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Abstract

Time-asymmetric evolution is derived from time-reversal invariant fundamental QFT-equations. Chrono-topology, the disconnected time topological space $\mathcal{J}_4$, is the playground for the generalized random and infinitely divisable quantum fields, a new development in time-asymmetry. Based on the properties of this time space and using the theory of random quantum fields previously developed a non-unitary, complexity evolution operator, $\mathcal{C}(\mathcal{J}_4)$, is derived. $\mathcal{C}(\mathcal{J}_4)$ breaks down, by Bohr-quantizing the field-action integral, alternatively into two, spontaneously *renormalized* parts: One, (*unitary*) $\mathcal{U}_u(\tau)$, implying $U - processes$ and one (*non-measure-preserving*), $\mathcal{U}_{nmp}(\tau)$, producing $R - processes$. $\mathcal{U}_{nmp}(\tau)$, *breaks time-symmetry* and provides a basis for *CP-violation* in QFT and in particular in the K^0-meson decay. Functional integrals arising in the theory have as a limit Feynman's path integral in accordance with the measure theoretical requirements. Irreversibility and time-symmetry are not incompatible (compare Boltzmann, Poincare) in chrono-topology.

1. Introduction

The reconciliation of the time reversal invariance of the basic equations of quantum theory with the overwhelming majority of irreversible phenomena in nature was for more than a century a puzzling issue for physicists. It is still a subject of the intensive research activities during the last decades [1-13].

One avenue of research was to introduce in quantum theory macroscopically successful methodologies, like diffusion, random processes, Wiener processes by using Ito's stochastic differential equations. Various Schroedinger equations were derived [2-11]. One characteristic feature was the propagation of the quantum processes with a diffusion constant $D = (\hbar / m)^{1/2}$. D is energy-independent and depends only on the rest mass, m, of the particle, both in relativistic and non relativistic theories. Those approaches maintain Boltzmann's and Poincare's views according to which irreversibility were not obtainable from time-symmetric equations.

On the other hand the experimental observation of chaos phenomena in nuclear physics [14] suggested the idea to some researchers that chaos and irreversibility may be connected by means of a not yet discovered fundamental relationship.

These developments in relation with the persisting well-known paradoxes of quantum theory made clear that a fundamental concept in physics was possibly not well-defined [15-17]. Time, lacking a physical definition, attracted attention as possibly being responsible for the long standing problems. Various time conceptions have been proposed.

The present work is part of a series of papers [18-22] in which a rigorous time definition was given and a special time topology, the *chrono-topology,* was developed, applied and obtained irreversibiliy from form time-symmetric equations. For example, quantum statistical mechanics follows from *QFT* without leaving Minkowski's space-time going over to a Euclidean metric [19]. Furthermore, the Schroedinger's cat and the wave packet decay paradoxes were solved [20], among others.

The purpose of the present paper is to demonstrate in the framework of chrono-topology the existence and to present constructively two distinct kinds

of evolution operators in quantum field theory which solve the mentioned problems:

i) *Non-measure preserving evolution* operators lacking the property of time-reversal invariance. In this case the probability measure of the evolving system is not preserved.

ii) *Unitary evolution* operators differing from the evolution operators in

standard *QFT* only by the property of being spontaneously renormalized. The renormalization appears in the theory by means of the field-action integral quantization using Bohrs method.

These results are direct consequences of the fundamental property of chrono-topology of disconnectedness and randomness which imply on the quantum level that the physical fields become *generalized random and infinitely divisible*.

The present work consists of nine sections. In sect. 2 the new time definition is given mathematically and the main properties of *chrono-topology* are presented together with the required notation.

This time definition which contains the time definitions given by Plato, by Aristotle, by Kant and by Bergson, differs from that given by Newton. It allows to explain, among other things, that time does not flow contrary to what has always been believed. The definition of the Newtonian universal time space, N_t^1, is given after the definition of the time elements ,$\{\tau_\lambda{}^j\}$, in chrono-topology.

In sect. 3 certain aspects of quantum statistical mechanics are discussed in the light of chrono-topology. It is shown that the Boltzmann and the Schroedinger factors $exp[-e/k_BT]$ and $exp[-iEt/\hbar]$ follow both from *QFT* in chrono-topology. This establishes a firm link between quantum field theories and thermodynamics.

In sect. 4 some basic properties of the *generalized random and infinitely divisible fields* [23] are introduced. In sect. 5 the proof of existence and the construction in a *fundamental quantum proposition* of the generalized and renormalized evolution operator, $C(\mathcal{T}_4)$, is presented. In sect. 6 it is demonstrated that *unitary evolution* (**U**) and *reduction* (**.R**) are derivable from

Schroedinger's equation in the framework of chrono-topology. Several authors, like Poincare, Boltzmann and others, had expressed the opinion that irreversibility were not derivable from time-symmetric equations.

In sect. 7 path integrals are found avoiding the measure theoretical problems connected with Feynman's path integral [41-42].

In sect. 8 the derivation of the Boltzmann factor from *QFT* together with a quantum definition of the thermodynamic temperature is given without recourse to the introduction of an imaginary time. The dispute between Bohr and Einstein about the character of quantum mechanics persists still today. In sect. 9 it is shown that Bohr and Einstein both were correct in their statements about the *statistical* or the *deterministic* nature of quantum physics. Their opposing views in Minkowski's space-time can be made compatible in the framework of chrono-topology in which quantum fields become stochastic. In sect. 9 the conclusions and their discussion are given.

Finally, a few words about the term *chrono-topology*: Important advances in quantum theory have been made possible by means of the division of the quantities into two classes: *Observables* and *non-observables*. The next decisive step was taken in [24] in recognizing the importance of the *anthropic principle* in cosmology.

A third step between these two principles is the recognition what is observable and what non-observable in physical reality. It is perhaps not much of an exaggeration if one says that the human brain physiology plays a certain part in this matter. There is a relationship between the anthropic principle and the set of Heisenberg's observables. The *meeting point* of them is the anatomy and the physiology of the human body and, in particular, of the human brain.

On the one hand, the physiology determines the life-time of the human observers which is decisive for the building-up of physical theories in cosmology. On the other hand, the human physiology determines the way the observer's brain functions and, hence, what is observable and not observable by him. By saying that, technological means of observation are included since they are also a product of the human brain.

These facts, of course, are not to mean that physics and cosmology are

sciences belonging to the medicine or to the humanities. It affirms simlpy- what is known since long time- that physics studies that part of the universe

which is accessible directly or indirectly to man's five senses' organs. These make the physical observation possible and lend it its perceived structure. These facts are combined in the theory of relativity which by means of the Lorentz transformation gives to the moving observer his space-time topology. In its framework is decided what is *observable* and what is *non-observable* in the physical experiment.

2. The time topology in quantum physics

In order to make precise the description and to facilitate the understanding, some notation and definitions are given needed for the presentation of the results.

Let a set $\mathcal{J}$, called the space, be given with a family $\{\tau\}$ of subsets $\tau \subseteq \mathcal{J}$ together with the empty set $\varnothing$. The elements of $\mathcal{J}$ are called points of the space and the elements τ are called open sets.

Definition 1

A pair $(\mathcal{J}, \tau)$ of $\mathcal{J}$ and τ represents a topological space, if the following conditions are satisfied [25] :

(i) $\varnothing \in \tau$ *and* $\mathcal{J} \in \tau$.

(ii) *If* $U_1 \in \tau$, *and* $U_2 \in \tau$,*then* $U_1 \cap U_2 \in \tau$.

(iii) *If* $\mathcal{A} = \{A_1, A_2, ...\}$ *is a family of elements of τ and I is a subset of the index set J such that $A_i \in \tau$, $\forall i \in I$, then* $\bigcup_{i \in I} A_i \in \tau$.

It is clear that the intersection $\bigcap_i A_i$ of a finite subset $\{A_i, i \in I \subseteq J\}$ of open subsets is open.

Definition 2

A space , $\mathcal{J}$,is called regular if and only if for every $x \in$ $\mathcal{J}$ and every

neighbourhood $\mathcal{V}$ of x in a fixed subbase $\mathcal{P}$ there exists a neighbourhood U of

x such that $\mathcal{U} \subset \mathcal{V}$, *where* $\mathcal{U}$ *is the closure of U* .

The topological spaces may be ordered in a hierarchy according to the restrictions which are imposed on them.These restrictions are called *axioms of separation*. Here are the axioms of separation concerning the fundamental interactions physics:

Definition 3

0. *A topological space,* $\mathcal{T}$*,is called a* $\mathcal{T}_0$ *-space,if for every pair of distinct points* $t_1, t_2 \in \mathcal{T}$ *there exists an open* τ' *containing exactly one of these points.*

1. *A topological space,* $\mathcal{T}$*,is called a* $\mathcal{T}_1$ *-space,if for* every pair of distinct *points* $t_1, t_2 \in \mathcal{T}$ *there exists an open* $\tau' \subset \mathcal{T}$ *such that either* $t_1 \in \tau', t_2 \notin \tau'$ or $t_1 \notin \tau', t_2 \in \tau'$.

2. *A topological space,* $\mathcal{T}$*,is called a* $\mathcal{T}_2$ *-space,or a Hausdorff space ,if for every pair of distinct points* $t_1, t_2 \in \mathcal{T}$ *there exist open sets* $\tau_1, \tau_2 \subset \mathcal{T}$ *such that* $t_1 \in \tau_1, t_2 \in \tau_2$ *and* $\tau_1 \cap \tau_2 = \varnothing$.

3. *A topological space,* $\mathcal{T}$*,is called a* $\mathcal{T}_3$ *-space or a regular space, if it is a* $\mathcal{T}_1$ *-space and for every* $t \in \mathcal{T}$ *and for every closed set* $\mathcal{F} \subset \mathcal{T}_3$ *such that* $t \notin \mathcal{F}$ *there exist open sets* τ_1, τ_2 *such that* $t \in \tau_1, \mathcal{F} \subset \tau_2$ *and* $\tau_1 \cap \tau_2 = \varnothing$.

4. *A topological space,* $\mathcal{T}$*,is called a* $\mathcal{T}_4$ *-space or a normal space, if* $\mathcal{T}$ *is a* $\mathcal{T}_1$ *-space and for every pair of disjoint closed subsets* τ_1, τ_2 *there exist open sets U and V such that* $\tau_1 \subset U, \tau_2 \subset V$ *and* $U \cap V = \varnothing$.

Clearly, a $\mathcal{T}_4$ -space is a $\mathcal{T}_3$ -space so that the hierarchy holds :

$$\mathcal{T}_0 \Rightarrow \mathcal{T}_1 \Rightarrow \mathcal{T}_2 \Rightarrow \mathcal{T}_3 \Rightarrow \mathcal{T}_4 .$$

Next, the three axioms are given of the time physics of the present work:
Axiom I.

All time measurements, classical or quantal, are based on an interaction process implementing a change of a physical or technical observable which generates, if observed, a corresponding time neighbourhood. The generated interaction

proper time neighbourhood (IPN) is a regular into-map of just this change. The change image is stored either in the observer's memory through one of his five senses or in the memory of an electronic device.

Axiom II.

Every fundamental interaction process is associated with (different among them, but) finite changes of the relevant physical observables. Sets of observables' changes are intrinsicly random character, as to their embedment

in the Newtonian time space. They start at irregular Newtonian times and have, within limits, stochastically distributed durations. They may be thought of as embedded in the Newtonian universal time space, N_t^1, but their union has not the topology of N_t^1.

Axiom III.

The elements of the empty set, $\varnothing$, of a class of sets $\{O^j \mid j \in J \subset Z^+\}$ of observables, O^j, are not observable, and their values are identically equal to zero.

A time definition satisfying the above axioms is based on *observational data*. If an observer observes **no change** in his external or internal environments, then he can neither have the impression of time nor does he need it. Based on this trivial but physically fundamental idea is the following time

Definition 4.

An interaction proper time neighborhood (IPN), τ_λ^j, is an injective map, f, of the λ-th change, ΔO_λ^j, of the j-th observable, O^j, caused by a fundamental interaction process:

$$f : \Delta O_\lambda^j \to f(\Delta O_\lambda^j) = \tau_\lambda^j \in \mathcal{T}_4 \subset N_t^1.$$

The index 'j' can in general be omitted in $\mathcal{T}_\Lambda = \bigcup_{\lambda=1}^{\Lambda} \tau_\lambda$, whenever it is not required to know to which particular observable, O^j, belongs the change ΔO_λ^j corresponding to τ_λ^j.

The Newtonian universal time space, N_t^1, which has the topology of the straight line, R^1, can also be defined mathematically in the framework of chrono-topology.

The union of all $\{\tau_\lambda^j\}$ corresponding to the j-th observable's changes and belonging to a closed particle system is a disconnected topological space [20], satisfying the separation axioms of $\mathcal{T}_4^{(j)}$. It can also be used to give the *Newtonian universal time space, N_t^1, a rigorous definition.*

In fact, it is the natural origin of the macroscopic time. This time, known from classical mechanics and from every-day life, comes about as the union of $\{\mathcal{T}_4^{(j)}\}$. It is a denumerable, part of its consecutive elements, τ_λ^j, which may be partially overlapping or not be subjectively discernible.

Here is the definition of the Newtonian universal time in the framework of chrono-topology:

Let be: S the number of observable systems in the universe, $K(s)$ the number observed particles in the s-th system, j the number of observed observables' changes of the κ-th particle, and Λ the number of observed interaction processes in the s-th system of the κ-th particle changing the j observable. Then the *Newtonian universal time space, N_t^1, is the union of the time spaces* $\mathcal{T}_4^{(j\kappa s)}$,

$$N_t^1 = \bigcup_{s \in S} \bigcup_{\kappa \in K(s)} \bigcup_{j \in J(\kappa,s)} \mathcal{T}_4^{(j\kappa s)}$$

for S, $K(s)$, $J(\kappa,s)$, $\Lambda(j,\kappa,s) \subset Z^+$.

The time space, $\mathcal{T}_4^{(j)}$, as defined in *chrono-topology* satisfies the three axioms stated above. They are deduced from operational observation.

An important property of an *IPN*, τ_λ, is that its change, $\Delta\tau_\lambda$, during creation is physically not observable for two reasons:

(i) The open set, τ_λ^j, is the mathematical map of a physical observable's observed change. τ_λ^j is not a physical quantity like, e.g., mass, charge or momentum etc. which are observable.

(ii) If a $\Delta\tau_\lambda$ were observable, then it would be possible to define a $\tau_\lambda' \subset \tau_\lambda$ contrary to the evidence that τ_λ is indivisible.

The indivisibility of τ_λ follows from the fact that $\Delta\tau_\lambda$ would be a map of a part of an observable's change, $\Delta(\Delta O_\lambda)$, due to a fundamental interaction process. But ΔO_λ cannot be divided, because the interaction process whose ΔO_λ is the result cannot be stopped. There is no experimental evidence for the possibility to stop a fundamental interaction process, once it has started.

Similarly, a quantum, ΔO_λ^j, produced by a fundamental interaction process, cannot be divided, at least not by the same interaction process. Hence, it can be observed after the completion of the interaction process.

3. Boltzmann and Schroedinger - Unification of $exp[\dfrac{-E}{k_B T}]$ and $exp[-\dfrac{iEt}{\hbar}]$ in chrono-topology

A particularly important position in the recent literature on the irreversibility problem takes the reduction of the wave function. In view of the difficulties to solve it, some authors expressed the opinion that possibly a fundamental physical concept has not been correctly understood. It is believed that the topology of the time space plays the most important part in solving these problems.

There are two simple, since long puzzling, fundamental facts in physics which resemble a confrontation of Boltzmann and Schroedinger.

- One of them is that statistical mechanics (QSM) is based [26] on the expression "$exp[-E / k_B T]$" or its various, formally different but physically similar expressions.

- The second fact is that standard quantum field theories (QFT) produce the Schroedinger factor "$exp[-iEt/\hbar]$" instead of "$exp[-E / k_B T]$".

The deeper reason for the difference of these two kinds of expressions resides in the space-time topology: Standard *QFT* is formulated in the Minkowski space, $(ict, x, y, z) \in M^4$ [26], while *QSM* is based in the Euclidean space [27], $(t, x, y, z) \in R^4$.

Many highly sophisticated theories (*C*-Algebra*, *KMS*-theory) aim at paving the way for *QSM* to move from the R^4 to M^4. *The target is to derive the statistical, or the Boltzmann factor, $exp[-E / k_B T]$, in standard QFT.* The practice of some authors is as follows:

1) In [28] the Boltzmann factor, $exp[-E / k_B T]$, is put in a direct, *ad hoc* way .

2) In another case [27] one obtains the factor $exp[-E / k_B T]$ directly from the classical Gibbs ensemble.

3) The expression $exp[-\beta(H - \mu N)]$ is also introduced ready in the books on *QFT* (e.g., [26]) as *a priori* given, where $\beta = 1 / k_B T$ and μ is the chemical potential.

4) Other authors apply the transformation, $t \to t' = -it$, on the evolution operator and introduce an imaginary time in Minkowski space-time physics, with a view of coming closer to the desired exponential function, $exp[-E / k_B T]$.

This procedure gives rise to a number of comments which are briefly discussed here:

4a) Time becomes and is kept complex in the physical results after the above mathematical operation, i.e., time remains in disagreement with quantum theory and foreign to the physical reality.

4b) The imaginary part of the Newtonian universal time is put in relation with an absolute temperature, $\Im m \, t = \beta = 1 / k_B T$:

First, until recently there was no clear definition of the time in physics at all. *Second*, an imaginary time has no physical counterpart in the reality as it is usually understood in physics; It is not observable, while temperature is observable.

Third, the quantity $i. \Im m \, t$ is not a time; there is no physical process to which it would correspond according to chrono-topology.

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4c) The principal feature of the Newtonian universal time is its *uniqueness* in dynamics of the universe. It will be shown presently that the uniqueness is spoiled by the assumption **4b**:

1) Let T *represent the complex,* **unique** *and* **universal** *Newtonian time .*

2) Let $f(T)$ *be a periodic or a non-periodic single-valued function of the Newtonian universal time* T.

3) Let $(\mathscr{I}m\,T)^{-1} \propto T$ *be proportional to the temperature, T, of quantum field - theoretical processes for* $T = T_1, T_2, ..., T_N$.

4) Let $\{T_1 \neq T_2 \neq, ..., \neq T_N.\}$ *be the temperatures of N different, but simultaneously proceeding isothermal processes of the kind described in 3).*

Then,

(a) premise 3) is incompatible with premises 1) and 4).
(b) any function $f(T)$, *independently of its structure and interpretation, does not justify the relation* $(\mathscr{I}m\,T)^{-1} \propto T$.

Proof 1

3) and 4) imply

$$(\mathscr{I}m\,T_1)^{-1} \neq (\mathscr{I}m\,T_2)^{-1} \neq ... \neq (\mathscr{I}m\,T_N)^{-1}. \qquad (A)$$

(A) and 1) imply

$$T_1 = t_1' + it_1 \neq T_2 = t_2' + it_2 \neq ..., \neq T_N = t_N' + it_N, \qquad (B)$$

$$f(T_1) \neq f(T_2) \neq ... \neq f(T_N). \qquad (C)$$

Relation (B) expresses the statement that the **unique** and **universal** Newtonian time, **takes simultaneously N different values.**

1) implies, however, the relations

$$T_1 = t_1' + it_1 = T_2 = t_2' + it_2 = ..., = T_N = t_N' + it_N. \qquad (D)$$

$$f(T_1) = f(T_2) = ... = f(T_N), \qquad (E)$$

because of the uniqueness and universality of the Newtonian time.

The relation pairs (B,D) and (C,E) are contradictory: (C,E) is and remains contradictory even if $f(T)$ *represents a field state periodical in imaginary time. Hence, the inverse temperature cannot be identified with any time period.*
Because either (A) is true and premise 1) is false, or (D) is true and premise 4) is false.
But this is not true, since 1) and 4) are in agreement with all observational evidence. Therefore, the contradiction arises because because 3) was assumed for the Newtonian universal time. Therefore, 3) is false, and $(\mathscr{I}m\,T)^{-1}$ is not proportional to N different temperatures of isothermal processes, because at

any time instant there exists one and only one Newtonian universal time-value , T .

N.B. The infinitely many time variables $\{t_n | n = 1,2,...\}$ in $H_I(t_N)...H_I(t_1), N \to \infty$ of

the evolution operator $U(t,t') = 1 - \dfrac{i}{\hbar}\int\limits_{t'}^{t}d\tau H_I(\tau)U(\tau,t')$ perturbation series expansion

take values in one and the same time interval, $t_n \in [t,t'] \subset \mathsf{T}, \forall n \in Z^+$. The same is true for all isothermal processes. The isothermal processes in question are driven by interaction processes causing transitions described by $U(t,t')$ between quanta or particles belonging to the field.

Proof 2

The Newtonian universal time flows eternally [see the *Principia*]. This is the fundamental fact implying the wave packet decay in quatum theory.Hence, it"

is impossible to keep time constant. Consequently, the relation $(\mathcal{I}m\mathsf{T})^{-1} \propto T$ excludes the existence of isothermal processes. This contradicts, again, all observational evidence.

Therefore, $(\mathcal{I}m\mathsf{T})^{-1}$ cannot be proportional to the temperature of an isothermal process.

4d) It is a tradition in thermodynamics as well as in *QFT* to make finite temperature calculations of thermodynamical quantities using the very useful quantum mechanical *partition function* $\mathcal{Z} = Tre^{-\beta(H-\mu N)}$ [29 to 36] which is

either postulated or derived from the evolution operator $U(t,t_0) = e^{-iH(t-t_0)/\hbar}$ by means of the transformation $t \to t' = -it$. H is the Hamiltonian of the relativistic field. The above time transformation implies an inacceptable consequence for special relativity:

$$t \to t' = -it \Rightarrow \{\gamma^{-1} = \sqrt{1-(v/c)^2} \to \sqrt{1+(v/c)^2} > 1\}.$$

Since the velocity of light, c, is a *reference-frame-independent universal constant*, leaves the time rotation c invariant and, hence, implies $\gamma < 1$.

Hence, $t \to t' = -it$ is not compatible with relativity.

4e) The general relativity does not admit a complex time variable, because it would lead to metrics not deriving from gravitational fields [37]. The method of analytic continuation is, of course, a powerful tool for the calculus. It can safely be applied in physics, if after calculation the mathematically introduced parameters either disappear spontaneously, or they do not destroy other physical theories (e.g., metric. Hyperbolicity is required in relativity).

The remarks about the transformation $t \to t' = -it$ are valid especially for relativistically covariant theories in which the Lorentz transformation must apply simultaneously. This is clearly the case in *QFT*. It is concluded, therefore, that the transformation in question may apply in many other cases, but it cannot be use for the derivation of quantum statistical mechanics from *QFT*.

Therefore, the above facts disprove the applicability of $t \to t' = -it$ for the deriviation of the Boltzmann factor, $exp[-E / k_B T]$, from $exp[-iEt / \hbar]$ in the framework of *QFT*. It would require simultaneously the relations

a) $t \to t' = -it$ and

b) $\sqrt{1-(v/c)^2} \to \sqrt{1-(v'/c)^2}$

which are incompatible.

The relation b) is tantamount to: $v' = \dfrac{dx}{dt'} = i\dfrac{dx}{dt} = \dfrac{dx}{dt} = v$ (!) which, of course, is impossible.

It is shown in the framework of chrono-topology that the temperature is related to the average interaction time in the system of the particles under consideration, $\langle \delta(\tau) \rangle$, [22].

Chrono-topology is based on the very simple and obvious observation that *a time parameter not associated with physical changes is neither needed nor definable.*

In order to give our main application of the chrono-topology we need the definitions given below.

4 Stochastic properties of quantum fields

It was stated in the introduction that there are several recent, direct methods by means of which stochasticity is introduced in quantum theory by means of diffusion methods. The quasi-inverse way is followed in the present method. An evolution operator is constructed

first by taking into account the stochastic character of the observed fields which is a consequence of the topology of space-time in chrono-topology and *second* by quantizing the field- action integral.

It turns out that the by-product of this procedure is more important than the main purpose itself: A *non-measure preserving evolution operator* [38] introducing *time asymmetry* in quantum field theory is derived for the first time in quantum theory resolving thereby a number of paradoxes and puzzles.

In addition, a series of functional integrals is derived which resemble the Feynman path integral. They are distinguished from the Feynman path integral by two very important properties. They have:

(i) *Countably additive measures.*

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(ii) *Finite normalization constants.*

As it has been shown by other authors (sect. 7) the normalization constant of the Feynman path integral becomes infinite under certain conditions.

I start this section with the introduction of the generalized random and infinitely divisible fields (*GRIDF*). This theory is the basis of the derivations in the present paper.

Definition 5

A field $\mathcal{L}=\mathcal{L}(\varphi(x,t),\partial\varphi(x,t)) \in R^1$ *is called a generalized random field, if for*

$\mathcal{L}<\xi \in R^1$ *a probability* $P(\xi)$ *is given such that the conditions are fulfilled:*

1. $P(\xi_1) = P(\xi_2)$, if $\xi_1 = \xi_2$,

2. $\lim_{\xi \to -\infty} P(\xi)=0$ and $\lim_{\xi \to \infty} P(\xi) =1,$ (2)

3. $\lim_{\xi \to a-0} P(\xi)=P(a).$

The limits $(-\infty,+\infty)$ in 2. above must in our case be replaced by some finite numbers *(a,b)*, because the field, $\mathcal{L}$, does not become infinite.

Definition 6

A generalized random field, $\mathcal{L}(\varphi(x,t)\partial\varphi(x,t))$, *is called infinitely divisible, if for*

every $\Lambda_\kappa \in Z^+$ *the decomposition is possible:*

$$\mathcal{L}=\mathcal{L}_1+\mathcal{L}_2+...+\mathcal{L}_{\Lambda_\kappa}, \; \forall \kappa \in Z^+ \text{ and } x \in \overline{M}_{\Lambda_\kappa}^4 \quad (3)$$

in which the $\{\mathcal{L}_\lambda(\varphi(x,t)\partial\varphi(x,t))\}$ *are mutually independent, have identical probability distributions,* $\{P(\xi_\lambda)\}$ *and are different from zero only in their corresponding IPNs,* $\tau_\lambda \in \mathcal{T}_4$.

The decomposition of the field Lagrangian density into an arbitrary number of identical terms with identical probability measures at any point of the space-time is mathematically perfect. Such a decomposition violate all conservation laws in the Minkowski or in the Euclidean space-time. This disappears in chrono-topology of the *many-fold space-times* $\overline{M}^4_{\kappa\lambda_\kappa}$, $\forall \kappa \in Z^+$. In every *IPN* the conservation laws hold separately.

1) $$\mathcal{L}=\mathcal{L}_1 +\mathcal{L}_2 \qquad\qquad \text{in } \overline{M}^4_{\kappa 2}, \quad (4)$$

2) $$\mathcal{L}=\mathcal{L}_1 +\mathcal{L}_2 +\mathcal{L}_3 \qquad\qquad \text{in } \overline{M}^4_{\kappa 3} \quad (5)$$

3) $$\mathcal{L}=\mathcal{L}_1 +\mathcal{L}_2 + \ . \ . \ . +\mathcal{L}_{\Lambda_\kappa}. \qquad \text{in } \overline{M}^4_{\kappa\Lambda_\kappa} \quad (6)$$

. . .

The range of $\{\mathcal{L}_\lambda\}$ is determined by the domains both of $x=(x,t)$. The

domain of x does not depend only on the problem at hand but also on the motion velocity of the observer with respect to the rest frame of the interacting particles. Because, according to the Lorentz transformation, the space-time topology depends on the topology of the time space. On the other hand, the domain of t depends on the strength of the interaction. Hence, in the defining condition (2) above of the generalized random field, the limit value $\overline{\xi}$ of ξ_λ, for which the conditions

$$\mathcal{L}_\lambda(\varphi(x,t),\partial\varphi(x,t))<\xi_\lambda \text{ for } t \in \tau_\lambda \qquad\qquad (7)$$

and

$$\lim_{\xi\to\overline{\xi}} P(\xi) = 1$$

are fulfilled, is not infinite.

The chrono-topology induced by the injective maps in the time consists of *IPN*s that are structured as follows:

(i) For systems with few *IPN*s time is given by the union

$$\mathcal{T}_{Disconnected} = \bigcup_{\lambda} \tau_{\lambda}, \lambda \in \Lambda. \qquad (8)$$

Λ is not very large and $\{\tau_{\lambda}\}$ are disconnectedly and randomly embedded in

the Newtonian time space, N_t^1. What is large in this context depends on the

resolving power of the sensors used in the observation.

In order to observe them, a Lorentz transformation is required.

(ii) The system-time topology may change radically in the rest frame of

reference of the interacting particles if Λ becomes very large . The union

$$\mathcal{T}_{\Lambda} = \bigcup_{\lambda} \tau_{\lambda}, \lambda \in \Lambda \qquad (9)$$

may become equal to the sum of some disconnected spaces $\{\mathcal{T}_{Disconnected}\}$ and

of some partitions $\{\mathcal{P}_{\Lambda_{\kappa}} \in R^1\}$ dense in disconnected subsets, T, of N_t^1.

If the cardinality of Λ approaches $\aleph_0$, then $\mathcal{T}_{\Lambda}$ may with high probability,

but not certainly, be densely embedded in $\mathcal{P}_{\Lambda} \subset R^1$, so that

$$Cardinality \; (\mathcal{T}_{\Lambda}) \rightarrow \mathbf{c}.$$

5 Random QFT and time asymmetry

After the above clarifications we are prepared to demostrate the following

Fundamental quantum proposition 1 (The superspace-times $\overline{M}_{\Lambda_{\kappa}}^4$ used in the

proofs are defined in [39,40]).

1. *Let $\mathcal{T}_{\kappa}$ be a set of IPNs due to interaction processes related to the*

Hamiltonian density $\mathcal{H}(\varphi(x), \partial\varphi(x))$ for any $\kappa \in \Lambda_{\kappa}$ such that

$$\mathcal{H}(\varphi(x),\partial\varphi(x)) \neq 0 \quad t \in \mathcal{T}_\kappa,$$

$$\mathcal{H}(\varphi(x),\partial\varphi(x)) \equiv 0 \quad \text{for} \quad t \notin \mathcal{T}_\kappa. \tag{10}$$

2. *Let the field Lagrangian density have the form*

$$\mathcal{L}(\varphi(x),\partial\varphi(x)) = \pi(x)\,\partial_0\varphi(x) - \mathcal{H}(\varphi(x),\partial\varphi(x)) \tag{11}$$

for $t \in \mathcal{T}_\kappa$ and $\mathcal{L}(\varphi(x),\partial\varphi(x)) \equiv 0$ for $t \notin \mathcal{T}_\kappa$

with $\quad \pi(x) = \dfrac{\partial}{\partial[\partial_0\varphi(x)]}\mathcal{L}(\varphi(x),\partial\varphi(x)).$

3. Let $\quad d\varphi(x,s) := \partial_0\varphi(x,t)dt = [\varphi(x,t+d\tau_\lambda) - \varphi(x,t)]$ *be the path variation*

for some $s \in \mathcal{T}_\kappa$ *and every* $x \in \overline{M}^4_\kappa.$

4. *Let* $\mathcal{L}(\varphi(x),\partial\varphi(x))$ *be a generalized, random and infinitely divisible field,*

satisfying

$$\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2 + \dots + \mathcal{L}_{\Lambda_\kappa}. \tag{12}$$

5. Let *the field action integral,* $\mathrm{A}_{\Lambda_\kappa}(S_\kappa)$, *be quantized by*

$$\mathrm{A}_{\Lambda_\kappa}(S_{\Lambda_\kappa}) = \int_{S_{\Lambda_\kappa}} \mathcal{L}(\varphi(x),\partial\varphi(x))\,d^4x = \hbar\Lambda(j,\sigma), \tag{13}$$

where

$$\Lambda(n,\sigma) = \pm\pi \begin{cases} n \pm 1/2, \sigma = 1 \\ \pm n, \sigma = 2 \end{cases}, \tag{14}$$

with $\quad n = 0,1,2,\dots.$ and $\quad S_{\Lambda_\kappa} \subseteq \overline{M}^4_{\Lambda_\kappa}.$

Then the time evolution operator ($\mathcal{C}$ is understood here as a set function) of

the system is

$$C(\mathcal{T}_{\Lambda_x}) = \exp\langle\; [(i\hbar)^{-1} \int_{S_{\Lambda_x}} d^4x\, \mathcal{H}(\varphi(x), \partial\varphi(x)) + i\Lambda(j,\sigma)]$$

$$\times [\cos[\Lambda(j,\sigma)] - i\sin[\Lambda(j,\sigma)]] \;\rangle, \qquad (15)$$

and breaks down into two parts:

(i) non- measure preserving (nmp)[38],

$$\mathcal{U}_{nmp}(\mathcal{T}_{\Lambda_x}) = \exp[\frac{\mp 1}{\hbar}\int H(s)ds \mp \Lambda(n,1)], \quad s \in \Lambda_x \qquad (16)$$

(ii) unitary (u):

$$\mathcal{U}_u(\mathcal{T}_{\Lambda_x}) = \exp[(i\hbar)^{-1}\int H(s)ds + i\Lambda(n,2)], \quad s \in \Lambda_x. \qquad (17)$$

Remark 1

$\Lambda(n,\sigma)$ *is just the renormalization parameter of the evolution operator and depends on the quantum number, n, of the field-action.*

The proof of the Fundamental Proposition 1 is given on the basis of the equation governing the time evolution of the state vector, Ψ, in the time space $\mathcal{T}_{\Lambda_x}$.

$$i\hbar\frac{\partial\Psi(t)}{\partial t} = H(t)\Psi(t), t \in \mathcal{T}_{\Lambda_x}. \qquad (18)$$

The difference between the time evolutions, according to equation (18) in chrono-topology and to equation (19) in the topology of the of the Newtonian time space, N_t^1, of standard *QFT*, is that in the second case it proceeds on the basis of the continuous group property.

In chrono-topology , equation (18), the continuous group property disappears on $\mathcal{T}_{\Lambda_x}$ and stochasticity implies U (unitary) + R (reduction) processes.

The topology of the Newtonian time space, N_t^1, cannot physically accommodate infinitely divisible fields, because there occurs violation of the conservation laws. For example, the infinite divisibility of the Hamiltonian densities would imply violation of the field energy conservation. Hence, the Newtonian time space topology leads only to U evolution according to

$$i\hbar\frac{\partial \Psi(t)}{\partial t} = H(t)\Psi(t), t \in R^1. \qquad (19)$$

Remark 2

It is interesting to observe, before starting the proof of the proposition, that there is a very close relationship between the theory of *GRIDF and* The Feynman approach for the derivation of his path integral is based on first principles . In fact, Feynman tacitly made extensive use of the infinite divisibility of the Hamiltonian. His derivation would be impossible whithout this property.

There is, however, an essential difference between the two approaches: Feynman's path functions are defined in a *commutative geometry*. This approach would conflict with Heisenberg's uncertainty principle if $\varphi(x), \pi(x)$ were operators. Because products of the form $\{ D\pi(x)D\varphi(x) \}$ are basic to the functional integration, and they are forbidden in quantum theory. In chrono-topology this problem does not exist, because here the measure is of the form $\pi(x)D\varphi(x)$ instead of $D\pi(x)D\varphi(x)$.

Proof of the fundamental proposition 1

We start the proof by giving the solution of (18)

$$\mathcal{U}(\mathcal{T}_A) = \{\exp[-i\hbar^{-1}\int \mathcal{H}(\varphi(x'), \partial\varphi(x'))d^4x']\}, \ x' \in \overline{M}_A^4 \qquad (20)$$

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$$= \left\{ \exp[-i\hbar^{-1} \int H(s)ds \right\}, s \in \mathcal{T}_\Lambda \qquad (21)$$

The steps followed are :

1) Combine the above integral with assertion 2.

2) Write the exponential of the sum of the terms as a product of exponentials of the terms of the sum and

3) Develop the factor $\exp[(i\hbar)^{-1} \int dx^4 \partial_0 \varphi(x)\pi(x)]$ in a power series. This gives

$$\mathcal{U}(\mathcal{T}_\Lambda) = \{1 + \sum_{\Lambda_\kappa=1}^{\infty} \frac{(i\hbar)^{-\Lambda_\kappa}}{\Lambda_\kappa!} \prod_{\lambda_\kappa=1}^{\Lambda_\kappa} \int_{\overline{M}_{\Lambda_\kappa}} \partial_0 \varphi(x'_{\lambda_\kappa})\pi(x'_{\lambda_\kappa})d^4x'_{\lambda_\kappa}\}$$

$$\times \exp[(-i\hbar)^{-1} \int_{x' \in \overline{M}_\Lambda} \mathcal{L}(\varphi(x'), \partial \varphi(x'))d^4x']. \qquad (22)$$

The above integral series is defined on $\overline{M}^4_{\Lambda_\kappa}$, $\Lambda_\kappa, \kappa = 1,2,\dots$, $\{\Lambda_\kappa\} = \{$partitions of $\Lambda\}$, because the infinite divisibility property is used. The integration of the last factor is made on $\overline{M}^4_\Lambda$.

Next, assertion 3 is used in the form $\partial_0\varphi(x')d^4x' = d\varphi(q,t')d^3q$ and inserted into (22) together with assertion 4. Λ_κ in the n-th term is put equal to n corresponding to the partition $\mathcal{T}_{\Lambda_\kappa=n}$ $\forall n \in Z^+$.

The λ_κ-th integration in (22) is carried out on the λ_κ-th sector (of the Λ_κ-fold in time) of the space-time super surface. The result is:

$$\mathcal{U}(\mathcal{T}_\Lambda) = \{1 + \sum_{\lambda_\kappa=1}^{\infty} \frac{(i\hbar)^{-\lambda_\kappa}}{\lambda_\kappa!} \prod_{\eta=1}^{\lambda_\kappa} \langle \int_{Q^3} d^3q' \int_{\varphi(\overline{q},0)}^{\varphi(\overline{q},\delta(\tau_\eta))} d\varphi(q',s'_\eta)\pi(q',s'_\eta)$$

$$\times \exp[(-i\hbar)^{-1} \int_{\overline{M}^4_{\lambda_\kappa}} \mathcal{L}(\varphi(x'_{\lambda_\kappa}), \partial \varphi(x'_{\lambda_\kappa}))d^4x'_{\lambda_\kappa}] \rangle\}, s'_\eta \in \tau_\eta. \qquad (23)$$

According to **Definition 5** of the infinitely divisible fields, all $\{\mathcal{L}_{\lambda_\kappa}\}$ have probability distributions independent of κ. Using this property and omitting,

therefore, the index $,\kappa,$ in λ_κ under the product sign in (23), we can sum-up the series.

Using again the relation

$$\pi(x)\,\partial_0\varphi(x)=\mathcal{L}(\varphi(x),\partial\varphi(x))+\mathcal{H}(\varphi(x),\partial\varphi(x)) \qquad (24)$$

the result gives after summation

$$C(\mathcal{T}_\Lambda)=\exp\!\Big((i\hbar)^{-1}\int_{Q^3}d^3q'\int_{\varphi(q,0)}^{\varphi(\bar{q},\delta(\tau_\eta))}d\varphi(q',s'_\eta)\,\pi(q',s'_\eta)$$

$$\times\exp[(-i\hbar)^{-1}\int_{\overline{M}_\Lambda^4}\mathcal{L}(\varphi(x''),\partial\varphi(x''))d^4x''\Big) \qquad (25)$$

$$=\exp\Big\langle\ (i\hbar)^{-1}\int_{\overline{M}_\Lambda^4}d^4x\Big[\mathcal{L}(\varphi(x),\partial\varphi(x))+\mathcal{H}(\varphi(x),\partial\varphi(x))\Big]$$

$$\times\exp[(-i\hbar)^{-1}\int_{\overline{M}_\Lambda^4}\mathcal{L}(\varphi(x'),\partial\varphi(x'))d^4x']\ \Big\rangle \qquad (26)$$

$$=\exp\Big\langle\ (i\hbar)^{-1}\int_{\overline{M}_\Lambda^4}d^4x\Big[\mathcal{L}(\varphi(x),\partial\varphi(x))+\mathcal{H}(\varphi(x),\partial\varphi(x))\Big]$$

$$=\exp\Big\langle\ (i\hbar)^{-1}\int_{\overline{M}_\Lambda^4}d^4x\Big[\mathcal{L}(\varphi(x),\partial\varphi(x))+\mathcal{H}(\varphi(x),\partial\varphi(x))\Big]$$

$$\times\{cos[\hbar^{-1}\int_{\overline{M}_\Lambda^4}\mathcal{L}(\varphi(x'),\partial\varphi(x'))d^4x']$$

$$-i\,sin[\hbar^{-1}\int_{\overline{M}_\Lambda^4}\mathcal{L}(\varphi(x'),\partial\varphi(x'))d^4x']\}\ \Big\rangle. \qquad (27)$$

After separation of the real and the imaginary parts in the exponent, we get the fundamental formula for the generalized time evolution operator, $C(\mathcal{T}_\Lambda)$, which is a realization of both quantum dynamical processes U and R.

$$C(\mathcal{T}_\Lambda)=\exp\Big\langle\ \{(i\hbar)^{-1}\int_{\overline{M}_\Lambda^4}d^4x\Big[\mathcal{L}(\varphi(x),\partial\varphi(x))+\mathcal{H}(\varphi(x),\partial\varphi(x))\Big]$$

$$\times\ cos[\hbar^{-1}\int_{\overline{M}_\Lambda^4}\mathcal{L}(\varphi(x'),\partial\varphi(x'))d^4x']\}$$

$$+\{(\hbar)^{-1}\int_{\overline{M}_\Lambda^4}d^4x\Big[\mathcal{L}(\varphi(x),\partial\varphi(x))+\mathcal{H}(\varphi(x),\partial\varphi(x))\Big]$$

$$\times\{-sin[\hbar^{-1}\int_{\overline{M}_\Lambda^4}\mathcal{L}(\varphi(x'),\partial\varphi(x'))d^4x']\}\ \Big\rangle. \qquad (28)$$

This is the new complex fundamental form of the evolution operator in random QFT.

6 Unitary, U, and Reduction, R, quantum processes

We continue the proof of the proposition 1 and apply the quantization condition (13) on the field-action integral. The critical parts are the terms with the "cos" and "sin" expressions in (28) . The result is

$$C(\mathcal{J}_\Lambda) = \exp\left\{ \; [(i\hbar)^{-1} \int_{\overline{M_\Lambda^4}} d^4x \, \mathcal{H}(\varphi(x), \partial\varphi(x)) + \Lambda(j,\sigma)] \right.$$

$$\left. \times \; [\cos[\Lambda(j,\sigma)] - i\sin[\Lambda(j,\sigma)]] \right\} \quad . \quad (28')$$

Remembering (13) we see that the above expression can be written as a product of two exponential factors. They represent:

a) U the unitary part of the evolution, $\mathcal{U}_u(\mathcal{J}_\Lambda)$.

b) R the incoherent part of the evolution, $\mathcal{U}_{nmp}(\mathcal{J}_\Lambda)$.

Hence, the general evolution operator, $C(\mathcal{J}_\Lambda)$, breaks down after the quantization of the field-action integral into the alternative:

$$C(\mathcal{J}_\Lambda) = \text{EITHER} \; \mathcal{U}_u(\mathcal{J}_\Lambda) \times 1 \quad \text{OR} \quad 1 \times \mathcal{U}_{nmp}(\mathcal{J}_\Lambda) \qquad (29)$$

This completes the proof of the *fundamental quantum proposition 1* .

This is the contemplated result by many authors in their publications. The most interesting feature of (29) is that it proves that Schroedinger's equation exhibits both U and R properties simultaneously in the framework of chrono-topology. This is just the form of evolution postulated and required for the

implementation of the *wave function reduction* by *the measurement problem* or by *Schroedinger's cat paradox*.

A further feature of the evolution operator (29) is the spontaneous renormalization of the field-action integral in the exponent by means of the term $\Lambda(n,\sigma)$ (see also (33) and (34) below).

7 Stochastic fields and their relationship with Feynamn's path integral

The integrals in series (23) are all finite. φ is a solution of the Euler- Lagrange equation, and it satisfies appropriate boundary conditions. The series converges because each term is the power of a definite integral. It can be demonstrated by majorization and is omitted.

The continuous parameter s'_k characterizes the integration path $d\varphi(q,s)$ in the space interval $\left[\varphi(x,t),\varphi(x,t+\tau)\right]$ with initial and the final values $\left[\varphi(x,t)\right]$ for $s'_k = t$, and $\left[\varphi(x,t+\tau)\right]$ for $s'_k = \delta(\tau)$ respectively. The field function φ coincides at these values of s with the lower and the upper limits of the φ-path integration.

Corollary 1

The contribution, $\mathcal{U}_{\lambda_\kappa}\left((\mathcal{T}_\Lambda)\right)$ *), to the evolution operator of the path integral in the expansion (23) vanishes in the limit* $\lambda_\kappa \to \infty$.

Proof

The λ_κ-*th* term in equation (23) is

$$\mathcal{U}_{\lambda_\kappa}\left((\mathcal{T}_\Lambda)\right) = \frac{(i\hbar)^{-\lambda_\kappa}}{\lambda_\kappa!} \prod_{\eta=1}^{\lambda_\kappa} \int_{Q^3} d^3q \int_{\varphi(q,t)}^{\varphi(\bar{q},t+\tau)} d\varphi(q,s_\eta)\pi(q,s_\eta)$$

$$\times \exp[(-i\hbar)^{-1} \int_{\overline{M}_\Lambda^4} \mathcal{L}(\varphi(x'),\partial\varphi(x'))d^4x'] \tag{30}$$

The integral over $d\varphi$ sums values along all paths. The integral over d^3q sums the function $\varphi(q,s)$-values along every selected individual path between the two fixed limit function values $\varphi(\underline{q},t)$ and $\varphi(\bar{q},t+\tau_\lambda)$.

The measures in the integrals of the product in (30) are well-defined and exist on the supports:

i) $\left[\varphi(0),\varphi(\delta(\tau_{\lambda_{\kappa}}))\right]$, and ii) $Q^3 \subseteq R^3$.

The factors $\{f_\eta\}$ of the product $\prod\limits_{\eta=1}^{\lambda_\kappa} \int\limits_{Q^3} d^3q \int\limits_{\varphi(\underline{q},t)}^{\varphi(\overline{q},t+\tau)} d\varphi(q,s_\eta)\pi(q,s_\eta)$ are

independent of η and $\{\prod\limits_{\eta=1}^{\lambda_\kappa} f_\eta = f^{\lambda_\kappa}\}$. Also, the factor $(\lambda_\kappa!)^{-1}$ decreases faster

than the power f^{λ_κ}, and for $\lambda_\kappa \to \infty$ there holds $\lim\limits_{\lambda_\kappa \to \infty} f^{\lambda_\kappa}/\lambda_\kappa! = 0$. It

follows, therefore, that the functional integral does not contribute to the evolution operator in the limit $\lambda_\kappa \to \infty$. This completes the proof **Corollary 1**

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The "phase" factor in (30) is identical to the one in Feynman's path integral. Differences appear in the functions to be integrated over dq and over $d\varphi(q,s)$. The following correspondances with Feynman's path integral are observed:

In the limit, $\lambda_\kappa \to \infty$, the integrals become functional integrals with the measure correspondences

$$Dq \;\Rightarrow\; \prod_{\lambda_\kappa=1}^{\infty} d^3q_{\lambda_\kappa} \;,q_{\lambda_\kappa} \in Q^3 \subseteq R^3 \tag{31}$$

$$Dp \;\Rightarrow\; \prod_{\eta=1}^{\infty} \pi(\varphi(q_\eta,s_\eta),\partial\varphi(q_\eta,s))d\varphi(q_\eta,s_\eta) \tag{32}$$

for $\varphi(q_\eta,s),\pi(\varphi(q_\eta,s_\eta),\partial\varphi(q_\eta,s_\eta)) \in L^2$.

The contributions of these integrals, which are similar to Feynman's path integral in the limit $\lambda_\kappa \to \infty$,are zero because of the normalization factor, $\lambda_\kappa!$.

This normalization factor implies that the path integral

$$\frac{(i\hbar)^{-\lambda_\kappa}}{\lambda_\kappa!} \prod_{\eta=1}^{\lambda_\kappa} \int_{Q^3} d^3q \int_{\varphi(\underline{q},t)}^{\varphi(\overline{q},t+\tau)} d\varphi(q,s_\eta)\pi(q,s_\eta)$$

$$\times \; exp[(-i\hbar)^{-1} \int_{M_\Lambda^4} \mathcal{L}(\varphi(x'),\partial\varphi(x'))d^4x']$$

takes finite values in the above limit.

Yourgrau and Mandelstam [41] have shown that the normalization constants

$$A_n = \int exp[iL(q_{r,(j+1)},q_\eta)(t_{j+1}-t_j)]dq_m$$
$$= \prod_r \{2\pi(t_{n+1}-t_n)/a_r(q_s)\}^{1/2}$$

in Feynman's path integral,

$$\int exp[i\sum_{j=1}^{n-1} L(q_{r,(j+1)},q_\eta)(t_{j+1}-t_j)]\frac{1}{A_1}\frac{dq_{r2}}{A_2}...\frac{dq_{r(n-1)}}{A_{n-1}},$$

vanish in the limit $lim_{n\to\infty}(t_{n+1}-t_n)=0$, if the Lagrangian, L, is of the form

$$L = \sum_{r=1}\frac{1}{2}a_r(q_s)\dot{q}_r^2 - V(q_s).$$

See, however, also results obtained by Klauder on the normalization constant of this integral [42].

Some of the similarities and some of the differences between Feynman's path integral and the present theory's integrals are compiled in

Table 1. *Comparison of Feynman's path integral properties with those of the present theory with some statistical and quantum properties.*

	Feynman	Present work
Spatial measure:	$Dq = \prod_{k=1}^{\infty} dq_k \Leftrightarrow$	$\prod_{k=1}^{\infty} d^3\mathbf{q}_k$
Functional measure:	$Dp = \prod_{k=1}^{\infty} dp_k \Leftrightarrow$	$\prod_{k=1}^{\infty} \pi(\varphi(\mathbf{q}_k,s),\partial\varphi(\mathbf{q}_k,s))d\varphi(\mathbf{q}_k,s)$
Normalization:	$? \Leftrightarrow$	$1/\kappa!$
Uncertainty Principle:	no	yes
Gibbs ensemble:	no	yes

The exponential in (28') becomes real for $j = 2n+1)/2$ and takes on a form reducible to the one known from *statistical mechanics*.

We thus have two kinds of evolution operators: One preserving the norm of the state vector, and one changing it. The canonical momentum, $\pi(q_\eta, s)$, enters the measure expression as a *weight factor - not as differential*. This makes the integration measures compatible with Heisenberg's uncertainty principle and allows to quantize the field-action integral.

8 The temperature in stochastic quantum field theory

The measure preserving evolution is implemented by means of the unitary operator

$$\mathcal{U}_u\,(\delta(\tau))=exp[\,(i\hbar)^{-1}\int_{\overrightarrow{M_\lambda}}d^4x\,\mathcal{H}(\varphi(x),\partial\varphi(x)+2i\pi n]\quad,\,n=0,1,2...\quad. \qquad (33)$$

If the norm of the state vector is not preserved during evolution, then it is described by

$$\mathcal{U}_{nmp}(\delta(\tau))=exp[\langle\,\frac{\mp 1}{\hbar}\int_{\overrightarrow{M_1^4}}d^4x\,\mathcal{H}(\varphi(x),\partial\varphi(x))\mp\pi(2n+1/2)\,\rangle]\,,$$

$$n = 0,1,2,3,...\,. \qquad (34)$$

If the state vector, Ψ, is expanded in a series of eigenstates of the Hamiltonian, and $\mathcal{U}_u\,(\delta(\tau))$ or $\mathcal{U}_{nmp}(\delta(\tau))$ act on Ψ, then (33) and (34) become respectively:

$$exp[-i\sum_{\lambda=1}^{\Lambda_\tau}E_\lambda.\delta(\tau_\lambda)/\hbar+2i\pi n]\,, \qquad (35)$$

$$exp[\mp\sum_{\lambda=1}^{\Lambda_\tau}E_\lambda.\delta(\tau_\lambda)/\hbar\mp\pi(2n+1/2)]\,. \qquad (36)$$

Corollary 2

The temperature of a particle system interacting via a fundamental interaction is proportional to Planck's constant and inversely proportional to the average diameter of the interaction proper-time neighbourhoods $\langle \delta(\tau) \rangle$,

$$\sum_{\lambda_\kappa=1}^{\Lambda_\kappa} E_{\lambda_\kappa} \delta(\tau_{\lambda_\kappa}) / \hbar = \frac{\langle E \rangle_{\Lambda_\kappa}}{k_B T_{\Lambda_\kappa}}. \tag{37}$$

Proof

We divide and multiply the sum in the exponent of

$$exp[\mp \sum_{\lambda=1}^{\Lambda_\kappa} E_\lambda . \delta(\tau_\lambda) / \hbar \mp \pi(2n+1/2)]$$

by $\sum_{\lambda=1}^{\Lambda_\kappa} \delta(\tau_\lambda) / \Lambda_\kappa$, and we write for the time averaged energy per particle the expression

$$\langle E \rangle_{\mathcal{I}_\lambda} \times \Lambda \times {<}\delta(\tau){>}_\Lambda$$

$$= \Lambda \times (\sum_{\lambda=1}^{\Lambda} \delta(\tau_\lambda) / \Lambda) \times \sum_{\lambda=1}^{\Lambda} E_\lambda . \delta(\tau_\lambda) / \sum_{\lambda=1}^{\Lambda} \delta(\tau_\lambda), \tag{39}$$

$${<}E{>}_{\mathcal{I}_\lambda} = \sum_{\lambda=1}^{\Lambda} E_\lambda \delta(\tau_\lambda) / \sum_{\lambda=1}^{\Lambda} \delta(\tau_\lambda),$$

$${<}\delta(\tau_\lambda){>}_\Lambda = \sum_{\lambda=1}^{\Lambda} \delta(\tau_\lambda) / \Lambda.$$

The factor (35) appearing in a state vector after the action of $\mathcal{U}_{nmp}(\delta(\tau))$ is the Schroedinger factor familiar to time dependent problems. The

corresponding factor in (36) becomes identical to the Boltzmann statistical factor, if the system temperature is defined by

$$k_B T^{(\Lambda)} = \frac{\hbar}{\langle \delta(\tau) \rangle_\Lambda} \quad , \qquad (40)$$

where k_B is the Boltzmann konstant, and the system energy is given by (39).

It should be observed that the temperature, $T^{(\Lambda)}$, given by (40) is not the global temperature of the system. It is the average temperature of the particle subset of the system which had their last cillisions for $\lambda = 1,2...\Lambda_\kappa$.

It is evident that the above definition of the temperature is valid for both the equilibrium and the non-equilibrium states of the system. It depends only on the average frequency of the collisions, $f_{coll.}$, in chrono-topology and it is given by $f_\Lambda^{coll.} = \langle \delta(\tau) \rangle_\Lambda^{-1}$. It allows to give a definition of the temperature in the framework of quantum theory. This completes the proof of *Corollary 2*.

Expression (40) has been derived in [19] for weaker conditions on the sysstem. The present definition of the temperature in thermodynamics is based on very clear physical processes and conditions. The role of the time in this definition is fully clearly related to the interaction time which is a real measurable quantity.

The main features of the present temperature definition are:

i) Relativity is respected by avoiding the introduction of an imaginary time by means of the transformation
$$t \rightarrow t' = -it | Tr \, exp[-i(H - \mu N)t] = Tr \, exp[-\beta(H - \mu N)] .$$

ii) The canonical ensemble and the partition function follow in the framework of quantum theory in Minkowski's metric but not in Minkowski's space-time.

iii) The temperature is related in a natural way to the collision frequency in

the framework of *QFT*.

Remark 3

The average energy is calculated over the time during which an enegry value is possessed by a system particle. It is not averaged over the number of the particles. This is quite natural because, if an energy value is possessed by a particle during zero time, then it contributes zero to the average energy of the system.

9 Einstein - Bohr. Both views are correct in chrono-topology

Some authors believe that Bohr was right and Einstein wrong or vice-versa, (e.g., [43]) in their dispute about the deterministic or the statistical character of quantum mechanics. It has been proved in the framework of chrono-topology, that both were right.

The reason for this fact is that Einstein's statement, according to which *God does not play dice*, regarded the quantum equations of motion (Schroedinger, Dirac etc.) *per se, i.e.,* inside a single *IPN* which in that time was the entire N_t^1. In this case - as is now clear - God, indeed, does not play dice for time $t \in N_t^1$ with $\delta(N_t^1) = \infty$, and Einstein was right. In fact, the quantum equations are per contruction *non-statistical* for $t \in N_t^1$ and, evidently, for $\tau_\lambda \subset N_t^1$.

Since the quantum equations are deterministic for $t \in N_t^1$, so are they for $t \in \tau_\lambda \subset N_t^1$, because τ_λ is dense in itself and N_t^1 is continuous.

Hence, Einstein's statement is true both in the Newtonian universal time space and in chrono-topology.

However, a quantum change does not correspond to a τ_λ of infinite extension, and the experimental description of a quantum emission is done for $t \in \bigcup_{\lambda \in \Lambda_\kappa} \tau_\lambda$ for any large Λ_κ. This is true for at least two reasons:

(i) The complete experimental measurement of an observable cannot be based on one single fundamental interaction process (e.g., the measurement of a cross section in nuclear or atomic reactions) and, hence, only inside one single interaction proper time neighbourhood, τ_λ, because of its very small diameter, $\delta(\tau_\lambda)$.

(ii) In the case of isolated events, as frequently are observed in high energy physics, the experimental measurement of an observable is purposefully repeated many times in order to obtain sufficient accuracy by minimizing statistical errors. A measured value of an observable never coincides with another value measured, however accurately, with the same method, the same apparatus, the same initial conditions for the relevant particles, etc. This is so not because of the errors in the experimental accuracy only. The stochasticity of the interaction introduces randomness in the interaction duration and in the impact parameter.

For these reasons observation is conventionally done for $t \in \mathcal{T}_\Lambda$, not for $t \in \tau_\lambda$. But for $t \in \mathcal{T}_\Lambda$, the interpretation of the solutions of the quantum equations becomes necessarily statistical. In order to get agreement between measured and calculated values, the former are *de facto* averaged by the detector. An averaging procedure is required for the latter by means of integrations over the space and time variables according to the case as follows:

(i) The time durations of many interaction processes, $t \in \bigcup_\lambda \tau_\lambda$.

(ii) The impact parameter, $\mathbf{r} \in S_\Lambda = \bigcup_\lambda \{\mathbf{r}_\lambda + \Delta\mathbf{r}_\lambda\}$.

Clearly, the two parameters, $(\tau_\lambda, \Delta\mathbf{r}_\lambda)$ do not appear in the solutions of the quantum equations as $\mathbf{r} + \Delta\mathbf{r}$ or as $t + \tau_\lambda, t \in N_t^1$, because the wave functions are solutions of *deterministic equations*. But, of course, it makes no difference whether the averaging integrations are over $\mathbf{r}_\lambda + \Delta\mathbf{r}_\lambda$ or over $\mathbf{r}$, etc.

Bohr's statement, on the other hand, regarded the quantum physics results of a measurement as a whole for a particular experimental value, because quantum physics observation's arena is not τ_λ. The physical 'playground' is rather $\mathcal{T}_\Lambda = \bigcup \tau_\lambda$, and the space-time, $\overline{M}_{\kappa\lambda_\kappa}^4$, resulting from it in accordance with Einstein's relativity.

Moreover, since $\{\delta(\tau_\lambda) | \lambda \in Z^+\}$ are within limits random numbers, an averaging process in $\mathcal{T}_\Lambda$ takes place to give any measured value. This is the way in which the statistical character of quantum physics emerges from the data point of view . Hence, Bohr was correct too (see also [21] sect. 4).

It becomes, thus, evident that both Bohr and Einstein were right in theirrespective statements. The link between their points of view could not be discerned within Minkowski's space-time topology. In chrono-topology, this is clear.

It should be emphasized that the possibility for the reconciliation of Bohr' s and Einstein' s views about the nature of quantum theory exists only in the framework of chrono-topology. Because only the space-time structure imposed by chrono-topology allows the coexistence of the time reversal invariance (inside τ_λ) of the fundamental equations of quantum theory with the statistical interpretation of the wave function (inside $\mathcal{T}_\Lambda$) .

In other words:

a) The quantum theory equations were historically developed in the Newtonian universal time space topolology, in which *de facto* researchers live.

b) The quantum experimental results are obtained in chrono-topology (which, is *de facto* a set of maps in observer's brain of observed observables' changes), and the events, leading to these results, must necessarily be of quantum structure.

The *living matter factor* as conjectured by E. P. Wigner [45] enters just in this way the into the physical measurement process according to chrono-topology.

Furthermore, the concept of the *infinite divisibility* of the Hamiltonian can be compatible with the energy conservation law only in the framework of chrono-topology's many-folded superspace-times, $\overline{M}^4_{\Lambda_r}$.

In Euclid's or in Minkowski's space-times the equation

$$H = H_1 + H_2 + ... + H_{\Lambda_r} \quad with \quad H = H_1 = H_2 = ... = H_{\Lambda_r}, \quad \forall \Lambda_r \in Z^+$$

is meaningless for the Hamiltonian of a quantum system. Physically, it violates the energy conservation law. Mathematically, it violates the definition of a function which is not identically equal to zero.

10. Conclusions and discussion

The chief conclusion following from the presented theory is that the probability measure of a quantum state can change in a time-asymmetrical way under well stated conditions.

The second major result is the proof that time asymmetry exists in quantum field theory and is consistent with the irreversibility in nature. The tantalizing issue of "*Time-symmetry* versus *Irreversibility*" is settled in the framework of chrono-topology.

In the time definition in the present paper the existence of the observer is, so to speak, integrated with physics. Wigner's conjecture [45] about the influence of living matter on the issue of a quantum measurement becomes now quite reasonable and understandable.

The Boltzmann probability factor, $exp[-E_n / k_B T]$, consistently introduces itself in quantum field theory in a disconnected space-time topology with the Minkowski metric in its compact subsets. Although this result is not fully independent of the first conclusion it should be separately appreciated, because it helps to explain the T- symmetry violation exemplified in nature by the CP- violation in $K_L^0 - K_S^0$ system discovered [44] in 1964. CP-violation exists also in the semi-leptonic decay of neutral *kaons* $K^0 \rightarrow l^+ \nu_l \pi^-, K^0 \rightarrow l^- \overline{\nu} \pi^+$, where the lepton l is either an electron, e, or a muon, μ, and the final states transform into one another. These problems have found their natural explanation in the framework of the chrono-topological structure of the space-time implying the time asymmetric evolution operator, $\mathcal{U}_{nmp}(\tau_\lambda)$.

The building-up of a unified theory for the description of some irreversible quantum and some macroscopic phenomena has been an issue of considerable research interest for a long time and for many researchers in many laboratories. In fact, the main problem was the deduction of *time-irreversible behavior* in macroscopic phenomena from the *time reversal invariant quantum equations*.

The achievement of an agreement [18 -22] between, on the one hand, the time

reversal invariant solutions of the basic equations of physics, like the Schroedinger, the Dirac equations and the quantum field theories (*QFT*), and, on the other hand, the overwhelming majority of the irreversible macroscopic phenomena in nature makes up a great unsolved problem in physics keeping research activities busy since long all over the world [1-13].

Moreover, the discovery of chaos phenomena in nuclear physics [14] induced the idea to many researchers that *chaos* and *irreversibility* may be connected

by means of a not yet discovered fundamental relationship.

It has been shown in the preceding sections that such a relationship, which may very well exist in other areas of physics, is not necessary for the description of the irreversibility. The existence of a non-measure preserving evolution operator deriving from Schroedinger's equation, after postulating the properties of the *generalized random and infinitely divisible fields,* shows conclusively that the supposed relationship- if it exists- must not have the character of a physical law. It may be either casual, or it is of a correlational character following from the random structure of chrono-topology with respect to the universal Newtonian time, in which are embedded all results of observation.

It is of great importance to note that the irreversibility property exhibited by the operator, $\mathcal{U}_{nmp}(\mathcal{T}_\Lambda)$, constructed in sect. 5 implements the twofold task:

a) It demonstrates the existence of irreversible or time-asymmetric processes *on the quantum level*, giving a possibility to understand the time asymmetry of the K^0-meson decay process.

b) It allows to explain the irreversibility in the behavior of the macroscopic system phenomena.

These developments, seen in relation with the persisting well-known paradoxes of quantum theory, make clear that the view is justified [40] to consider the Newtonian time topology as responsible for some interpretational issues in quantum theory.

In a series of papers [18-22], the idea of *chrono-topology* was advanced, the partition function was obtained in the framework of the quantum theory and

a quantum definition of the thermodynamic temperature was obtained even for non-equilibrium states. In the simplest form it is given by the expression $T = \dfrac{\hbar}{k_B <\tau>}$, where $<\tau>$ is the collision transition time of the system's molecules averaged over the collisions' set within the interaction proper time neighbourhoods, τ_λ^j, considered.

As far as the explanation of the state vector reduction is concerned, the novel conclusion is that "reduction" by no means implies the vanishing of all state vector components exept one. It rather means:

i) Very strong reduction of some amplitude values with respect to the surviving state's amplitude.

ii) The number, N, of surviving states after the reduction process is not necessarily one. It may very well take also values higher than one, $N>1$, with various probability amplitudes [40].

Finally, Bohr's and Einstein's adverse views about the statistical or the deterministic character of quantum mechanics are compatible in the framework of chrono-topology. Both great physicists were right in their respective statements, and their reconciliation is appreciated as a proof of the correctness of the chrono-topological structure of the present theory about the time physics.

Finally, it is interesting to remark that the interelationships between the different dynamics disciplines change radically in the framework of chrono-topology. It becomes, now obvious that thermodynamics and statistical mecanics (SM) come closer to quantum theory (QT), while the distance further increases between classical dynamics (CD) and thermodynamics (TD) (Fig. 1).

This is so not only because QFT share irreversibility with TD and SM, but also because the Boltzmann factor, $e^{-\frac{E_n}{kT}}$, and the absolute temperature, T, can now be derived in the framework of QFT. This amounts to the quasi-unification of these hitherto - with an exception- unrelated theories. In addition, it is expected that some phenomena considered as having different origins will turn out to be related.
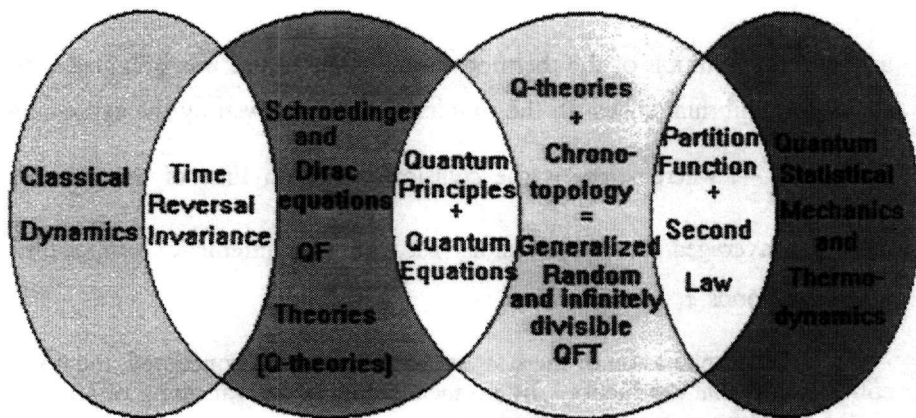
Fig. 1. Relationships and intersections of the different disciplines of physical dynamics. Standard QT in Minkowski's space-time is separated from TD and, e.g., the partition function is introduced *ad hoc* in the form $e^{-\beta H}$ as an extraneous element to QFT. In chrono-topology, QFT and TD have a non-empty intersection; it contains the partition function, the irreversibility property and the second law. CD is most distant from TD due to the time reversal invariance. Standard QT is closer to TD than to CD because it shares the quantum equations with chrono-topological QFT. The latter has randomness in common with SM.

References

[1]   T. Petrosky and I. Prigogine, *Phys. Letters*, **A182** (1993) 5.

[2]   E.Nelson, *Quantum fluctuations* (Princeton University, Princeton, N.J. ,1985),

[3]   F. Guerra, and L. M. Morato, *Phys. Rev.* **D27** (1983) 1786.

[4]   I. Bonzani, *Math. Comput. Modelling*, **17**, No. 6 (1993) 37.

[5]   V.Yu. Podlipchuk, *J. Moscow Phys.* Soc. , **3** (1993) 37.

[6]   V.N.Kolokol' tsov, *J.Math. Phys.* , **36** (1995) 2741.

[7]   N. C. Petroni and F. Guerra, *Foundations of Physics*, **25** (1995) 297.

[8]   M. Pavon, *J. Math. Phys.*, **36** (1995) 6774.

[9]   M. Keller, and G. Mahler, *Quantum Semiclass. Opt.*, **8** (1996) 223.

[10] S. Stenholm, *Quantum Semiclass. Opt.*, **8** (1996) 297.

[11] V. P. Belavkin, and O. Mehlsheimer, *Quantum Semiclass. Opt.*, **8** (1996) 167.

[12] J. C. Zambrini, *J. of Mathemetical Physics*, **27** (1986) 2307.

[13] M. C. Gutzwiller, *"Chaos in Classical and Quantum Mechanics"*, (Springer-Verlag, 1990).

[14] O. Bohigas and H.A.Weidemueller, *Ann. Nucl. Part. Sci.* **38**(1988) 421.

[15] J. A. Wheeler, *Einstein's Vision.* Springer, 1968, p. 51.

[16] C. Syros, *Lettere al Nuovo Cim.* **10**, (1974), p. 718-723.

[17] R. Douglas, in *Time's arrows today*, ed. S.F.Savitt , (Cambridge University Press ,1995), p. 173-188.

[18] C.Syros, *Mod. Physics Letters* **B4**, (1990) 1089.

[19] C.Syros, *Int. J. Mod. Phys.* **B5** (1991), p. 2909.

[20] C.Syros and C. Schulz-Mirbach, *Chrono-topology in quantum field theory and the solution of Schroedinger's cat paradox, Int. J. Mod. Phys.* A. In press.

[21] C. Syros, The time concept in atomic and sub-atomic systems-Reconciliation of the time-reversal-invariance and the macroscopic arrow of time, in *Advances in Nuclear Physics*, eds. C. Syros and C. Ronchi , (European Commission, Luxenburg, 1995), p. 242-287.

[22] C. Syros, The space-time topology of nuclear and sub-nuclear reactions, Proceedings of the *7-th Nuclear Physics Symposium of the Hellenic Nuclear Physics Society*, (Athens, 6-8 May, 1996 , Ed. A. Aravantinos).

[23] I.M.Gel'fand, and N.Ya.Vilenkin, *Generalized Functions*, Vol.4 (Academic Press, New York, 1964), p. 238.

[24] R. H. Dicke, *Rev. Mod. Phys.* **29** (1977) 35.

[25] R.Engelking, *General Topology*, (Sigma Series in Pure Mathematics, Haldermann Verlag Berlin, 1989)

[26] C. Itzykson and J.-B. Zuber, *Quantum Field Theory*, (McGraw-Hill International Editions, Physics Series, 1988), p.125.

[27] J. Glimm and A. Jaffe, *Quantum Physics*, (Springer-Verlag, New York, 1981), p.32.

[28] N. N. Bogoliubov, *Lectures in Quantum Statistics*, Vol. I (McDonald Technical and Scientific, London, 1967), p. 12.

[29] Taizo Muta, *Foundations of Quantum Chromodynamics*, (World Scientific, Singapore, 1987), p. 42.

[30] L.S.Schulman, *Techniques and applications of path integration*, (J.Wiley & Sons, New York, 1981), p. 237.

[31] Michio Kaku, *Quantum field theory*, (Oxford University Press,

1993), p. 571.

[32] D. Bailin and A. Love, *Introduction to gauge field theory*, (Institute of
Physics Publishing in Assoc. with University of Sussex Press,1993),p. 80.

[33] D.J.Amit, *Field theory, the rewnormalization group, and critical
phenomena*, (World Scientific, Singapore, 1984), p.31.

[34] S.Hawking, and R. Penrose, *The nature of space and time*, (Princeton
University Ptress,1996), p. 47.

[35] H. Kleinert, *Path integrals*, (World Scientific, Singapore, 2nd ed. ,1995),

p. 59.

[36] A. Connes, *Noncommutative geometry*, (Academic Press,Inc. San Diego,

1994), p. 40.

[37]. L. Landau and E. Lifchitz, *Theorie des champs*,(Editions MIR,
Moscou, 1970), 309.

[38] P. R. Halmos, *Ergodic Theory*, (Chelsea New York, 1956), p.43.

[39] C. Syros and G. S. Ioannidis, The wave packet stability in nuclear
reactions, *European Conference in Advances in Nuclear
Physics and Related Areas*, Thessaloniki 8-12 July 1997.
In press.

[40] C. Syros and C. Schulz-Mirbach, *Quantum Chrono-topology of
Atomic Nuclear and Sub-nuclear Reactions,* Preprint,
TU Harburg-Hamburg, hep-th/9609093 11 Sep 1996.

[41] W. Yourgrau and S. Mandelstam, *Variational Principles in Dynamics
and Quantum Theory*, (Dover Publications, Inc.,New York, 1968),p.131.

[42] J.R.Klauder, in *Progress in Quantum Field Theory*, eds. E. Ezawa and S.
Kamefuzi (North-Holland, Amsterdam, 1986), p.51.

[43] J. S. Bell, *Speakable and unspeakable in quantum mechanics,*
Cambridg University Press(1987).

[44] J. H. Christensen, J. Cronin, V. Fitch and R. Turly, *Phys.Rev.Lett.* 13
(1964) 138.

[45] E. P. Wigner, Remarks on the mind-body question. In *The scientist
speculates*, ed. by I.J.Good, (Heineman, London, 1961), reprinted in E.Wigner
Symmetries and reflections . Indiana University Press, Bloomington, and in
Quantum theory and measurement, ed. J.A.Wheeler and W.H.Zurek, Princeton
University Press 1983).