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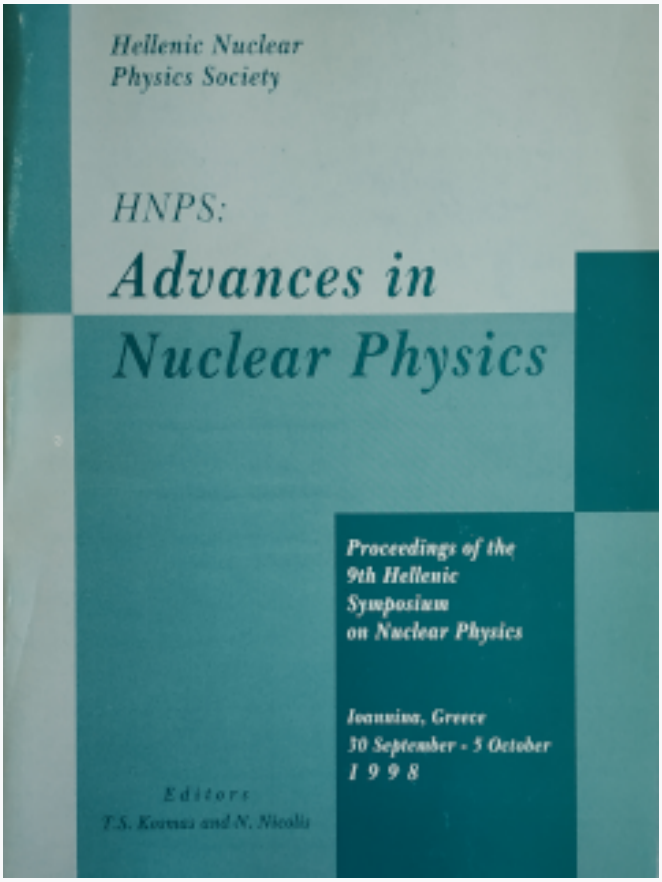

Systematic study of the effect of short range correlations on the form factors and densities of s-p-d shell nuclei

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Abstract

Analytical expressions of the one– and two–body terms in the cluster expansion of the charge form factors and densities of the s–p and s–d shell nuclei with N=Z are derived. They depend on the harmonic oscillator parameter b and the parameter β which originates from the Jastrow correlation function. These expressions are used for the systematic study of the effect of short range correlations on the form factors and densities and of the mass dependence of the parameters b and β. These parameters have been determined by fit to the experimental charge form factors. The inclusion of the correlations reproduces the experimental charge form factors at the high momentum transfers ($q \geq 2$ fm^{-1}). It is found that while the parameter β is almost constant for the closed shell nuclei, ^{4}He, ^{16}O and ^{40}Ca, its values are larger (less correlated systems) for the open shell nuclei, indicating a shell effect in the closed shell nuclei.

1 Introduction

The calculation of the charge form factors, $F_{ch}(q)$, and density distributions, $\rho_{ch}(r)$ of nuclei, is a challenging and appealing problem. A possibility to face this problem is by means of an independent particle model. This approach, which is particularly attractive because of its simplicity, fails to reproduce the high momentum transfer data from electron scattering in nuclei [1–8]. For this reason a modification of the single particle (SP) potentials has to be suitably made. In fact a short range repulsion in this potential seems advisable for light nuclei [9]. For example, with an harmonic oscillator (HO) potential having in addition an infinite soft core, the $F_{ch}(q)$ of ^4He can be well reproduced, but for the heavier nuclei, such as ^{12}C and ^{16}O, state dependent potentials seem necessary and even then the fit is not so good for higher q–values [9]. Another way is the introduction of the short range correlations (SRC) in the Slater

determinant. Many attempts have been made in this direction, concerning mainly light closed shell nuclei in the framework of the Born–approximation [1–8,10]. Ciofi Degli Atti, using the "single pair approximation" [1] and the Iwamoto–Yamada cluster expansion [2] in s–p shell nuclei, showed that the elastic electron scattering at high momentum transfers seems to give a strong indication of the presence of SRC in nuclei. Bohigas and Stringari [5] and Dal Ri et al [6] evaluated the effect of SRC on the one– and two–body densities by developing a low order approximation (LOA) in the framework of the Jastrow formalism. Stoitsov et al [8] generalized the model of Jastrow correlations, suggested by Bohigas and Stringari [5] within the LOA of Ref. [4], to heavier nuclei like ^{16}O, ^{36}Ar and ^{40}Ca reproducing very well the experimental data. The motivation of the present work is the systematic study of the effect of SRC on the s–p and s–d shell nuclei by completely avoiding the approximation made in earlier work [10] for open shell nuclei. General expressions for the $F_{ch}(q)$ and $\rho_{ch}(r)$ were found using the factor cluster expansion of Clark et al [11–13] and Jastrow correlation functions [14] which introduce SRC. These expressions are functionals of the SP wave functions and not of the wave functions of the relative motion of two nucleons as was the case in many previous works [1,7,10]. Because of that, it is easy to extrapolate them to the case of open shell nuclei and use them either for analytical calculations with HO wave functions or for numerical calculations when more realistic SP wave functions are used. An advantage of the present method is that the mass dependence of the HO parameter b (with the presence of correlations) and the correlation parameter β can be studied. These parameters have been determined, for the various s–p and s–d shell nuclei by fit of the theoretical $F_{ch}(q)$ to the experimental ones. It is found that while the parameter β is almost constant for the closed shell nuclei, ^{4}He, ^{16}O and ^{40}Ca, it takes larger values (less correlated systems) in the open shell nuclei, indicating a shell effect for the closed shells.

2 Correlated density distributions and form factors

If we denote the model operator, which introduces SRC, by $\mathcal{F}$, an eigenstate Φ of the model system corresponds to an eigenstate $\Psi = \mathcal{F}\Phi$ of the true system. Several restrictions can be made on the model operator $\mathcal{F}$, as for example, that it depends on (the spins, isospins and) relative co–ordinates and momenta of the particles in the system, it is a scalar with respect to rotations e.t.c. [15]. Further, it is required that $\mathcal{F}$ is translationally invariant and symmetrical in its argument $1 \cdots i \cdots A$ and possesses the cluster property. That is if any subset, $i_1 \cdots i_p$, of the particles is removed far from the rest, $i_{p+1} \cdots i_A$, $\mathcal{F}$ decomposes into a product of two factors, $\mathcal{F}(1 \cdots A) = \mathcal{F}(i_1 \cdots i_p)\ \mathcal{F}(i_{p+1} \cdots i_A)$ [13]. In the present work $\mathcal{F}$ is taken to be of the Jastrow–type [14]: $\mathcal{F} = \prod_{i<j}^{A} f(r_{ij})$, where $f(r_{ij})$ is the state-independent correlation function of the form:

$$f(r_{ij}) = 1 - \exp[-\beta(\mathbf{r}_i - \mathbf{r}_j)^2] \ . \tag{1}$$

The charge form factor of a nucleus, in Born–approximation, can be written: $F_{ch}(q) = f_p(q)\ f_{DF}(q)\ f_{CM}(q)\ F_p(q)$, where $f_p(q)$ and $f_{DF}(q)$ are the correction for the finite proton size and the Darwin–Foldy relativistic correction, respectively [16], $f_{CM}(q)$ is the Tassie–Barker [17] center–of–mass correction and $F_p(q)$ the point form factor of the nucleus which is the expectation value of the one–body operator,

$$\mathbf{O}_q \ = \ \sum_{i=1}^{A} \mathbf{o}_q(i) \ = \ \sum_{i=1}^{A} \exp[i\mathbf{q}\ \mathbf{r}_i] \ . \tag{2}$$

That is,

$$F_p(q) \ = \frac{\langle\Psi|\mathbf{O}_q|\Psi\rangle}{\langle\Psi|\Psi\rangle} \ = \ N\langle\Psi|\mathbf{O}_q|\Psi\rangle \ = \ N\langle\mathbf{O}_q\rangle \ , \tag{3}$$

where $N = \langle\Psi(r_1, r_2, \cdots, r_A)|\Psi(r_1, r_2, \cdots, r_A)\rangle^{-1}$ is the normalization factor which is determined so that $F_{ch}(0) = F_p(0) = 1$ or $4\pi \int_0^\infty \rho(r) r^2 dr = 1$.
The point density distribution has the form

$$\rho_p(r) = \frac{\langle\Psi|\mathbf{O}_r|\Psi\rangle}{\langle\Psi|\Psi\rangle} \ = \ N\langle\Psi|\mathbf{O}_r|\Psi\rangle \ = \ N\langle\mathbf{O}_r\rangle \ , \tag{4}$$

where

$$\mathbf{O}_r \ = \ \sum_{i=1}^{A} \mathbf{o}_r(i) \ = \ \sum_{i=1}^{A} \delta(\mathbf{r} - \mathbf{r}_i) \ . \tag{5}$$

In order to evaluate the point density distribution, $\rho_p(r)$, we consider, first, the generalized normalization integral,

$$I(\alpha) = \langle\Psi| \exp[\alpha I(0) \mathbf{O}_r]|\Psi\rangle \ , \tag{6}$$

corresponding to the operator $\mathbf{O}_r$, from which we have,

$$\langle\mathbf{O}_r\rangle = \left[\frac{\partial \ln I(\alpha)}{\partial\alpha}\right]_{\alpha=0} . \tag{7}$$

For the cluster analysis of equation (7), following the factor cluster expansion of Ristig and Clark [11–13], we consider the sum–product integrals, $I_i(\alpha)$, $I_{ij}(\alpha)$, $\cdots$, for the subsystems of the A–nucleon system and a factor cluster decomposition of these integrals. The expectation value of the density distribution operator is written in the form,

$$\langle\mathbf{O}_r\rangle = \langle\mathbf{O}_r\rangle_1 + \langle\mathbf{O}_r\rangle_2 + \cdots + \langle\mathbf{O}_r\rangle_A \ , \tag{8}$$

where

$$\langle \mathbf{O}_r \rangle_1 = \sum_{i=1}^{A} \left[\frac{\partial \ln I_i(\alpha)}{\partial \alpha} \right]_{\alpha=0} = \sum_{i=1}^{A} \langle i \mid \mathcal{F}_1^{\dagger} \mathbf{o}_r(1) \mathcal{F}_1 \mid i \rangle \ , \tag{9}$$

$$\langle \mathbf{O}_r \rangle_2 = \sum_{i<j}^{A} \frac{\partial}{\partial \alpha} \left[\ln I_{ij}(\alpha) - \ln I_i(\alpha) - \ln I_j(\alpha) \right]_{\alpha=0} \tag{10}$$

$$= \sum_{i<j}^{A} \langle ij \mid \mathcal{F}_{12}^{\dagger} (\mathbf{o}_r(1) + \mathbf{o}_r(2)) \mathcal{F}_{12} \mid ij \rangle_a - \sum_{i<j}^{A} [\langle i \mid \mathbf{o}_r(1) \mid i \rangle + \langle j \mid \mathbf{o}_r(2) \mid j \rangle],$$

and so on. $\mathcal{F}_1$ is chosen to be the identity operator. The cluster expansion establishes the separation of one–body, two–body, $\cdots$, A–body correlation effects on the density. Three– and many–body terms will be neglected in the present analysis. After some algebra the density $\rho_p(r)$ takes the form:

$$\rho_p(r) \approx N[\langle \mathbf{O}_r \rangle_1 - 2O_{22}(r, \beta) + O_{22}(r, 2\beta)] \ . \tag{11}$$

The terms $\langle \mathbf{O}_r \rangle_1$ and $O_{22}(r, z)$ and the density $\rho_p(r)$ can be expressed also in the convenient form:

$$\langle \mathbf{O}_r \rangle_1 = \rho_{SD}(\mathbf{r}) \ , \tag{12}$$

$$O_{22}(r, z) = \int g(r, r_2, z)[\rho_{SD}(\mathbf{r})\rho_{SD}(\mathbf{r}_2) - \rho_{SD}^2(\mathbf{r}, \mathbf{r}_2)]\mathbf{dr}_2 \ , \tag{13}$$

$$\rho_p(r) \approx N\left[\rho_{SD}(\mathbf{r}) + \int [g(r, r_2, 2\beta) - 2g(r, r_2, \beta)][\rho_{SD}(\mathbf{r})\rho_{SD}(\mathbf{r}_2) - \rho_{SD}^2(\mathbf{r}, \mathbf{r}_2)]\right] \mathbf{dr}_2 \tag{14}$$

where $\rho_{SD}(\mathbf{r}_1, \mathbf{r}_2)$ is the uncorrelated density matrix associated with the Slater determinant,

$$\rho_{SD}(\mathbf{r}_1, \mathbf{r}_2) = \sum_{i=1}^{A} \phi_i^*(\mathbf{r}_1)\phi_i(\mathbf{r}_2) \tag{15}$$

and

$$g(r_1, r_2, z) = \exp[-zr_1^2] \exp[-zr_2^2] \exp[2zr_1 r_2 \cos \omega_{12}] \ , \quad z = \beta, 2\beta \ . \tag{16}$$

The diagonal elements of this gives the one body density distribution, $\rho_{SD}(\mathbf{r}_1) = \rho_{SD}(\mathbf{r}_1, \mathbf{r}_1)$.
It should be noted that, a similar expression for $\rho_p(r)$, given by equation (14), was derived by Gaudin et al. [4] in the framework of LOA. This expansion contains one– and two–body terms and a part of the three–body term which

was chosen so that the normalization of the wave function was preserved. Expression (14) of the present work has only one– and two–body terms and the normalization of the wave function is preserved by the normalization factor N. In the above expression of $\rho_p(r)$, the one–body contribution to the density is well known and given by the equation,

$$\langle \mathbf{O}_r \rangle_1 = 4 \sum_{nl} \eta_{nl}(2l+1) \frac{1}{4\pi} \phi^*_{nl}(r) \phi_{nl}(r) \ , \tag{17}$$

where η_{nl} is the occupation probability of the state nl (0 or 1 in the case of closed shell nuclei) and $\phi_{nl}(r)$ is the radial part of the SP wave function.
An expression for the two–body term is usually found by making a transformation to the relative and the center–of–mass coordinates of the two interacting nucleons [1,7,10]. This is because the Jastrow function $f(r_{ij})$ depends on the relative coordinates of the two nucleons. Here, the expression for the two–body term, that is of the term $O_{22}(r,z)$, will be found by expanding the factor $\exp[2zr_1r_2\cos\omega_{12}]$ in spherical harmonics. That is

$$\exp[2zr_1r_2\cos\omega_{12}] = 2\pi \sum_{km_k} U_k(2zr_1r_2) Y^*_{km_k}(\Omega_1) Y_{km_k}(\Omega_2) \ , \tag{18}$$

where $U_k(2zr_1r_2) = \int_{-1}^{1} \exp(2zr_1r_2\cos\omega_{12}) P_k(\cos\omega_{12}) \mathrm{d}(\cos\omega_{12}) = 2i_k(2zr_1r_2)$, $i_k(x)$ is the modified spherical Bessel function. Using the algebra of spherical harmonics, the term: $O_{22}(r,z)$ takes the form:

$$O_{22}(r,z) = 4 \sum_{n_il_i,n_jl_j} \eta_{n_il_i}\eta_{n_jl_j}(2l_i+1)(2l_j+1) \times \left[4A^{n_il_in_jl_jk}_{n_il_in_jl_j}(r,z) - \sum_{k=0}^{l_i+l_j} \langle l_i0l_j0 \mid k0\rangle^2 A^{n_jl_jn_il_ik}_{n_il_in_jl_j}(r,z) \right] , \tag{19}$$

where

$$A^{n_3l_3n_4l_4k}_{n_1l_1n_2l_2}(r,z) = \frac{\phi^*_{n_1l_1}(r)\phi_{n_3l_3}(r)e^{-zr^2}}{4\pi} \int_0^\infty \phi^*_{n_2l_2}(x)\phi_{n_4l_4}(x) e^{-zx^2} i_k(2zrx) x^2 \mathrm{d}x \tag{20}$$

Thus the expression of the term $O_{22}(r,z)$ depends on the SP wave functions and so it is suitable to be used either for analytical calculations with the HO potential or for numerical calculations with more realistic SP potentials. Expressions (17) and (19) were derived for the closed shell nuclei with N=Z, where η_{nl} is 0 or 1. For the open shell nuclei (with N=Z) we use the same expressions, where now: $0 \leq \eta_{nl} \leq 1$. In this way the mass dependence of the correlation parameter β and the HO parameter b can be studied. Finally, using the known values of the Clebsch–Gordan coefficients and provided that

$\phi^*_{nl}(r) = \phi_{nl}(r)$, i.e.: $A^{n_3l_3n_4l_4k}_{n_1l_1n_2l_2}(r,z) = A^{n_1l_1n_2l_2k}_{n_3l_3n_4l_4}(r,z)$, equation (19), for the case of s–p and s–d shell nuclei, takes the form:

$$
\begin{aligned}
O_{22}(r,z) = 4\Big[& 3A^{00000}_{0000}\eta^2_{1s} + \left(33A^{01010}_{0101} - 6A^{01012}_{0101}\right)\eta^2_{1p} + 3A^{10100}_{1010}\eta^2_{2s} + \Big(95A^{02020}_{0202} \\
& - \frac{50}{7}A^{02022}_{0202} - \frac{90}{7}A^{02024}_{0202}\Big)\eta^2_{1d} + \left(12A^{00010}_{0001} + 12A^{01000}_{0100} - 6A^{01001}_{0001}\right)\eta_{1s}\eta_{1p} \\
& + \left(20A^{00020}_{0002} + 20A^{02000}_{0200} - 10A^{02002}_{0002}\right)\eta_{1s}\eta_{1d} + \left(60A^{01020}_{0102} + 60A^{02010}_{0201}\right. \\
& \left. - 12A^{02011}_{0102} - 18A^{02013}_{0102}\right)\eta_{1p}\eta_{1d} + \left(4A^{00100}_{0010} + 4A^{10000}_{1000} - 2A^{10000}_{0010}\right)\eta_{1s}\eta_{2s} \\
& + \left(12A^{01100}_{0110} + 12A^{10010}_{1001} - 6A^{10011}_{0110}\right)\eta_{1p}\eta_{2s} \\
& + \left(20A^{02100}_{0210} + 20A^{10020}_{1002} - 10A^{10022}_{0210}\right)\eta_{1d}\eta_{2s}\Big] \ , \qquad (21)
\end{aligned}
$$

where $A^{n_3l_3n_4l_4k}_{n_1l_1n_2l_2} \equiv A^{n_3l_3n_4l_4k}_{n_1l_1n_2l_2}(r,z)$. The point form factor $F_p(q)$ can be derived in two equivalent ways. The first one is to follow the same cluster expansion as in the case of the density distribution and the second one is to take the Fourier transform of the density distribution $\rho_p(r)$, $F_p(q) = \int \exp[i\mathbf{q}\,\mathbf{r}]\rho_p(r)\mathbf{dr}$. In both cases, the form factor takes the following form:

$$
F_p(q) \approx N[\langle \mathbf{O}_q \rangle_1 - 2\tilde{O}_{22}(q,\beta) + \tilde{O}_{22}(q,2\beta)] \ . \qquad (22)
$$

In the above expression, the one–body term is given by the equation

$$
\langle \mathbf{O}_q \rangle_1 = 4\sum_{nl} \eta_{nl}(2l+1) \int_0^\infty \phi^*_{nl}(r)\phi_{nl}(r) j_0(qr) r^2 dr \ , \qquad (23)
$$

while the two–body term $\tilde{O}_{22}(q,z)$ is given by the right hand side of equations (19) and (21) by replacing the matrix elements $A^{n_3l_3n_4l_4k}_{n_1l_1n_2l_2}(r,z)$ by $\tilde{A}^{n_3l_3n_4l_4k}_{n_1l_1n_2l_2}(q,z)$ given by the equation,

$$
\begin{aligned}
\tilde{A}^{n_3l_3n_4l_4k}_{n_1l_1n_2l_2}(q,z) = & \int_0^\infty \phi^*_{n_1l_1}(r_1)\phi_{n_3l_3}(r_1)\exp[-zr_1^2] j_0(qr_1) r_1^2 \mathrm{d}r_1 \times \\
& \int_0^\infty \phi^*_{n_2l_2}(r_2)\phi_{n_4l_4}(r_2)\exp[-zr_2^2] i_k(2zr_1r_2) r_2^2 \mathrm{d}r_2 \ . \qquad (24)
\end{aligned}
$$

3 Analytical expressions

In the case of the HO wave functions analytical expressions of the one–body term and of the matrix elements $A^{n_3l_3n_4l_4k}_{n_1l_1n_2l_2}(r,z)$ and $\tilde{A}^{n_3l_3n_4l_4k}_{n_1l_1n_2l_2}(q,z)$, defined by equation (20) and (24), can be found. From these expressions, the analytical expressions of the terms $O_{22}(r,z)$ and $\tilde{O}_{22}(q,z)$, defined by equation (21), can

also be found. The expression of the one–body term of the density and form factor has the form:

$$\langle \mathbf{O}_x \rangle_1 = C\ \mathrm{e}^{-\xi^2}\ \sum_{k=0}^{2} C_{2k}\xi^{2k}\ , \quad x = r, q\ , \tag{1}$$

where for the $\langle \mathbf{O}_r \rangle_1$, $\xi = r/b$, $C = \frac{2}{\pi^{3/2}b^3}$ and the coefficients C_{2k} are:

$$C_0 = 2\eta_{1s} + 3\eta_{2s},\ \ C_2 = 4(\eta_{1p} - \eta_{2s}),\ \ C_4 = \frac{4}{3}(2\eta_{1d} + \eta_{2s})$$

while for the $\langle \mathbf{O}_q \rangle_1$ the corresponding quantities are: $\xi = bq/2$, $C = 2$ and

$$C_0 = 2(\eta_{1s} + \eta_{2s} + 3\eta_{1p} + 5\eta_{1d}),\ C_2 = -\frac{4}{3}(3\eta_{1p} + 10\eta_{1d} + 2\eta_{2s}),\ C_4 = \frac{4}{3}(2\eta_{1d} + \eta_{2s})$$

The analytical expression of the matrix element $A^{n_3l_3n_4l_4k}_{n_1l_1n_2l_2}(r,z)$, which are given by equation (20), has the form:

$$\begin{aligned} A^{n_3l_3n_4l_4k}_{n_1l_1n_2l_2}(r,z) = &\left(\prod_{i=1}^{4} N_{n_il_i}\right) \frac{y^k}{16\sqrt{\pi}b^3}\ \xi^{l_1+l_3+k}\ L_{n_1}^{l_1+\frac{1}{2}}(\xi^2)\ L_{n_3}^{l_3+\frac{1}{2}}(\xi^2) e^{-\frac{1+2y}{1+y}\xi^2} \times \\ &\sum_{w=0}^{n_2}\sum_{s=0}^{n_4} \frac{(-1)^{w+s}}{w!s!} \begin{pmatrix} n_2 + l_2 + \frac{1}{2} \\ n_2 - w \end{pmatrix} \begin{pmatrix} n_4 + l_4 + \frac{1}{2} \\ n_4 - s \end{pmatrix} \times \\ &\frac{(\frac{1}{2}(l_2 + l_4 - k) + w + s)!}{(1+y)^{\frac{1}{2}(l_2+l_4+k+3)w+s}}\ L^{k+\frac{1}{2}}_{\frac{1}{2}(l_2+l_4-k)+w+s}\left(\frac{-y^2}{1+y}\xi^2\right) \end{aligned} \tag{2}$$

where $\xi = r/b$ and $y = zb^2$ ($z = \beta,\ 2\beta$) and $N_{nl} = (2n!/\Gamma(n+l+3/2))^{1/2}$. The above expression of $A^{n_3l_3n_4l_4k}_{n_1l_1n_2l_2}(r,z)$ is of the form: $f(\xi^2)\exp[-\frac{1+2y}{1+y}\xi^2]$, where $f(\xi^2)$ is a polynomial of ξ^2. The substitution of the expression of $A^{n_3l_3n_4l_4k}_{n_1l_1n_2l_2}(r,z)$ to the expression of $O_{22}(r,z)$, which is given by equation (21), leads to the analytical expression of the two–body term of the density. This expression, for the case of s–p and s–d shell nuclei, is again of the form $f(\xi^2)\exp[-\frac{1+2y}{1+y}\xi^2]$ where $f(\xi^2)$ is a polynomial of ξ^2 of order 4.
The corresponding analytical expression of the matrix element $\tilde{A}^{n_3l_3n_4l_4k}_{n_1l_1n_2l_2}(q,z)$, which are given by equation (24), has similar form with that of equation (2) where $\xi = qb/2$.
The analytical expressions of the form factor and the density, which were found previously, will be used in section 4 for the fit of the theoretical charge form factors to the experimental ones and for the calculations of the charge density distributions for various N=Z (s–p and s–d shell) nuclei.

4 Results and discussion

The calculations of the charge form factors for various s–p and s–d shell nuclei, with N=Z, have been carried out on the basis of equation (22) and the analytical expressions of the one– and two–body terms which were given in section 3. Two cases have been examined, named Case 1 and Case 2, which correspond to the analytical calculations with HO wave functions without and with SRC respectively. In Case 1 there is one free parameter, the HO parameter b, while in Case 2 there are two free parameters, the parameter b and the correlation parameter β. The parameters, in both cases, have been determined, for each nucleus separately, by least squares fit to the experimental $F_{ch}(q)$. The best fit values of the parameters as well as of the values of χ^2, are displayed in Table I. In the same table the calculated root mean square (RMS) charge radii $\langle r^2_{ch}\rangle^{1/2}$ and the contribution of the SRC to them, $\langle r^2\rangle_2 = \langle r^2_{ch}\rangle - \langle r^2_{ch}\rangle_1/A$, are displayed and compared with the corresponding experimental RMS radii. It is noted that $\langle r^2\rangle_2$ is independent from the center–of–mass correction and finite proton size.

From the values of χ^2, which have been found in Cases 1 and 2 (see Table I) it can be seen that the inclusion of the correlations improves the fit of the form factor of all the nuclei we have examined. Almost all the diffraction minima which are known from the experimental data are reproduced in the correct place. There is a disagreement in the fit of the form factor of the open shell nuclei ^{24}Mg, ^{28}Si and ^{32}S for $q \approx 3.5$ fm^{-1} where it seems that there is a third diffraction minimum in the experimental data, which cannot be reproduced in both cases.

It is seen from Table I that the parameter b has the same behavior as function of the mass number A in the HO and the correlated model, while the following inequality holds: $b(HO) > b(SRC)$. This is due to the fact that the introduction of SRC tends to increase the relative distance of the nucleons i.e. the size of the nucleus, while the parameter b, which is (on the average) proportional to the (experimentally fixed) radius of the nucleus, should become smaller. It is also noted that the difference: $\Delta b = b(HO) - b(SRC)$, is almost constant for the open shell nuclei and it is larger for the closed shell nuclei ^{4}He, ^{16}O and ^{40}Ca. This can also be seen from figure 1 where the values of Δb versus the mass number A has been plotted. The behavior of Δb as function of A indicates that the SRC are stronger for the closed shell nuclei than in the open shell ones. In Figure 2 the values of the correlation parameter β versus the mass number A have been plotted. From this figure it is seen that the parameter β is almost constant for ^{4}He, ^{16}O and ^{40}Ca and takes larger values (less correlated systems) in the open shell nuclei. The behavior of the two parameters, b and β, indicates that there should be a shell effect in the case of closed shell nuclei. That is, there is a shell effect not only on the values of the harmonic oscillator spacing $\hbar\omega$, as has been noted in Refs. [23,24] but also on the values of the correlation parameter β.

In the above analysis, the nuclei ^{24}Mg, ^{28}Si and ^{32}S were treated as 1d shell

Table 1
The values of the parameters b and β, of the χ^2, and of the RMS charge radii $\langle r_{ch}^2\rangle^{1/2}$: contribution of the one body density (column HO), contribution of SRC (column SRC) and of the total RMS charge radii (column Total), for various s–p and s–d shell nuclei, determined by fit to the experimental $F_{ch}(q)$. Case 1 refers to the HO form factor, Case 2 when SRC are included. The experimental RMS charge radii are from Ref. [18], while the experimental $F_{ch}(q)$ for ^4He is from Ref. [19], for ^{12}C and ^{16}O from Ref. [20], for ^{24}Mg, ^{28}Si and ^{32}S from Ref. [21] and for ^{40}Ca from Ref. [22].

Case	Nucleus	b [fm]	β [fm^{-2}]	χ^2	$\langle r_{ch}^2\rangle^{1/2}$ [fm]			
					HO	SRC	Total	Exper.
1	^4He	1.4320	–	31	1.7651	–	1.7651	1.676(8)
2	^4He	1.1732	2.3126	3.5	1.5353	0.5277	1.6234	
1	^{12}C	1.6251	–	177	2.4901	–	2.4901	2.471(6)
2	^{12}C	1.5923	3.7051	110	2.4463	0.2566	2.4597	
1	^{16}O	1.7610	–	199	2.7377	–	2.7377	2.730(25)
2	^{16}O	1.6507	2.4747	120	2.5853	0.7070	2.6802	
1	^{24}Mg	1.8495	–	188	3.1170	–	3.1170	3.075(15)
2	^{24}Mg	1.8270	6.6112	161	3.0823	0.3009	3.0969	
1	^{28}Si	1.8941	–	148	3.2570	–	3.2570	3.086(18)
2	^{28}Si	1.8738	8.2245	114	3.2249	0.2438	3.2341	
1	^{32}S	2.0016	–	320	3.4830	–	3.4830	3.248(11)
2	^{32}S	1.9810	9.1356	270	3.4497	0.2114	3.4561	
1	^{36}Ar	1.8800	–	–	3.3270	–	3.3270	3.327(15)
2	^{36}Ar	1.8007	2.2937	–	3.1970	0.9470	3.3343	
1	^{40}Ca	1.9453	–	229	3.4668	–	3.4668	3.479(3)
2	^{40}Ca	1.8660	2.1127	160	3.3353	1.1115	3.5156	

nuclei. We have also considered the Case 2* in which the occupation probability η_{2s} of the nuclei ^{24}Mg, ^{28}Si and ^{32}S is taken to be a free parameter besides the other two parameters b and β. We found that while the χ^2 values become better, comparing to those of Case 2, the third diffraction minimum is not reproduced either and the behavior of the parameters b and β as functions of the mass number A remains the same.

From the determined mass dependence of the parameters b and β, the values of these parameters for other s–p or s–d shell nuclei can found. This has been done in the nucleus ^{36}Ar treated as 1d closed shell nucleus. As there are no experimental data for $F_{ch}(q)$ for high q values, the value of the parameter β

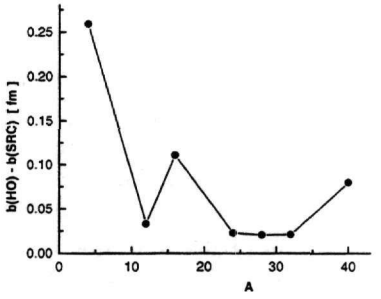

Fig. 1. The difference $\Delta b = b(HO) - b(SRC)$ versus the mass number A. $b(HO)$ and $b(SRC)$ are the HO parameter in Cases 1 (HO without SRC) and 2 (HO with SRC) respectively.

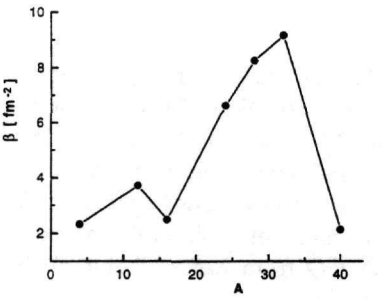

Fig. 2. The correlation parameter β versus the mass number A in Case 2 (HO+SRC).

is taken to be the mean value of the corresponding values of ^{16}O and ^{40}Ca, that is : $\beta_{36} = 2.2937$ fm^{-2}, while the parameter b is determined assuming that $\Delta b_{36} = \Delta b_{40}$ where $\Delta b_A = b_A(HO) - b_A(SRC)$. Using the values of the parameters $b_{40}(HO) = 1.9453$ fm, $b_{40}(SRC) = 1.8660$ fm from Table I and choosing the parameter $b_{36}(HO) = 1.8800$ fm in order to reproduce the experimental RMS charge radius of ^{36}Ar ($< r^2 >^{1/2}_{exp} = 3.327 \pm 15$ fm [18]) the value $b_{36}(SRC) = 1.8007$ fm was found. These values of β_{36} and $b_{36}(SRC)$ have been used for the calculations of the correlated $F_{ch}(q)$ and $\rho_{ch}(r)$ of ^{36}Ar. The calculated RMS charge radius, $< r^2 >^{1/2} = 3.3343$ fm, which was found, is within the experimental error.

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