when the distances of the interacting particles of the system are allowed to change only slightly. A more complete explanation is given in [4] in conjunction with the lattice space discussed there.
With the above premises in mind, we shall demonstrate the following

Proposition 3

Let

1. *PS = P X Q be the phase space.*

2. $\{\mathcal{p}^{(n)} \in P, \mathcal{g}^{(n)} \in Q, n = 1,2,\ldots N\}$ *be the phase space coordinates of an N-particle system interacting via given forces* $\{\mathcal{F}^{(n)} | n = 1,2,\ldots N\}$.
3. *The elementary solution for N particles be*

$$g(\mathcal{g}, \mathcal{p}, t) = \sum_{n=1}^{N} [i\lambda \varepsilon_n t \pm \frac{\mu_n}{2}\{\mathcal{F}^{(n)}.\mathcal{g}^{(n)} + \mathcal{g}^{(n)}.\mathcal{F}^{(n)}\}$$

$$\mp \frac{\mu_n}{2}\{\mathcal{p}^{(n)} m_n^{-1/2} \pm i\nu_n \mathcal{F}^{(n)})^2\}, \qquad (5)$$

where $\{\varepsilon_n, \mu_n, \nu_n \,|\, n = 1,2,\ldots N\}$ parameters to be determined and m_n the mass of the n-th particle.

4. The Liouville operator be written in the form

$$\mathcal{L} = \vartheta_1 + \sum_{n=1}^{N} (\mathcal{p}^{(n)} / m_n . \nabla^{g^{(n)}} + \mathcal{F}^{(n)}. \nabla^{p^{(n)}}) \qquad (6)$$

5. F be a differentiable function satisfying the conditions

$$\mathcal{Im}\, F(g) = 0 \qquad \text{(reality).} \qquad (7)$$

$$F(g_1).F(g_2) = F(g_1 + g_2) \qquad \text{(additivity)} \qquad (8)$$

The, upon appropriate determination of λ *and* $\{\varepsilon_n, \mu_n, \nu_n \,|\, n = 1,2,\ldots N\}$ the functions $g(\mathcal{g},\mathcal{p}, t,)$ and $F(g)$ *satisfy*

(a) $$\mathcal{L}g(\mathcal{g},\mathcal{p},t) = 0 \qquad (9)$$

(b) $$\mathcal{L}F(g) = 0 \qquad (10)$$

(c) *The time variable, t, takes values in* J_4 *given by*

$$t = \pm 2\pi[j_n - l_n(1 - \frac{\delta(\tau_n)}{S_n})]/(\lambda \varepsilon_n), j_n, l_n \in Z^+. \qquad (11)$$

Proof of (9)

Application of $\mathcal{L}$ on $g(\mathcal{g},\mathcal{p},t)$ yields [2]

$$\sum_{n=1}^{N} [i\lambda \varepsilon_n \pm \frac{\mu_n}{2}\{\mathcal{F}^{(n)}.\mathcal{p}^{(n)} + \mathcal{p}^{(n)}.\mathcal{F}^{(n)}\} m_n^{-1/2}$$

$$\mp \frac{\mu_n}{2}\{\mathcal{F}^{(n)} m_n^{-1/2}.(\mathcal{p}^{(n)} m_n^{-1/2}$$

$$\pm i\nu_n \mathcal{F}^{(n)}) \mp (\mathcal{p}^{(n)} m_n^{-1/2} \pm i\nu_n \mathcal{F}^{(n)}).\mathcal{F}^{(n)} m_n^{-1/2}\} \qquad (12)$$

If the *n*-th particle energy is given by the equation

$$\varepsilon_n \lambda = \mu_n \nu_n m_n^{-1/2} (\mathcal{F}^{(n)})^2 \qquad (13)$$

then (12) vanishes identically and $g(\mathcal{g},\mathcal{p}, t)$ satisfies the Liouville equation. This completes the proof of (9).

Proof of (10)

Since $\mathcal{L}$ is a linear superposition of first-order differential operators, it follows that

$$\mathcal{L}\partial F(g) = F'_g(g) x \mathcal{L}g(\mathcal{g}, \mathcal{p}, t).$$

From the proof of (9) equation (10) follows immediately and the proof of it is complete.

4 Liouville's time and energy quantization

In this section, we shall prove a number of propositions concerning the foundations of quantum mechanics are implied by Liouville's theorem as applied in classical statistical mechanics.

Proof of (11)

An appropriate function fulfilling (8) is *F(g)* = exp[g]. Applying (7) one gets the condition

$$\sin\langle \sum_{n=1}^{N} \{\lambda \varepsilon_n t - \mu_n \nu_n [\mathcal{F}^{(n)}.\mathcal{p}^{(n)} + \mathcal{p}^{(n)}.\mathcal{F}^{(n)}](2 m_n^{1/2})^{-1}\}\rangle = 0 .$$

This is equivalent to

$$\sum_{n=1}^{N} \{\lambda \varepsilon_n t - \mu_n \nu_n [\mathcal{F}^{(n)}.\mathcal{p}^{(n)} + \mathcal{p}^{(n)}.\mathcal{F}^{(n)}](2 m_n^{1/2})^{-1}\}$$

$$= \pm 2\pi \sum_{n=1}^{N} j_n = \pm 2\pi J_N, j_n, J_N \in Z^+, \tag{14}$$

where the factor '2' in front of π insures that the *d.f.* is positive.

By adding *to* and subtracting *from* the expression in angled brackets '[]' in the lhs of (14) the expression $\frac{\mu_n \nu_n}{2 m_n^{1/2}}(\mathcal{p}_0^{(n)}.\mathcal{F}^{(n)} + \mathcal{F}^{(n)}.\mathcal{p}_0^{(n)})$, where $\mathcal{p}_0^{(n)}$ is the momentum of the *n*-th particle just before its last interaction, i.e., before time $\delta(\tau_n) + t_{ff}$ foregoing expression can be simplified to the form

$$\sum_{n=1}^{N} \{\lambda \varepsilon_n t - \frac{\mu_n \nu_n}{m_n^{1/2}} \delta(\tau_n)(\mathcal{F}^{(n)})^2 - \frac{\mu_n \nu_n}{2 m_n^{1/2}}(\mathcal{p}_0^{(n)}.\mathcal{F}^{(n)} + \mathcal{F}^{(n)}.\mathcal{p}_0^{(n)})\}$$

$$= \pm 2\pi J_N, J_N \in Z^+. \tag{14'}$$

The time t_{ff} is an interval of the Newtonian time $t_{ff} \subset R_t^1, t_{ff} \not\subset \mathcal{T}_4$ separating τ_n^{λ} from $\tau_n^{\lambda+1}$ when they are embedded in R_t^1.

The interaction proper time neighbourhood's diameter is $\delta(\tau_\lambda)$ and there the force definition has been used

$$\mathcal{F}^{(n)} = (\mathcal{p}^{(n)} - \mathcal{p}_0^{(n)}) / \delta(\tau_n) .$$

Having still the parameters $\{\mu_n, \nu_n \mid n = 1,2,...N\}$ free, we equate

$$-\frac{\mu_n \nu_n}{2 m_n^{1/2}}(\mathcal{p}_0^{(n)}.\mathcal{F}^{(n)} + \mathcal{F}^{(n)}.\mathcal{p}_0^{(n)}) = \pm 2\pi l_n, l_n \in Z^+$$

and restrict the product of the free parameter set, $\{\mu_n \nu_n\}$, to the values

$$-\mu_n \nu_n = \pm 4\pi l_n m_n^{1/2}.[\mathcal{F}^{(n)}.\mathcal{p}_0^{(n)} + \mathcal{p}_0^{(n)}.\mathcal{F}^{(n)}]^{-1}, l_n \in Z^+ \tag{15}$$

From (14') and (11) it follows that

$$\sum_{n=1}^{N} \{\lambda \varepsilon_n t \mp 2\pi l_n \delta(\tau_n)(\mathcal{F}^{(n)})^2 \times [\mathcal{F}^{(n)}.\mathcal{p}_0^{(n)} + \mathcal{p}_0^{(n)}.\mathcal{F}^{(n)}]^{-1} \pm 2\pi l_n\} \tag{16}$$

$$= \pm 2\pi \sum_{n=1}^{N} j_n,$$

or for each n-value

$$\lambda \varepsilon_n t \mp 2\pi l_n \delta(\tau_n) \mathcal{F}^{(n)})^2 \times [\mathcal{F}^{(n)} . \mathcal{P}_0^{(n)} + \mathcal{P}_0^{(n)} . \mathcal{F}^{(n)}]^{-1} = \pm 2\pi (j_n - l_n) \quad (17)$$

We define time

$$s_n = [\mathcal{F}^{(n)} . \mathcal{P}_0^{(n)} + \mathcal{P}_0^{(n)} . \mathcal{F}^{(n)}] \times (\mathcal{F}^{(n)})^{-2} \quad (18)$$

which, as we see, it is a constant, characteristic of the n-th particle interaction, and (17) can be written in the form

$$t = \pm 2\pi \{ j_n - l_n [1 - \frac{\delta(\tau_n)}{d_n}] \} / (\lambda \varepsilon_n) . \quad (11')$$

This completes the proof of (11).

Remark 5

The parameter λ in (11), remains still undetermined. We shall calculate the value of λ in the next section with the help of {11') and from experimental data in two cases (see sect. 6, eq. 21) and we shall find

$$\lambda^{-1} \cong 1{,}0544 \times 10^{-34} Js .$$

This entitles us, while anticipating the above value for λ^{-1}, to write (11') in the form

$$t_n = \pm 2\pi\hbar \{ J_n - l_n [1 - \frac{\delta(\tau_n)}{S_n}] \} / \varepsilon_n \quad (11'')$$

which shows that time is quantized following the premises of the present theory.

Remark 6

If it is assumed that $S_n = \delta(\tau_n)$, then (11'') takes the particularly simple form

$$t_n = \pm 2\pi\hbar \, j_n / \varepsilon_n . \quad (11''')$$

The above result follows also from the relation $\mathcal{F}^{(n)} | \mathcal{P}_0^{(n)}$.

Remark 7

Combining (11'') with (12') of sect. 5 and requiring $t_n = \delta(\tau_n)$ we find for $\delta(\tau_n)$ the expression

$$\delta(\tau_n) = S_n \frac{j_n - l_n}{l_n + 1}$$

which proves both the time quantization and the possibility for the time to take positive or negative values, just like in chrono-topology, where time can be defined as a positive or a negative map of observed observables' changes.

5 A quantum Maxwell-Boltzmann distribution

The *d.f.* *g*($\mathcal{g}$,$\mathcal{p}$, *t*) (eq. 5) acquires, after the time and the energy quantization, a set of quantum numbers one for each particle of the system and it becomes time independent

$$g(\mathcal{g}, \mathcal{p}, t) = \sum_{n=1}^{N} \{ \mu_n [\mp \frac{(\mathcal{p}^{(n)})^2}{2m_n} \pm \frac{1}{2} (\mathcal{F}^{(n)} . \mathcal{g}^{(n)} + \mathcal{g}^{(n)} . \mathcal{F}^{(n)})] \pm \frac{\mu_n v_n^2}{2} (\mathcal{F}^{(n)})^2 \} \quad (5')$$

Relation (15) shows that the *d.f.* depends on a set of quantum numbers, $L_N=\{l_1,l_2,...l_N\}$, precisely like a many-particle wave function absolute square $|\Psi_{l_N...l_N}|^2$ depends on the relevant quantum numbers.

If the product $\mu_n v_n$ is restricted to the values given by (15) above, then (5') takes according to (18) the form

$$F(g)=\exp\langle\sum_{n=1}^{N}\{\mu_n[\mp\frac{(p^{(n)})^2}{2m_n}\pm\frac{1}{2}(\mathcal{F}^{(n)}.\mathcal{g}^{(n)}+\mathcal{g}^{(n)}.\mathcal{F}^{(n)})]$$

$$\pm\frac{8\pi^2}{\mu_n s_n}l_n^2 m_n(\mathcal{F}^{(n)}.\mathcal{p}_0^{(n)}+\mathcal{p}_0^{(n)}.\mathcal{F}^{(n)})^{-1}\}\rangle, \qquad (5'')$$

where a set of N quantum numbers $L_N=\{l_1,l_2,...l_N\}$ enters the *d.f.* .

From the general relation $\mathcal{F}^{(n)}=-\nabla^{q^{(n)}}V(\mathcal{g}^{(n)},\mathcal{g}^{(n)},...\mathcal{g}^{(n)}$ and from its inverse

$$\mathcal{F}^{(n)}.\mathcal{g}^{(n)}=\int d\,\mathcal{g}^{(n)}.\mathcal{F}^{(n)}=-\int d\,\mathcal{g}^{(n)}.\nabla^{q^{(n)}}V(\mathcal{g}^{(n)},\mathcal{g}^{(n)},...\mathcal{g}^{(n)})$$

$$=-V^{(n)}(\mathcal{g}^{(n)},\mathcal{g}^{(n)},...\mathcal{g}^{(n)})$$

we see that the second term in the *d.f.* is, up to an arbitrary constant of integration, equal to the potential energy of the n-th particle.

Remark 8

It is observed that *F(g)* acquires an element of reality in the sense of *EPR*[10] after the application of the condition $Im\mathrm{F(g)}=0$.

This condition (7) implies-surprisingly enough- the quantization of three quantities:

(i) *The time variable, t.* It is even more surprising that the reality condition leads to the quantization and the full elimination of the time variable in the *d.f.* . This makes *F(g)* accessible to experimental measurement and, hence, an observable. This fact may be considered an indication that perhaps time is not a physical quantity, and it cannot appear in an observable quantity[4].

This is in accordance with quantum theory in which only eigenvalues are observable, which are time-independent quantities. This is also in Keeping with chrono-topology in which time is defined as a map of observed observable's changes, as in ***Definition 1***, above.

(ii) *The total energy,* E_N, of the system described by (5), as well as the energy

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[4] This would suggest thinking that, for example, the energy-time uncertainty relation $\Delta E.\Delta t\rangle\hbar$ (a) may, after all, not be fundamental at all, as it could be derived from $\Delta p_x.\Delta x\rangle\hbar$ (b)

According to chrono-topology, every change is due to an interaction. Hence, Δp_x is the result of a force, $\mathcal{F}$, and it is related to it by Newton's relation $\mathcal{F}=\frac{\Delta p_x}{\Delta t}$. The time, Δt, is nothing else but the time given in ***Definition 1*** where there holds:

$$\Delta p_x\to f(\Delta p_x)=\tau^{p_x}=\Delta t. \qquad \text{(c)}$$

By multiplying and dividing (b) by τ^{p_x}, the relation (a) follows. The above derivation is meaningful only, if (c) is true.

ε_n of each particle belonging to the same system, become quantized, (see proof of eq. 12 below), after the elimination of the time variable.

(iii) *The Maxwell-Boltzmann d.f.,F(g),* becomes a time-independent function. The fact is in full agreement with quantum mechanics, of atomic and sub-atomic systems, according to which only (time-independent) eigenvalues and functions are experimentally measurable and observable. In addition, it becomes dependent on a set, $\{l_1,\dots l_N\}$, of quantum numbers with a term proportional to the temperature, an unexpected form of temperature dependence,

$$F(g)=\exp\langle\sum_{n=1}^{N}\{[\mp\frac{(\mathcal{P}^{(n)})^2}{2m_nk_BT}\pm\frac{V(\mathcal{G}^{(n)})}{k_BT}]\pm\frac{8\pi^2l_n^2m_n}{S_n^2|\mathcal{F}^{(n)}|^2}k_BT\}\rangle \qquad (5''')$$

Remark 9

The elementary *d.f.*, *g,* equals the total energy (dimensionless though it is) plus a quantum term which may be positive of negative. The expression is also multiplied by μ_n, which has the inverse dimensions of energy.

Remark 10

By comparing the *d.f.* F(g) with that of the classical statistical mechanics [11] it becomes clear that putting $\mu_n=(k_BT)^{-1}$ is one possibility to determine the parameters $\{\mu_n\}$. In this case it is noted that the rhs first and second terms of (5') are, of course, inversely proportional to the temperature and correspond to the Maxwell-Boltzmann *d.f.* .

The last term, which is unknown to classical statistical mechanics, is proportional to the temperature and represents a quantum term in the Maxwell-Boltzmann distribution function.

Since the obtained *d.f.* (5' or 5'') depends on quantum numbers, it is natural to expect that the energy be quantized. It will be shown presently that, indeed, this is the case for the energy as well as for the time. We give first the

Proof of (12)

From (13) and from (15) it follows that

$$\lambda\varepsilon_n=2\pi l_n\frac{(\mathcal{F}_n)^2}{\mathcal{P}_0^{(n)}\cdot\mathcal{F}^{(n)}+\mathcal{F}^{(n)}\cdot\mathcal{P}_0^{(n)}}=2\pi l_n/S_n$$

The last expression is the appropriate equation for the determination of λ, where ε_n is taken from experimental data for various real gases. Putting in anticipation $\lambda^{-1}=\hbar$, we find

$$\varepsilon_n=2\pi\hbar l_n/S_n \qquad (12')$$

and the proof of the energy quantization is complete.

Remark 11

In the expression for S_n, eq. (18), we write for the absolute values of the vectors

$$\mathcal{P}o^{(n)}\cdot\mathcal{F}^{(n)}=\left|\mathcal{P}o^{(n)}\right|\left|\mathcal{F}^{(n)}\right|\cos\theta=\sqrt{2\varepsilon_nm}.\frac{e^2}{4\pi\varepsilon_0r_n^2}\cos\theta$$

we see that there exists an interval $\theta_1 \le \theta \le \theta_2$ for which indeed the equality $S_n = \delta(\tau_n)$ is satisfied. This equality is quite natural because if it is supposed that the time required for the n-th gas molecule, starting from rest, to acquire the momentum $\wp o^{(n)}$ is $\delta(\tau_n)$, then eq. (18) becomes an identity.

Remark 12

Based on (11'') and (12') below and requiring $t_n = \delta(\tau_n)$ we got

$$\frac{\delta(\tau_n)}{S_{(n)}} = \frac{2\pi(j_n - l_n)}{2\pi l_n - 1} \tag{11'''}$$

Expression (11''') explains the empirical fact that the higher the interaction energy (higher l_n for given j_n) the shorter the interaction proper time

Eq. (11'') can be used to obtain the *system-time*. If we sum both sides of it over n for $l_n = 0$, we find either the average energy per particle, $\langle\varepsilon\rangle_N$ or the average interaction time $\langle\tau\rangle_N$:

$$\sum_{n=1}^{N} \varepsilon_n t_n = \langle\varepsilon_n\rangle T_N = \pm 2\pi\hbar \sum_{n=1}^{N} j_n = \pm 2\pi\hbar J_N, T_N = \sum_{n=1}^{N} t_n \tag{19.a}$$

$$\sum_{n=1}^{N} \varepsilon_n t_n = \langle t_n\rangle E_N = \pm 2\pi\hbar \sum_{n=1}^{N} j_n = \pm 2\pi\hbar J_N, E_N = E_N = \sum_{n=1}^{N} \varepsilon_n \tag{19.b}$$

where $\langle\varepsilon_n\rangle = \sum_{n=1}^{N} \varepsilon_n t_n / \sum_{n=1}^{N} t_n$ and $\langle t_n\rangle = \sum_{n=1}^{N} \varepsilon_n t_n / \sum_{n=1}^{N} \varepsilon_n$.

It seems that nothing prevents from taking the total action of the system either as positive, or as negative, but the total energy, E_N, is conserved. Eq. (19b) shows that since the total energy is conserved, the time $\langle t_n\rangle$ can change in steps according to J_N. This completes the proof of ***Proposition 1***. The proofs of the following ***Corollaries 1 to 6*** are obvious:

Corollary 1

The fundamentals of quantum theory are implied by the Liouville equation and conditions (7) and (8) of reality and additivity.

Corollary 2

Microscopic time, t_n, cannot change continuously.

Corollary 3

The average interaction proper time, $\langle t_n\rangle$, can change for constant E_N, only if different partitions of $\{j_n\}$ of J_N are possible according to (19b)[5].

Corollary 4

If time changes for constant total energy, E_N, it does so in steps, Δt, (Liouvillian time) at least as large as given by $\Delta\langle t_n\rangle \ge \pm 2\pi\hbar / E_N$.

Corollary 5

Quantum processes are the faster (shorter $\Delta\langle t_n\rangle$), the higher their energies, E_N, are

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[5] J_N can be the sum of N different positive integers j_n.

Corollary 6

$g(\mathcal{g},\mathcal{p}, t)$ is form invariant with respect to an increase of the particles number, N
(a C*-algebra property)

The second condition (8) is equivalent to: The action integral of the sum of two particle systems interacting by means of constant forces is equal to the sum of the action integrals of the separate particle systems, whose particles also interact by means of constant forces.

The conditions (7) and (8) when imposed on the distribution functions, entail the quantization of the time which then changes by steps.

These conclusions modify our picture of the time, even in a classical theory of atomic systems. They indicate that quantization is a fundamental property of the elements of matter as well as those of radiation (*Planck black-body radiation*).

6 Determination of Planck's constant as a verification of chrono-topology

In the present section we shall calculate the parameter λ entering the elementary *d.f., $g(\mathcal{g},\mathcal{p}, t)$*. This will be found from experimental data and from the equation (12') for the energy quantization.

It will be shown that λ equals the inverse Planck constant, $\lambda^{-1}=\hbar$. This is a very exciting finding, given that Liouville's equation is a classical one.

We consider this fact not as mere accident and we believe that it entitles us to think about it as an experimental verification of our chrono-topology. We use for the calculation of the λ^{-1}-value expression

$$\lambda^{-1}=\frac{\varepsilon_1\tau_1}{2\pi l_1},$$

where τ_1 is the smallest time element corresponding to the gas molecule. The energy, ε_1, is considered to be the thermal translation energy of the gas molecule

$$\varepsilon_1=\frac{3}{2}k_B T,$$

The following values for the natural constants are used:

$e=1.602\times10^{-19}C$ $\quad R_B=5.297\times10^{-11}m$

$m_e=9.109\times10^{-31}Kg$ $\quad m_p=1.673\times10^{-27}Kg$ $\quad K_B=1.38\times10^{-23}J/K$

ATOMIC HYDROGEN

$l^{atomic}_{inter}=2.906\times10^{-10}m$ $\quad \varepsilon_n=\frac{3}{2}k_B T=6.21\times10^{-21}J$

$q=5.4860$ $\quad v=[2\varepsilon_n/(m_p+m_e)]^{1/2}=2723.022m/s$

$T=300$ K $\quad \tau_{intr.}=l^{atomic}_{intr.}/v=1.06688\times10^{-13}s$

$l_1=1$

In order to find the value of the interaction time, $\tau_{intr.}$, first for the atomic hydrogen gas we put, *n=1*, and use the above atomic parameters. The atom's

interaction length, $l_{intr.}$, is taken to be a multiple, q = 5.4860, of the Bohr radius of the atom.
The average interaction time, $\tau_{intr.}$, is calculated from:

$$\tau_{intr.} = \frac{l_{intr.}}{v}, \qquad (20)$$

where the denominator is the average thermal velocity, of the atoms with the mass (chemical atomic mass unit = 1.007593).
The above parameters for the atomic hydrogen gas give for λ^{-1} the value

$$\lambda^{-1} = \frac{\varepsilon_n \times \tau_{intr.}}{2\pi l_2} = 1.05446 \times 10^{-34} Js, \qquad (21)$$

This value of λ^{-1} is, clearly, equal to Planck's constant, and we put
$\lambda^{-1} = \hbar = 1.05446 \times 10^{-34} Js$ *from atomic hydrogen*
which is very close to the experimental value.
We consider molecular light hydrogen gas, where $M_{Molec.} = 2M_{Atomic}$, and the degrees of freedom are 5.
MOLECULAR HYDROGEN

$l_{inter.}^{molecular} = 3.184 \times 10^{-10} m$

$\varepsilon_n = \frac{5}{2} k_B T = 1.035 \times 10^{-20} J$

T = 300 K

$v = [2\varepsilon_n/(2m_p + 2m_e)]^{1/2} = 2486.588 m/s$

$l_2 = 2$

$t_{intr.} = l^{molecular}_{intr.}/v = 1.28 \times 10^{-13} s$

$\lambda^{-1} = 1.05457 \times 10^{-34} Js.$

Hence:

$$\lambda^{-1} = \frac{\varepsilon_n \times \tau_{intr.}}{2\pi l_2} = \hbar \qquad (22)$$

The values (20) and (21) agree for the selected data.

7 Conclusions and discussion

Indications that the topology of the Newtonian time space, cannot correspond to systems of atomic, nuclear and sub-nuclear particles were available at least as early as in 1974. It became clear, that quanta need not take notice of any observers or anything else, except their own interactions with other quanta or particles. What is, then, time for the quanta? If at all, should the particles not have, as Dirac proposed [31], their own times?
The increasing number of paradoxes in theoretical physics generally and in nuclear and sub-nuclear theory in particular, suggests the view that something about the fundamental physical concepts should be revised. This view is enforced while the experimental quantum techniques become more and more sophisticated resulting in higher accuracy.
A systematic examination showed that most uncertainties in physics are associated with the time concept. This variable, time, being interwoven with the space through relativity, imposes its topology to the space-time. It determines, in this way, the evolution in nuclear and sub-nuclear interactions among others.
A new space-time topology is proposed, the *Chrono-topology*. It is based on the concept of the *interaction proper-time neighbourhood, τ (IPN)*. The space-time topology on the quantum level is determined by the number of the interacting

particles in every particular system. For small numbers of interacting particles the new time space turns out to be a J_4 topological space.

The new space-times, $\overline{M^4_{\kappa\lambda_\kappa}}$ being in general $\kappa x \lambda - fold$ in time, are defined as the Cartesian products, $R^3 x J_4^{\kappa\lambda_\kappa}$, of $J_4^{\kappa\lambda_\kappa}$. The later is a $\kappa \times \lambda_\kappa - fold$ cal space, satisfying the separation axioms of a J_4 space whose elements are the interaction proper-time neighbourhoods, $\{\tau_{\lambda_\kappa} | \forall \lambda_\kappa \in Z^+\}$.

$\overline{M^4}$ is not related to Hawking's *space-time foam,* neither as regards to the cell magnitude nor as to its creation process. While Hawking explains, as does Wheeler for his space-time foam, creation by means of the field fluctuations in Planck time-scale, $\overline{M^4}$ is created as maps of observed changes of physical observables, caused by fundamental interactions. They are mapped as *IPNs* into the observer's brain into the memory of an electronic device in every single case.

Although general relativity is not discussed in this paper, it is left, knowing the results of chrono-topology, that general relativity, being based on the Newtonian time space topology of N_t^1, is *per construction* a non-quantizable theory. A reformulation of the field equations in the framework of chrono-topology may lead to a quantum theory of space-time whose time average will give the Einstein field equations of gravity for macroscopic space-time neighbourhoods.

Many famous authors have given various answers to the question about the nature of time: Plato, Aristotle, Newton, Kant, Bergson, and many others. The quest for the meaning of the time by the above researchers and philosophers was rather of a knowledge-theoretical character, such that no direct physical judgement was possible -except for a logical one- of the practical applicability to modern problems of physics. Also, Eddington, Whitehead, Einstein, Dirac, Prigogine, Wheeler and others have shown concern in the elucidation of the nature and properties of time.

It is extremely interesting to verify, after a debate of many decades, Einstein's terrifically strong insight: Now, it is known that, in fact, God does not play dice (Gott wuerfelt nicht) in matters of quantum theory. It has been shown, in fact, that quantum mechanics is not *per se* a statistical theory. The statistical character of quantum physics is imposed on the wave function mainly by the topology of the space-time, $\overline{M^4_{\kappa\lambda_\kappa}}$. This would not be possible in the Minkowski space-time, M^4.

Therefore, both Bohr and Einstein were fully right in their statements.

Meanwhile, new problems appeared in theoretical physics that are not solvable in the frame of the classical understanding of time's nature. Bell, Hawking, Penrose, Unruh, Stamp, Legget, Douglas have published important works on this area, but the time issue remained open.

The beautiful researches of all above authors are only a very small sample of the world literature on time's nature. However, there exist still some very serious remaining problems, in particular in quantum theory, which make this issue central to the atomic, to the nuclear and to the elementary particle theoretical physics.

Some spectacularly successful results have been reached in these areas of physics during the now century now coming to the end, and a high degree of maturity both in experiment and in theory. Nevertheless, some important questions remained open:

i) Can the wave packet's decay be understood in absence of interactions?
ii) Can the reduction of the wave function be understood in the framework of the Schroedinger equation?
iii) Can quantum statistical mechanics (*QSM)* be derived rigorously, in particular the Boltzmann factor, from quantum field theory *(QFT)*?
iv) Can the microscopic and the macroscopic irreversibility be explained, starting from *QFT*?
v) Why is there a tunnel effect?
vi) Why is there an ergodicity?
vii) Why is there a Poincare returning?
viii) Why quarks are not directly observable?
ix) Have the non-locality, related to the Bell theory, and the interpretation of the Aspect et al. experiment to do with the topology of the time?. Etc., etc, ...

It became clear after the publication of Einstein's relativity (and due to the Lorentz transformation) than time and space are interwoven in the Minkowski space-time, M^4. The recognition was provided by relativity that each e-space point is associated with its own time (event), the proper-time.

Accordingly, one would expect that these facts should, normally, impose the replacement in modern physics of the universal Newtonian time, N_t^1, by the new Einsteinian time. This would make justice to Dirac's early proposal that every particle in the many-particle Schroedinger equation should have its own time variable. Dirac's proposal, being related to the topology of the space-time has not yet found in physics the place that it deserves.

It is important to note that, whichever is the topology adopted for the time space, a transformation like

$\{x' = L_x(x,t), t' = L_t(x,t)\}; x \in X \subset R^1$, induces on the space-time the topology of that time.

In the same way the Lorentz transformation $\{(x,t) \mapsto (x',t')\}$ induces the topology of the Newtonian time space, N_t^1, on each space point, *x'*, in the neighbourhood associated with that time, *t*, and space point, *x*.

Space-time topologies resulting from solutions of the Einstein field equations were not mentioned in this paper. However, one cannot tacitly bypass the fact that, in general, relativity the proper-time is a function of the Newtonian time. For example, in the Schwartzschild metric

$$ds^2 = c^2\left(1 - \frac{r_g}{r}\right)dt^2 - r^2 \sin^2\theta\, d\varphi^2 - r^2\, d\vartheta^2 - \frac{dr^2}{1 - \frac{r_g}{r}},$$

the time variable *t* takes values as $t \in T = R^1$.

Also in some text books of general relativity, for example, one reads: "Any monotonic parameter, increasing from the past to the future (i.e., $t \in [-\infty,+\infty]$) might be used to measure time on the world line of a material particle". This is clearly correct for a macroscopic theory. Is it correct for the discontinuous quantum phenomena?

Nevertheless, this attitude reflects the view of some researchers according to which time had nothing to do with fundamental interactions and with the changes induced by them in the different neighbourhoods of the universe.

This attitude has not been adopted in the present work.

In view of these facts one may speculate, if not reasonably conjecture, that the well-known paradoxes themselves in relativity and quantum theory, as well as the possibility for their appearance in physical theories are due to the space-time topology imposed by the Newtonian time.

Despite the obvious necessity to replace the Newtonian time and its topology by *IPNs* as defined precisely in the present work for each event, the Newtonian time remains until today generally dominant in classical and in quantum theory.

The present paper is dealing with the derivation of some consequences of a new type of time topology discovered earlier.

The new topology derives from the fundamental observation teaching that no time would be definable, if nothing changed in the universe.

Since the universe for a non-interacting, structureless particle is the particle itself, no time exists for it. Moreover, since the nuclear and sub-nuclear interaction processes factually are, each one, of finite duration, i.e., they are related to finite changes of the observables involved in the interaction, it is clear that *IPN* cannot de identified with the Newtonian time.

Because the latter is homeomorphic to the whole R^1, while $IPN \in T_4 \subset R^1$, and T_4 is disconnected.

It is also important to remark, that the time for, e.g., a nucleon is related to its corresponding interaction, and it does change as long as the interaction lasts. Just this time emerged in this work for the Liouville equation in connection with constant interaction forces. This time can 'flow' within the corresponding *IPN* as long as the interaction is going on in the rest reference frame of the interacting particle.

On the contrary, for an observer the reaction time (t') may, but must not, flow further, depending, according to Lorentz transformation, on whether he changes its position (x') or not with respect to the rest frame of the particle. This stresses the importance of the interaction for the changes in any system. A nucleon's reaction time, for example, cannot be identified with the universal time that consists, according to chrono-topology, of the union of the maps of all individual interactions occurring in the entire observable universe. On the other hand, the free-field quantum equations of physics, mathematically so instructive they do not provide us with information concerning physical changes due to dynamical processes.

Interaction free quantum equations do describe fundamental particle properties. Famous examples of such equations without interactions are the Dirac spinor equation for the electron spin and the Klein-Gordon equation for the zero spin of the π-meson. However, the interaction free Dirac equation does not predict the existence of positive charge particles. The charge conjugation transformation reveals the positron existence only in the presence of the four vector potential.

One must stress, however, that macroscopic motion, and in particular inertial motions, are correctly expressed either in terms of the Newtonian time or in terms of unions of large numbers of *IPNs* ($\bigcup\limits_{\kappa,\lambda\in Z^{+}}\tau_{\kappa\lambda}$) deriving from interactions in the observable neighbourhood of the universe.

It seems that the way to pave for general relativity towards quantization is to redefine the space-time topology by taking into account the topology of the *IPNs* corresponding to gravity and to reformulate the field equations in the topology of $t\in J_4$. In such a case, the quantization of the theory can most easily be carried out by means of the field-action-integral quantization.

It was judged worth reviewing some, in our view, important time topologies used in the past of proposed recently to describe the phenomena or to explain the interpretational problems in quantum theory.

Some researchers look for time traces in the past. They believe that time is *reversable*, that this mathematical operation which proves so useful in mathematics and in mathematical physics can be implemented in the physical experiment. There is –from the standpoint of this work quite obviously- a small misunderstanding. It may become clear by trying to answer the following questions:

- If the time elements are positive (negative) maps into observer's brain of observed physical observables' changes, could one make them negative (positive) by changing the observable's change sign?
- Could one get negative time by means of any action prepared for the future?
- Could one travel in the past, if every elementary action in the present is an initial condition for the next action in future?
- Becomes time negative, if all clocks' hands move the opposite way around?

Even if one succeeds in reverting a series of ordered events by which positive (negative) time intervals have been defined –an operation physically perfectly feasible- the time defined by the series of the reversely ordered events will still be strictly correspondingly positive (negative) time intervals, provided one keeps the same way of defining time.

Hence, time cannot be inversed in physical reality, and Aristotle was right in teaching that time is a series of 'nows'. Everything occurs at its corresponding 'now'. Every process of observation by a given observer occurs at his corresponding 'now'; not earlier and not later that that. This is the reason why we cannot change the past. Because an intention to change something, e.g., the past, means exclusively to act at a future's 'now'.

Nor can we change the future, once the conditions for it have been fixed, without acting at every 'now' before the 'future' in such a way as to create the new conditions for the 'changed' future. Because to determine the future means to prepare at every 'now' the conditions required for the next 'now'.

These considerations, correct as they may be, do not change anything in the mathematical correctness of, e.g., *Goedel's* solution to *Einstein's* equations. What probably should change is the belief that the time resulting from an elementary physical interaction process might take all values form the 'infinite past' until the 'infinite future' or vice-versa.

The quest for the nature of time is almost three millennia old. Also, since the systematic use of mathematical methods in physics started, (and reaching a peak in more recent times) it was possible for physicists and mathematicians to find various descriptions of the time idea, which are satisfactory to variable degrees [12-20].

This was so not only in mathematics and physics where the correctness of the definition of a physical quantity is more or less easily checked. Even in philosophy [21-26], many different time descriptions have been given which covered a considerable part of the physics requirements. All those time descriptions, although they were descriptive and not definitions in the rigorous sense, helped solve all problems of macroscopic technology and almost all problems in physics.

However, there were a few problems, mainly in physics, which led to situations characterized by some physicists and mathematicians as puzzling or as paradoxical [27-28]. The puzzles and the paradoxes refer to classical as well as to quantum theory and are well-known and described in the literature (See for example [29]). Recently a rigorous definition was obtained [30] relating time to observed changes of physical observables, and this admitted for the first time a mathematical formulation. This mathematical definition of the time is in agreement *with* and contains almost *all* time descriptions known to the authors to the present paper.

Moreover, the new time definition give the same results in the case of all already solved problems and, in addition, it helps to eliminate some puzzles and some paradoxes in quantum theory. It also makes compatible the time reversal invariance of the fundamental equations of physics compatible with the irreversibility of the overwhelming majority of the physical phenomena.

The problems solved in the framework of chrono-topology or partially derived, in this paper, from Liouville's theorem are:

1) The discontinuity of the time [2,4,30].
2) The imaginary impression of the time flow [7,30].
3) The relativity proof that time does not flow [7,30].
4) The relation of the time with Zermelo's well-ordering theorem [7,30].
5) The wave packet stability in absence of interactions [6,7].
6) The finite evolution of the wave packet as a consequence of an interaction process [7,30].
7) The calculation of Planck's constant ([30], and present work)
8) The measurement problem in quantum mechanics [7]
9) The Schroedinger's cat paradox [8].

The calculation of Planck's constant is considered as a major result (of [30] and of the present paper) and it verifies in the authors view the correctness of:

a) The time definition.
b) The chrono-topology.
c) The stochasticity of the quantum fields.

d) The existence of an evolution operator [7,8,30] implying time-asymmetric evolution in quantum theory.

Having stated some advantages of the mathematical time definition, we cannot be tacit about a restriction of the present paper: It solves the Liouville equation only with interaction forces that are independent of the inter-particle distances. This is, doubtlessly, a property of the method used, which is inapplicable to problems of a more general nature.

However, although the calculated *d.f.*'s form may change considerably for systems involving non-constant forces, it is not expected that the *quantum character* of the obtained results would change and become classical. It would be difficult to assert that time or energy would cease to be quantized, if the interaction forces $\{F^{(n)} \mid n = 1,2,...N\}$ depend on the inter-particle distances.

If that is right, it would not be an exaggeration to say that quantum theory together with conditions (7) and (8) reveal the quantum character of the phenomena on the atomic and sub-atomic scale.

Since (A) the time and energy quantizations transform a complex and, hence, unobservable *d.f.* to a real one and, hence, an observable one: Since (B) in addition, they eliminate the time variable altogether from *F(g)*: Since (C) they spontaneously introduce a number 1.05446 X 10^{-34} *j.s.* = $\hbar$, we, therefore, are entitled to think that there exists a strong relationship between quantization and observability on the atomic and sub-atomic scale of phenomena in nature.

Since all results presented follow directly from the classical Liouville theorem, quantum theory might very well have been discovered by Liouville himself or even by Boltzmann, long before Planck did it on the occasion of the black-body radiation problem.

Finally, it is our intention to develop the present method further so that it becomes applicable to systems with any type of interaction forces. We also aim to study the relationship between the form of the various interactions and the corresponding chrono-topologies deriving from them.

Appendix

To clarify the description and to facilitate understanding, it is useful to have herein some definitions and fix the notation used in general topology, as these are required for the presentation of the results. This should by no means be taken as a substitute to reading a book on general topology, which is recommended to the more interested reader.

Let a set *J*, called the space, be given together with a family $\{\tau\}$ of subsets $\tau \subseteq J$ together with the empty set $\varnothing$. The elements of *J* are called points of the space and the elements τ are called open sets.

Definition A1

A pair (J, τi) *of* J *and* τ *represents a topological space, if the following conditions are satisfied:*

(i) $\varnothing \in \tau$ and $J \in \tau$.

(ii) *If* $U_1 \in \tau$, *and* $U_2 \in \tau$, *then* $U_1 \cap U_2 \in \tau$.

(iii) *If* $A=\{A_1,A_2,...\}$ *is a family of elements of* τ *and I is a subset of the index set J such that* $A_i\in\tau,\forall i\in I$, *then* $\bigcup_{i\in I}A_i\in\tau$.

It is clear that the intersection $\bigcap_i A_i$ of a finite subset $\{A_i i\in I\subseteq J\}$ of open subsets is open.

Definition A2

A space, J, is called refular if and only if for every $x\in J$ *and every neighbourhood U of x in a fixed sub-base P there exists a neighbourhood U of x such that* $u\subset\!\!\!\!\!\!\;\; \lnot U$, *where u is the closure of U.*

The topological spaces may be ordered in a hierarchy according to the restrictions that are imposed on them. These restrictions are called *axioms of separation*. Here are the axioms of separation concerning the fundamental interactions physics:

Definition A3

0. *A topological space, J, is called a* J_0*-space, if for every pair of distinct points* $t_1,t_2\in J$ *there exists an open* τ\` *containing exactly one of these points.*

1. *A topological space, J, is called a* J_1*-space, if for every pair of distinct points* $t_1,t_2\in J$ *there exists an open* $\tau'\subset J$ *such that either* $t_1\in\tau',t_2\notin t'$ *or* $t_1\notin\tau',t_2\in\tau'$.

2. *A topological space, I, is called a* J_2 *space, or a Hausdorff space, if for every pair of distinct points* $t_1,t_2\in J$ *there exists open sets* $\tau_1,\tau_2\subset J$ *such that* $t_1\in\tau_1,t_2\in\tau_2$ *and* $\tau_1\cap\tau_2=\varnothing$.

3. *A topological space, J, is called a* J_3*-space or a regular space, if it is a* J_1*-space and for every* $t\in J$ *and for every closed set* $F\subset J$ *such that* $t\notin F$ *there exist open sets* τ_1,τ_2 *such that* $t\in\tau_2,F\subset\tau_2$ *and* $\tau_1\cap\tau_2=\varnothing$.

4. *A topological space, J, is called a* J_4*-space or a normal space, if J is a* J_1 *space and for every pair of disjoint closed subsets* τ_1,τ_2 *there exist open sets U and V such that* $\tau_1\subset U,\tau_2\subset V$ *and* $U\cap V=\varnothing$

Clearly, a J_4-space is a J_3-space so that the hierarchy holds:

$$J_0\Rightarrow J_1\Rightarrow J_2\Rightarrow J_3\Rightarrow J_4.$$

There are still the axioms of separation for the spaces $J_{3\frac{1}{2}},J_5,J_6$ whose definitions are not given here. The topology of the time mainly considered in this paper is just that of J_4. This time topology is generated by distinct finite interactions.

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