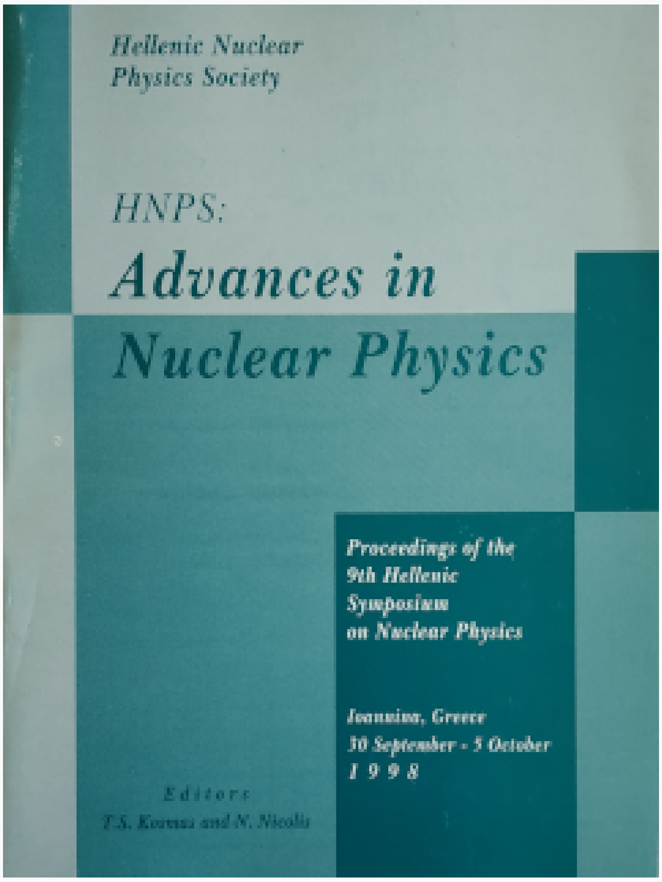
HNPS Advances in Nuclear Physics

Vol 9 (1998)

HNPS1998



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doi: 10.12681/hnps.2789

Description of the generalized momentum distribution in finite nuclei within the independent-particle shell model

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Abstract

A method for calculating the generalized momentum distribution in finite nuclear systems is presented and discussed within the context of the independent particle shell model. Application to the light closed-shell nuclei ^{16}O and ^{40}Ca is included.

1 Introduction

During the last few years great effort has been put in the treatment of final-state interactions (FSI) aiming to properly interprete the experimental data of quasi-elastic inclusive scatterings (e, e') [1], (p, p') [2], exclusive scatterings $(e, e'N)$[3], $(p, 2p)$[4], (γ, N)[5] etc. and to extract reliable values for quantities like momentum distribution $\eta(p)$, spectral function $S(k, E)$, transparency T and other quantities. The half diagonal two-body density matrix (2DM) $\rho_{2h}(\vec{r}_1, \vec{r}_2, \vec{r}_{1'})$ and its Fourier transformation in the variables $\vec{r}_1 - \vec{r}_{1'}$ and $\vec{r}_{1'} - \vec{r}_2$, namely the generalized momentum distribution $\eta(\vec{p}, \vec{Q})$ (GMD), have received increasing interest in this context, as they appear into quantitative microscopic treatments of the FSI of struck nucleons propagating through the nuclear medium. (See for example refs. [6,7] in the case of inclusive (e, e') scattering.) They are also key descriptors of the nucleon-nucleon correlations prevailing in the nuclear medium and are involved in fundamental sum rules that furnish insight into the nature of elementary excitations [8].

Both quantities $\eta(\vec{p}, \vec{Q})$ and $\rho_{2h}(\vec{r}_1, \vec{r}_2, \vec{r}_{1'})$ have been calculated for the system of nuclear matter (NM) by considering short-range correlations within the framework of variational theory [9,10]. In this work we are presenting a simple method for calculating the GMD of finite nuclear systems. It is a first step towards a more realistic calculation including correlations. The method has been applied to the magic nuclei ^{16}O and ^{40}Ca. A comparison of our calculations with the results for NM and for an infinite, non-interacting Fermi gas (FG) of appropriate wave number will reveal the role of the finite size of the nuclei and of the statistical correlations present in the model.

The method is reliable at certain kinematical regions of the momentum variables p and Q. That was the case when a similar method was applied for the calculation of the momentum distribution $\eta(p)$ and the charge form factor $F(Q)$. It can also be used to calculate other two-body quantities, such as the two-body momentum distribution $\eta(\vec{p}_1, \vec{p}_2)$ [11].

2 Brief description of the formalism for the GMD

In a system of A ($A \geq 2$) identical particles in a unit-normalized state $|\Psi\rangle$ the generalized momentum distribution $\eta(\vec{p}, \vec{Q})$ is defined as the expectation value

$$\eta(\vec{p}, \vec{Q}) = \langle\Psi| \sum_{\vec{s},\vec{s}'} \sum_{\vec{k}} a^\dagger_{\vec{k}+\vec{Q},\vec{s}'} a^\dagger_{\vec{p}-\vec{Q},\vec{s}} a_{\vec{p},\vec{s}} a_{\vec{k},\vec{s}'} |\Psi\rangle. \tag{1}$$

An alternative expression is obtained as the Fourier transform of the half-diagonal 2DM $\rho_{2h}(\vec{r}_1, \vec{r}_2, \vec{r}_{1'})$ which is given by:

$$\rho_{2h}(\vec{r}_1, \vec{r}_2, \vec{r}_{1'}) = A(A-1) \sum_{\vec{s}_1, \vec{s}_2} \int \Psi^*(x_1, x_2, \ldots, x_A) \times \Psi(x_{1'}, x_2, \ldots, x_A) \delta x_3 \ldots \delta x_A \tag{2}$$

($x_i \equiv \vec{r}_i, \vec{s}_i$).

$$\eta(\vec{p}, \vec{Q}) = \frac{1}{(2\pi)^3} \int \rho_{2h}(\vec{r}_1, \vec{r}_2, \vec{r}_{1'}) \epsilon^{-i\vec{p}\cdot(\vec{r}_1 - \vec{r}_{1'})} \epsilon^{-i\vec{Q}\cdot(\vec{r}_{1'} - \vec{r}_2)} \delta^3 r_1 \delta^3 r_{1'} \delta^3 r_2. \tag{3}$$

The role of $\eta(\vec{p}, \vec{Q})$ in the description of the FSI becomes more clear if we rewrite this quantity as

$$\eta(\vec{p}, \vec{Q}) = \langle\Psi| \hat{\rho}_Q a^\dagger_{\vec{p}-\vec{Q}} a_{\vec{p}} |\Psi\rangle - \eta(p) \tag{4}$$

where $\hat{\rho}_Q$ denotes the density fluctuation operator, $\hat{\rho}_Q = \sum_{\vec{k}} a^\dagger_{\vec{k}+\vec{Q}} a_{\vec{k}}$.

From the definitions of $\eta(\vec{p},\vec{Q})$, and $\rho_{2h}(\vec{r}_1,\vec{r}_2,\vec{r}_{1'})$ we can deduce the following properties for the GMD.

- If $\vec{r}_1=\vec{r}_{1'}$ then $\rho_{2h}(\vec{r}_1,\vec{r}_2,\vec{r}_1)=\rho_2(\vec{r}_1,\vec{r}_2)$, where $\rho_2(\vec{r}_1,\vec{r}_2)$ is the two-body distribution function. From eq. (3) we obtain the p-sum rule

$$\frac{1}{A}\int \eta(\vec{p},\vec{Q})\delta^3 p = AF^2(Q)+S(Q)-1 \tag{5}$$

where $F(Q)$ and $S(Q)$ are the elastic form factor and the static structure function respectively.

- The sequential relation for the half diagonal 2DM, $\int \rho_{2h}(\vec{r}_1,\vec{r}_2,\vec{r}_{1'})\delta^3 r_2 = (A-1)\rho_1(\vec{r}_1,\vec{r}_{1'})$, where $\rho_1(\vec{r}_1,\vec{r}_{1'})$ is the one-body density matrix, yields the corresponding relation for the GMD:

$$\eta(\vec{p},\vec{Q}=0)=(A-1)\eta(p). \tag{6}$$

In eq. (6) $\eta(p)$ is the momentum distribution. We use the normalization: $\frac{1}{(2\pi)^3}\int \eta(p)\delta^3 p = A$.

In the case of an infinitely extended, non-interacting boson gas being in its ground state, we have

$$\eta^{\mathrm{B}}(\vec{p},\vec{Q}) \propto A(A-1)\delta_{Q0}\delta_{p0}. \tag{7}$$

Similarly for an infinite, non-interacting fermion gas in the ground state with level degeneracy ν and Fermi wave number k_F we get

$$\rho^{\mathrm{F}}_{2h}(\vec{r}_1,\vec{r}_2,\vec{r}_{1'})=\rho^2\{\ell(k_{\mathrm{F}}r_{11'})-\frac{1}{\nu}\ell(k_{\mathrm{F}}r_{12})\ell(k_{\mathrm{F}}r_{1'2})\} \tag{8}$$

and

$$\frac{(2\pi k_{\mathrm{F}})^3}{6\pi^2}\eta^{\mathrm{F}}(\vec{p},\vec{Q})=A[(A-1)\delta_{Q0}\theta(k_{\mathrm{F}}-p)- \\ -(1-\delta_{Q0})\theta(k_{\mathrm{F}}-p)\theta(k_{\mathrm{F}}-|\vec{p}-\vec{Q}|)] \tag{9}$$

(ρ is the particle density and $\ell(x)$ the Slater function). The second, negative-signed terms in eqs. (8), (9) reflect the antisymmetric character of the wave-function. In the case of an infinite gas of bosons at non-zero temperature T occupying the same states as the fermions above one gets the same expressions as (8), (9) with a positive sign on the right hand side. Selected values of the GMD of an infinite, non-interacting fermi gas, as given by eq. (9) and of an infinite non-interacting boson gas, as described above, for $\vec{p}\parallel\vec{Q}$ ($\vec{Q}=Q_p\hat{p}$), are displayed in Fig. 1a.

As the GMD has been previously calculated in the case of Jastrow-correlated infinite nuclear matter using Fermi-hypernetted chain (FHNC) procedures [10], some results are shown for the sake of comparison in Fig. 1b.

3 Method of Calculation

Our method for calculating the GMD in the case of finite nuclear systems is based on the one used in [12–14] for the study of the nuclear form factor, the nuclear charge/matter and momentum distributions and the one-body density matrix in closed sub-shell nuclei. At first, a compact analytical expression for $\eta(\vec{p},\vec{Q})$ is derived in the context of the independent particle model (IPM). We consider a system of A identical non-interacting fermions in its ground state. The fermions occupy the lowest single-particle energy eigenstates $|n_j\rangle$ $(j = 1, 2, ..., A/\nu)$ described by the wavefunctions ψ_{n_j}. Then, $\eta(\vec{p},\vec{Q})$ is written as:

$$\eta(\vec{p},\vec{Q}) = AF(Q)\eta_1(\vec{p},\vec{p}-\vec{Q}) - \frac{1}{\nu}\int \eta_1(\vec{p},\vec{k}+\vec{Q})\eta_1(\vec{k},\vec{p}-\vec{Q})\delta^3 k \qquad (1)$$

or:

$$\eta(\vec{p},\vec{Q}) = AF(Q)\eta_1(\vec{p},\vec{p}-\vec{Q}) - \\ -\frac{1}{\nu}\sum_{n_j,n_k} \nu^2\tilde{\psi}_{n_j}(\vec{p})\tilde{\psi}^*_{n_k}(\vec{p}-\vec{Q})\int \psi^*_{n_j}(\vec{r})\psi_{n_k}(\vec{r})\epsilon^{i\vec{Q}\cdot\vec{r}}\delta^3 r \qquad (2)$$

where $\eta_1(\vec{p}_1,\vec{p}_{1'})$ is the one-body density matrix in momentum space. In the case of the nucleus, we have $A = Z$ or N for protons or neutrons respectively, and $\nu = 2$ for the degeneracy due to the nucleon spin. The second term on the right in either eq. (1) or (2) is an exchange term that arises from the statistical correlations among the non-interacting fermions, generated by the Pauli exclusion principle.

In order to obtain closed analytical expressions for $\eta(\vec{p},\vec{Q})$ we have assumed that the nucleons move in an isotropic harmonic oscillator potential and that the following approximations hold:

1. The center-of-mass and finite nucleon size corrections are small.
2. The Coulomb interaction (relevant for protons) is small.

First, we have ignored the spin-orbit coupling. For $\vec{p}$ parallel to $\vec{Q}$ ($\vec{Q} = Q_p\hat{p}$) the expression for $\eta(\vec{p},\vec{Q})$ for protons (the calculation is similar for neutrons) can be cast in the following form:

$$\eta(\vec{p},\vec{Q}) = \frac{b^3}{\pi^{3/2}}\epsilon^{-\frac{p^2b^2}{2}}\epsilon^{-\frac{w_p^2b^2}{2}}\epsilon^{-\frac{Q_p^2b^2}{4}} \times$$
$$\times \sum_{\mu=0}^{N_{\max}} (pb)^{\mu} \sum_{\mu'=0}^{N_{\max}} (w_p b)^{\mu'} \sum_{\rho=0}^{2N_{\max}} (Q_p b)^{\rho} (K_{\mu\mu'}\theta_{\frac{\rho}{2}} - C_{\mu\mu'\rho}) \tag{3}$$

where $\vec{w} \equiv \vec{p} - \vec{Q} = w_p\hat{p}$. $K_{\mu\mu'}$ and $\theta_{\frac{\rho}{2}}$ are rational numbers that enter the corresponding expressions of $\eta_1(\vec{p}_1, \vec{p}_{1'})$ and $F(Q)$ respectively. They are different from zero when the indices $\mu + \mu'$ and ρ are even. The coefficients $C_{\mu\mu'\rho}$ are equal to zero for $\mu + \mu' + \rho$ =odd. $N_{\max} = (2n + \ell)_{\max}$ is the number of energy quanta of the highest occupied $n\ell$ level. The corresponding expression for the momentum distribution $\eta(p)$ is

$$\eta(p) = \frac{b^3}{\pi^{3/2}}\epsilon^{-p^2b^2} \sum_{\lambda=0}^{N_{\max}} (pb)^{2\lambda} f_{\lambda}. \tag{4}$$

It has been shown that eq. (3) for $\eta(\vec{p},\vec{Q})$ satisfies the property (6).

The above expression (3) has been generalized by considering $\vec{p}$ not parallel to $\vec{Q}$ and including spin-orbit coupling. The result takes the form

$$\eta(\vec{p},\vec{Q}) = AF(Q)\eta_1(\vec{p},\vec{w}) - \frac{b^3}{\pi^{3/2}}\epsilon^{-\frac{p^2b^2}{2}}\epsilon^{-\frac{w^2b^2}{2}}\epsilon^{-\frac{Q^2b^2}{4}} \times$$
$$\times \sum_{\mu=0}^{N_{\max}} (pb)^{\mu} \sum_{\mu'=0}^{N_{\max}} (wb)^{\mu'} \sum_{\rho=0}^{2N_{\max}} (Qb)^{\rho} C_{\mu\mu'\rho}(\hat{p},\hat{w},\hat{Q}). \tag{5}$$

Analytical expressions can be found in ref. [15]. It is readily verified that for ℓ-closed shell nuclei, $\eta(\vec{p},\vec{Q})$ given by the above expressions equals the corresponding one derived by omitting the spin-orbit coupling.

4 Results and discussion

The analytical expression (3) has been applied to the calculation of the GMD of protons in the magic nuclei ^{16}O and ^{40}Ca for the special case that $\vec{p}$ and $\vec{Q}$ are parallel and both lie on the z-axis. Consideration of $\vec{\ell}\cdot\vec{s}$ coupling does not alter the results, since these nuclei have closed $n\ell$ shells. The HO parameter b for each nucleus has been determined in such a way as to reproduce the experimental value of the charge r.m.s. radius, $\langle r^2\rangle^{1/2}$ [16]. ($\langle r^2\rangle_{\exp} = 2.737$ fm and 3.45 fm for ^{16}O and ^{40}Ca respectively.) Some results are shown in Figs. 2 and 3. Comparing with the results for infinite homogeneous systems the following remarks may be drawn.

In the case of the systems of Fig. 1, one observes discontinuities at $Q_p = 0$ and at certain values of p, Q_p. More particularly, due to the infinite size of the systems, $\eta(p, Q_p)$ goes to infinity at $Q_p = 0$. For finite nucleon systems (see Figs. 2 and 3) the discontinuities are removed, while there exists a positive bump at $Q_p = 0$ for $p = 0$, shifted to higher values of Q_p for higher p. The GMD of the infinite boson($T \neq 0$)/fermion systems of Fig. 1 exhibits a finite positive/negative part at $Q_p > 0$ (Figs. 1a,b). It appears modified when correlations are present (nuclear matter, Figs.1b,3). Similarly, in the case of finite nuclei within the HO model, the second term in eq. (3) (the exchange term) gives rise to a negative part at $Q_p \geq 0$. It seems that the positive bump and the negative part at positive Q_p are bulk properties of the GMD and are due to fermi statistics.

As it has already been mentioned, in our present calculation of the GMD we have ignored the effect of dynamical correlations. The GMD of infinite nuclear matter has been calculated within a Fermi hypernetted-chain procedure in ref. [10]. Departures from ideal Fermi gas behaviour in certain kinematic domains provide signatures of the short-range correlations (namely, for $p < k_{\rm F}$ deviations from minus one or zero for $Q < p + k_{\rm F}$ and $Q > k_{\rm F}$ respectively, and for $p > k_{\rm F}$ deviations from zero). In Fig. 3 we make a comparison of the GMD per particle of ^{16}O and ^{40}Ca calculated within the harmonic oscillator model and of infinite nuclear matter at density $\rho^{\rm NM} = 0.182$ fm^{-3} ($k_{\rm F}^{\rm NM} = 1.3915$ fm^{-1}) as calculated in [10]. We have chosen the values of $p = 0$ and $\frac{3}{2}k_{\rm F}^{\rm NM}$ and present the GMD per particle as well as the exchange term per particle as a function of Q_p. The effect of dynamical correlations is observed in the above mentioned kinematical regions in the deviations of the results of ^{16}O and ^{40}Ca in the harmonic oscillator model from the ones of correlated nuclear matter (one should take into account that the values of the Fermi momentum for ^{16}O and ^{40}Ca are equal to 1.1 fm^{-1} and 1.2 fm^{-1} respectively).

The inclusion of correlations in our formalism can not be trivially done. One way is to use as input the results in nuclear matter [10] over a range of densities and apply a suitable local-density approximation. Another one is to consider Jastrow correlations and evaluate the GMD using some low-order approximation.

5 Summary and conclusions

In summary, the generalized momentum distribution $\eta(\vec{p}, \vec{Q})$, a momentum space transform of the half-diagonal two-body density matrix of finite, closed-shell nuclei in their ground state was studied in the independent-particle model with a harmonic oscillator basis. Closed analytical expressions have been extracted. The results for two examples, the magic nuclei ^{16}O and ^{40}Ca, exhibit

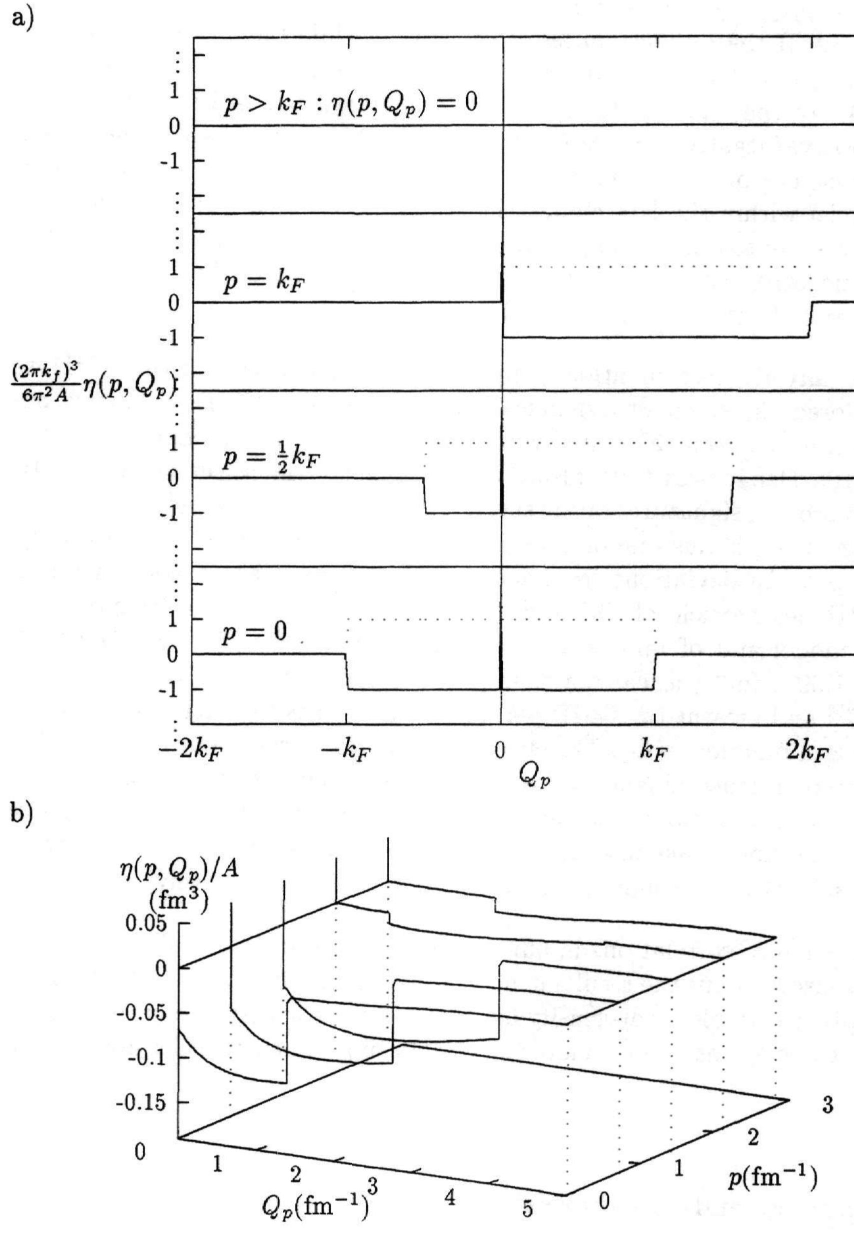

Fig. 1. The generalized momentum distribution per particle of infinite homogeneous systems. a) infinite, non-interacting fermi gas (solid lines) as compared to a gas of bosons occupying the same momentum states (dotted lines); b)nuclear matter (results of an FHNC calculation [10]).

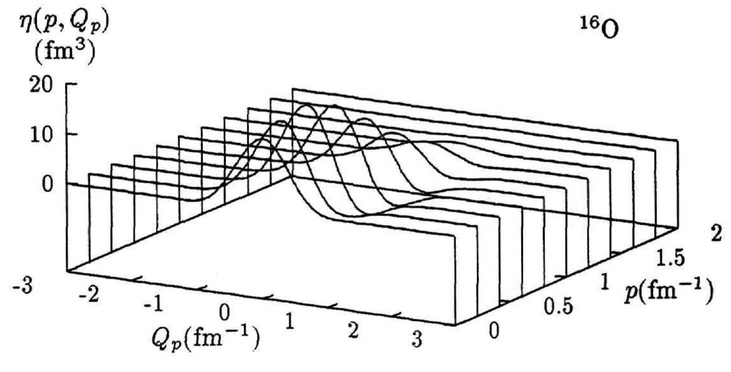

Fig. 2. The generalized momentum distribution of the ^{16}O nucleus in the harmonic oscillator model.

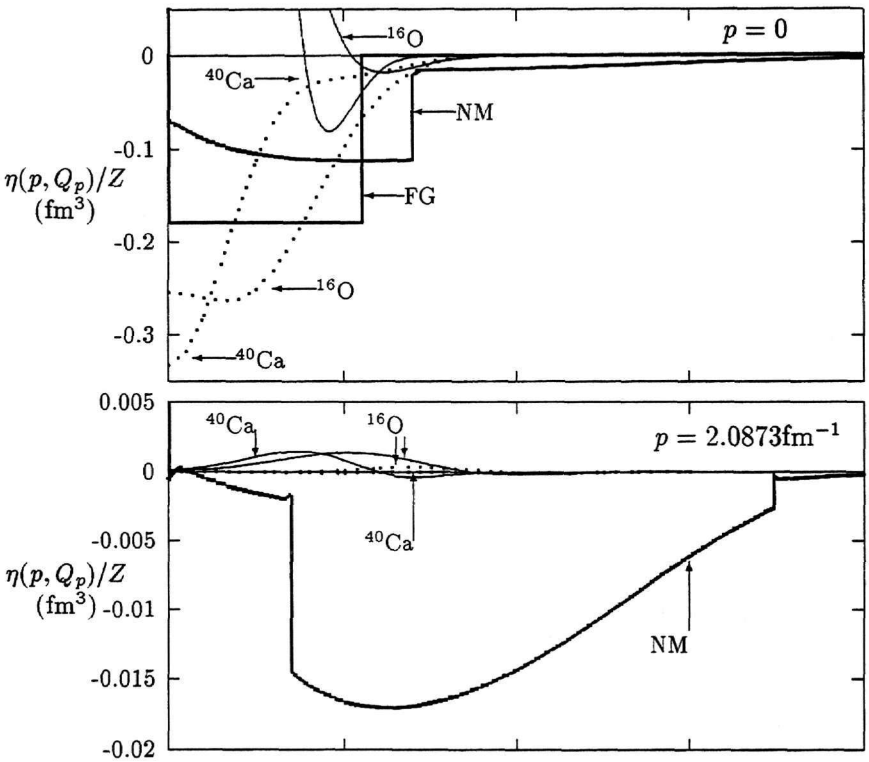

Fig. 3. Comparison of the generalized momentum distribution per particle of the ^{16}O and ^{40}Ca nuclei in the harmonic oscillator model (thin solid lines: total; dotted lines: exchange term) to that of nuclear matter (NM), as calculated in [10] (k_F^{NM} = 1.3915 fm^{-1}), and of an infinite non-interacting fermi gas (FG) with k_F^{FG} = 1.1 fm^{-1} (= fermi momentum of ^{16}O) for p = 0 and 2.0873 fm^{-1}. The GMD of this FG for $p = 2.0783$fm^{-1} (lower panel) equals zero and therefore is not displayed.

interesting features stemming from the finite size and the Fermi statistics. They are expected to be valid in certain regions of momenta p and Q where dynamical correlations do not play a significant role. Further investigations of $\eta(\vec{p}, \vec{Q})$ should consider in some way correlations.

References

[1] I. Sick. *Prog.Nucl.Part.Phys.* **34** (1995) 323; J. Arrington et al. *Phys.Rev.Lett.* **82** (1999) 2056.

[2] R.E. Chrien et al. *Phys.Rev.* **C21** (1980) 1014.

[3] D. Abbott et al. *Phys.Rev.Lett.* **80** (1998) 5072; J.H. Morrison et al. *Phys.Rev.* **C59** (1999) 221.

[4] I. Mardor et al. *Phys.Rev.Lett.* **81** (1998) 5085.

[5] G.J. Miller. *Nucl.Phys.* **A586** (1995) 125; B.E. Anderson et al. *Phys.Rev.* **C51** (1995) 2553.

[6] J.W. Clark and R.N. Silver. *Proceedings of the 5th International Conference on Nuclear Reaction Mechanisms*, ed. by E.Gadioli (Università degli Studi di Milano, Milan 1988) p.531.

[7] O. Benhar et al. *Phys.Rev.* **C44** (1991) 2328; O. Benhar et al. *Phys.Lett.* **359B** (1995) 8.

[8] S. Stringari. *Phys.Rev.* **B46** (1992) 2974.

[9] M.L. Ristig and J.W. Clark. *Phys.Rev.* **B41** (1990) 8611.

[10] E. Mavrommatis, M. Petraki, and J.W. Clark. *Phys.Rev.* **C51** (1995) 1849.

[11] A.N. Antonov, P.E. Hodgson, and I.Zh. Petkov. *Nucleon Correlations in Nuclei.* Springer–Verlag, 1993.

[12] T.S. Kosmas and J.D. Vergados. *Nucl.Phys.* **A510** (1990) 641.

[13] T.S. Kosmas and J.D. Vergados. *Nucl.Phys.* **A536** (1992) 72.

[14] P. Papakonstantinou. Diploma thesis, University of Athens, 1995.

[15] P. Papakonstantinou, E. Mavrommatis, and T.S. Kosmas. submitted for publication; P. Papakonstantinou. MSc thesis, University of Athens, 1998.

[16] H. de Vries, C.W. de Jager, and C. de Vries. *At.Data and Nucl.Data Tables* **36** (1987) 495.