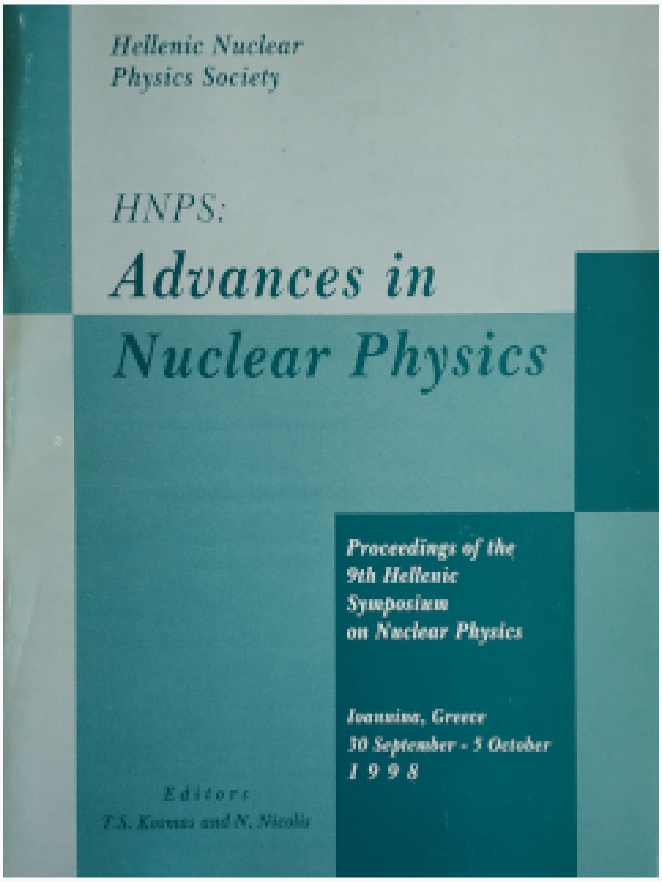
HNPS Advances in Nuclear Physics

Vol 9 (1998)

HNPS1998



New developments in neutrino physics

W. M. Alberico, S. M. Bilenky

doi: [10.12681/hnps.2775](https://doi.org/10.12681/hnps.2775)

New developments in neutrino physics

W.M. Alberico [a,b] and S.M. Bilenky [c,d]

[a] *INFN, Sezione di Torino, via P. Giuria 1, I-10125 Torino, Italy*

[b] *Dipartimento di Fisica Teorica, Università di Torino, Via P. Giuria 1, I-10125 Torino, Italy*

[c] *Joint Institute for Nuclear Research, 141980 Dubna, Moscow Region, Russia*

[d] *Institute for Theoretical Physics, University of Vienna, Boltzmanngasse 5, A–1090 Vienna, Austria*

Abstract

A review of the problems of neutrino mass, mixing and oscillations is given. Possible phenomenological schemes of neutrino mixing are discussed. The most important consequences of neutrino mixing–neutrino oscillations are considered in some detail. The data of atmospheric, solar and LSND experiments are discussed. The results of phenomenological analysis of the data under the assumption of the mixing of three and four massive neutrinos are briefly presented.

1 Introduction

The strong evidence in favor of the oscillations of atmospheric neutrinos, which was recently obtained by the Super-Kamiokande Collaboration [1], has attracted the attention of many physicists working on neutrinos. The problem of masses and mixing of neutrinos is at present the central problem of elementary particle physics. The investigation of this problem is one of the major tools of searching for new physics beyond the standard model. Vergados [2] understood this many years ago and gave very important contributions to such investigations.

Indications in favor of neutrino oscillations were obtained in all modern atmospheric-neutrino experiments (IMB [3], Soudan-2 [4], Kamiokande [5], Super-Kamiokande [1] and MACRO [6]), in all solar-neutrino experiments (Homestake [7], GALLEX [8], SAGE [9], Kamiokande [10], Super-Kamiokande [11]) and in the accelerator LSND experiment [12].

From all existing data it follows that there are three different scales of neutrino mass-squared differences, Δm^2: $\Delta m^2_{\text{solar}} \simeq 10^{-5}$ eV^2 (MSW) or 10^{-10} eV^2 (vacuum oscillations), $\Delta m^2_{\text{atm}} \simeq 10^{-3}$ eV^2 and $\Delta m^2_{\text{LSND}} \simeq 1$ eV^2. This implies that at least four massive neutrinos exist in nature, i.e. the number of massive neutrinos is larger than the number of flavor neutrinos (ν_e, ν_μ, ν_τ). Thus, if future neutrino-oscillation experiments confirm the existing results, it will imply that neutrino mixing and quark mixing are of a different origin.

Many new neutrino-oscillation experiments are in preparation. The region of Δm^2 of atmospheric neutrinos (Δm^2_{atm}) will be investigated in the long-baseline (LBL) experiments MINOS [13], ICARUS [14], OPERA [15] and others, and the solar-neutrino region ($\Delta m^2_{\text{solar}}$) in SNO [16], Borexino [17] and the reactor LBL experiment KAMLAND [18]. Finally, the LSND region of Δm^2 (Δm^2_{LSND}) will be investigated in the future short-baseline (SBL) experiment BooNE [19].

Neutrino masses, mixing and nature (Dirac or Majorana) are of fundamental importance for the theory. It is a common belief (see [20]) that neutrino masses and mixing are generated by a mechanism beyond the standard model. The main reason for that is the experimental fact that neutrino masses are much smaller than the masses of all the other fundamental fermions (leptons and quarks). The full understanding of the origin of neutrino masses and mixing will require, however, many new experiments.

Neutrinos are very important in astrophysics: massive neutrinos are plausible candidates for hot dark matter particles, the number of neutrino species plays a crucial role in the big bang nucleosynthesis, and so on. We will not discuss here these relevant issues.

In Sect. 2 we will consider the general phenomenological framework for neutrino masses and mixing. In Sect. 3 we will discuss neutrino oscillations. In Sect. 4 the latest experimental data will be examined. Finally in Sect. 5 the analysis of the data will be presented.

2 Neutrino Mixing

The neutrinos ν_e, ν_μ, ν_τ, which are produced in weak processes like pion and muon decays, nuclear beta decays etc., are called *flavor neutrinos*. In the interaction with nucleons a flavor neutrino ν_ℓ ($\ell = e, \mu, \tau$) produces the lepton ℓ^- and hadrons (CC) or the same neutrino ν_ℓ and hadrons (NC). From all the available data it follows that the interaction of flavor neutrinos is perfectly described by the Lagrangian of the standard model

$$\mathcal{L}_{\mathrm{I}} = \left(-\frac{g}{2\sqrt{2}} j_\alpha^{\mathrm{CC}} W^\alpha + \mathrm{h.c.} \right) - \frac{g}{2\cos\theta_{\mathrm{W}}} j_\alpha^{\mathrm{NC}} Z^\alpha \,. \tag{1}$$

Here the charged (j_α^{CC}) and neutral (j_α^{NC}) currents are given by

$$j_\alpha^{\mathrm{CC}} = 2 \sum_{\ell=e,\mu,\tau} \overline{\nu}_{\ell L} \gamma_\alpha \ell_L + \dots \,, \tag{2}$$

$$j_\alpha^{\mathrm{NC}} = \sum_{\ell=e,\mu,\tau} \overline{\nu}_{\ell L} \gamma_\alpha \nu_{\ell L} + \dots \,. \tag{3}$$

In (1) W^α and Z^α are the fields of the vector bosons $W^\pm$ and Z^0, θ_{W} is the weak mixing angle and g is the coupling constant. From LEP data it follows that the number of light flavor neutrinos, $n_{\nu_{\mathrm{f}}}$, is equal to three [21]

$$n_{\nu_{\mathrm{f}}} = 2.994 \pm 0.012 \,. \tag{4}$$

The Lagrangian (1) conserves the additive electron L_e, muon L_μ and tauon L_τ lepton numbers

$$\sum L_e = \mathrm{const}, \qquad \sum L_\mu = \mathrm{const}, \qquad \sum L_\tau = \mathrm{const}. \tag{5}$$

According to the *neutrino mixing hypothesis* this law is an approximate one: it is violated by the *neutrino mass term.*

The neutrino mass term (see the review article [22] and references therein) can be completely different from the corresponding lepton and quark mass terms. This is connected with the fact that neutrinos with definite masses can be *Dirac or Majorana particles*; charged leptons and quarks are *Dirac particles.* The neutrino mass term can be written in the following, general form

$$\mathcal{L} = -\overline{n}_R M n_L + \mathrm{h.c.} \,, \tag{6}$$

where $n_{L,R}$ are columns of the neutrino fields and M is a matrix. There are two general possibilities for n_L.

2.1 Case I

The column n_L contains only flavor neutrino fields

$$n_L = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} . \tag{7}$$

In this case M is a 3×3 matrix and for the mixing we have

$$\nu_{\ell L} = \sum_{i=1}^{3} U_{\ell i} \nu_{iL} , \qquad (\ell = e, \mu, \tau) , \tag{8}$$

where $U^\dagger U = 1$ and ν_i is the field of neutrinos with mass m_i. Only transitions between flavor neutrinos $\nu_\ell \rightleftharpoons \nu_{\ell'}$ are possible in this case.

The nature of ν_i depends on n_R. If

$$n_R = \begin{pmatrix} \nu_{eR} \\ \nu_{\mu R} \\ \nu_{\tau R} \end{pmatrix} , \tag{9}$$

where $\nu_{\ell R}$ are right-handed neutrino fields, global gauge invariance

$$\nu_{\ell L} \to \mathrm{e}^{\mathrm{i}\alpha} \nu_{\ell L} , \quad \nu_{\ell R} \to \mathrm{e}^{\mathrm{i}\alpha} \nu_{\ell R} , \quad \ell \to \mathrm{e}^{\mathrm{i}\alpha} \ell , \tag{10}$$

(α being a real constant, the same for all fields) takes place. Hence, in this case the total lepton charge

$$L = L_e + L_\mu + L_\tau \tag{11}$$

is conserved and fields of neutrinos with definite masses, ν_i, are *Dirac fields* (ν_i and the charge-conjugated field, $\nu_i^{\mathrm{C}} = \mathrm{C}\overline{\nu}_i^{\mathrm{T}}$ are independent). The corresponding mass term is called the Dirac mass term. Note that the Dirac mass term can be generated in the framework of the standard Higgs mechanism which is responsible for the generation of the masses of charged leptons and quarks.

If, instead

$$n_R = \begin{pmatrix} (\nu_{eL})^{\mathrm{C}} \\ (\nu_{\mu L})^{\mathrm{C}} \\ (\nu_{\tau L})^{\mathrm{C}} \end{pmatrix} , \tag{12}$$

where $(\nu_{\ell L})^{\mathrm{C}} = \mathrm{C}\overline{\nu}_{\ell L}^{\mathrm{T}}$ is the right-handed component, then there are no conserved lepton numbers and the fields of neutrinos with definite masses are *Majorana fields* ($\nu_i^{\mathrm{C}} = \nu_i$).[1] The corresponding mass term is called the Majorana mass term.

Note that if massive neutrinos are Majorana particles, a process like neutrinoless double-beta decay

$$(A, Z) \to (A, Z+2) + e^- + e^- ,$$

in which the total lepton number is not conserved, becomes possible. There is no difference in neutrino oscillations for the case of Dirac or Majorana masses.

2.2 Case II

In the most general case not only the three flavor neutrino fields enter into n_L, but also other fields ν_{sL} ($s = s_1, \ldots$), which are not contained in the standard Lagrangian of weak interactions, (1), and hence are called *sterile fields*

$$n_L = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \\ \nu_{s_1 L} \\ \vdots \end{pmatrix} . \tag{13}$$

Sterile fields can be right-handed neutrino fields ($\nu_{sL} = (\nu_{sR})^{\mathrm{C}}$) and/or fields of SUSY particles.[2]

[1] For Majorana neutrinos not only the electric charge, but also all lepton charges are equal to zero.

[2] If the ν_{sR} fields enter only into the neutrino mass term, then the corresponding particles are really sterile. Should there be a right-handed interaction, then 'sterile'

For the mixing we have, in this case,

$$\nu_{\alpha L} = \sum_{i=1}^{3+n_s} U_{\alpha i} \nu_{iL} \,, \qquad (\alpha = e, \mu, \tau, s_1, \ldots) \,, \tag{14}$$

where ν_i is the neutrino field with mass m_i and U is a $(3+n_s) \times (3+n_s)$ unitary mixing matrix. The number of sterile fields, n_s, can only be fixed by a model. For $\nu_{sL} = (\nu_{sR})^{\mathrm{C}}$ it is natural to assume that $n_s = 3$.

If the neutrino masses m_i are small ($i = 1, \ldots, 3+n_s$), then not only oscillations between flavor neutrinos $\nu_\ell \rightleftharpoons \nu_{\ell'}$, but also oscillations between flavor and sterile neutrinos $\nu_\ell \rightleftharpoons \nu_s$ will take place.

The nature of the massive neutrinos ν_i depends on n_R. If $n_R = (n_L)^{\mathrm{C}}$ the neutrinos ν_i are Majorana particles and neutrinoless double-beta decay is possible. The corresponding mass term is called the Dirac–Majorana mass term.

Majorana neutrino masses can be generated only in the framework of models beyond the standard model. In the case of the Dirac–Majorana mass term there exists a plausible (the so called seesaw) mechanism for neutrino mass generation [23]. It is based on the assumption that lepton numbers are violated by the right-handed Majorana mass term at a scale M much larger than the electroweak scale.

The spectrum of masses of Majorana particles in the seesaw case contains three light neutrinos ν_i with masses m_i and three very heavy Majorana particles with masses $M_i \simeq M$. The two sets of masses are connected by the seesaw relation

$$m_i \simeq \frac{(m_f^i)^2}{M_i} \ll m_f^i \qquad (i = 1, 2, 3) \tag{15}$$

where m_f^i is the mass of a quark or a lepton in the 'ith' family. The seesaw mechanism connects the smallness of Majorana neutrino masses with the violation of lepton numbers at very large mass scales. Note that in the seesaw case the neutrino masses satisfy a hierarchy relation

$$m_1 \ll m_2 \ll m_3 \,. \tag{16}$$

particles could experience a much weaker interaction than the standard electroweak interaction.

3 Neutrino Oscillations

In this section we will discuss the phenomenon of oscillations in neutrino beams [24] which can occur if neutrino masses are different from zero and flavor neutrino fields are mixtures of massive fields [see (8) and (14)]. In this case for a state with momentum $\vec{p}$ we have

$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha i} |i\rangle \ , \qquad (\alpha = e, \mu, \tau, s_1, \ldots) \ , \tag{17}$$

where $|i\rangle$ is the state of a neutrino with mass m_i, momentum $\vec{p}$ and energy

$$E_i = \sqrt{p^2 + m_i^2} \simeq p + \frac{m_i^2}{2p} \ , \qquad (pg^2 m_i) \ . \tag{18}$$

The state $|i\rangle$ is an eigenstate of the free Hamiltonian H_0

$$H_0 |i\rangle = E_i |i\rangle \ . \tag{19}$$

Relation (17) implies that the states of flavor neutrinos (and eventually the sterile ones) are *coherent superpositions* of neutrino states with different masses. This is valid only when the neutrino mass differences are small and, due to the uncertainty principle, different mass components cannot be distinguished in production and detection processes (for a recent discussion of this problem see [25]).

If at $t = 0$ the state of neutrinos is $|\nu_\alpha\rangle$, the probability amplitude of the transition into the state $|\nu_\beta\rangle$ after a time t is given by

$$\begin{aligned} \langle \nu_\beta | \mathrm{e}^{-\mathrm{i} H_0 t} | \nu_\alpha \rangle &= \sum_i \langle \nu_\beta | i \rangle \mathrm{e}^{-\mathrm{i} E_i t} \langle i | \nu_\alpha \rangle \\ &= \sum_i U_{\beta i} \mathrm{e}^{-\mathrm{i} E_i t} U^*_{\alpha i} \ . \end{aligned} \tag{20}$$

From (20), using the unitarity of the mixing matrix U, the following general expression for the probability of the transition $\nu_\alpha \to \nu_\beta$ can be obtained

$$P(\nu_\alpha \to \nu_\beta) = \left| \delta_{\beta\alpha} + \sum_i U_{\beta i} \left[\exp\left(-\mathrm{i} \Delta m^2_{i1} \frac{L}{2p} \right) - 1 \right] U^*_{\alpha i} \right|^2 \ . \tag{21}$$

Here $L \simeq t$ is the distance between the neutrino source and the neutrino detector, and $\Delta m^2_{i1} = m_i^2 - m_1^2$ (we have assumed that $m_1 < m_2 < \ldots$). The probability for an antineutrino transition $\overline{\nu}_\alpha \to \overline{\nu}_\beta$ is given by

$$P(\overline{\nu}_\alpha \to \overline{\nu}_\beta) = \left| \delta_{\beta\alpha} + \sum_i U_{\beta i}^* \left[\exp\left(-\mathrm{i}\Delta m_{i1}^2 \frac{L}{2p} \right) - 1 \right] U_{\alpha i} \right|^2 . \tag{22}$$

Obviously, as a consequence of CPT invariance

$$P(\nu_\alpha \to \nu_\beta) = P(\overline{\nu}_\beta \to \overline{\nu}_\alpha). \tag{23}$$

Note that from CP invariance it follows that

$$P(\nu_\alpha \to \nu_\beta) = P(\overline{\nu}_\alpha \to \overline{\nu}_\beta). \tag{24}$$

The probability of the transition $\nu_\alpha \to \nu_\beta$ ($\overline{\nu}_\alpha \to \overline{\nu}_\beta$) depends, in the general case, on $n-1$ neutrino mass-squared differences ($n = 3 + n_s$), $n(n-1)/2$ mixing angles, $(n-1)(n-2)/2$ phases and on the parameter L/p. When, for all values of i, $\Delta m_{i1}^2 \ll p/L$ then neutrino oscillations cannot be observed $[P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta}]$. In order to observe neutrino oscillations is it necessary that for some value of $\kappa \geq 2$, $\Delta m_{\kappa i}^2 \geq p/L$.

Let us consider the simplest case of the mixing of two neutrino species. Then

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} , \tag{25}$$

where θ is the mixing angle. From (21) and (22) one gets

$$\begin{aligned} P(\nu_\alpha \to \nu_\beta) &= P(\overline{\nu}_\alpha \to \overline{\nu}_\beta) \\ &= \frac{1}{2}\sin^2 2\theta \left(1 - \cos\frac{\Delta m^2 L}{2p}\right) , \quad (\beta \neq \alpha) , \end{aligned} \tag{26}$$

$$\begin{aligned} P(\nu_\alpha \to \nu_\alpha) &= P(\nu_\beta \to \nu_\beta) = 1 - P(\nu_\alpha \to \nu_\beta) \\ &= 1 - \frac{1}{2}\sin^2 2\theta \left(1 - \cos\frac{\Delta m^2 L}{2p}\right) . \end{aligned} \tag{27}$$

In the above, $\Delta m^2 = m_2^2 - m_1^2$ and α, β can assume the values e, μ or μ, τ and so on. The expressions (26), (27) are written in units $\hbar = c = 1$. The transition probability can also be written in the form

$$P(\nu_\alpha \to \nu_\beta) = \frac{1}{2}\sin^2 2\theta \left(1 - \cos 2.54\frac{\Delta m^2 L}{E}\right) , \tag{28}$$

where L is the distance in m, E is the neutrino energy in MeV, and Δm^2 is the neutrino mass-squared difference in eV^2. For the oscillation length we have, from (26) and (28)

$$L_0 = 4\pi \frac{E}{\Delta m^2} = 2.47 \frac{E\ (\mathrm{MeV})}{\Delta m^2\ (\mathrm{eV}^2)}\ \mathrm{m}\,. \tag{29}$$

In the simplest case of two-neutrino mixing, the probability of neutrino transitions depends on the parameters $\sin^2 2\theta$ (the amplitude of the oscillations) and Δm^2, which characterizes the oscillation length. The necessary condition for the oscillations to be observable is

$$\Delta m^2 \frac{L}{E} \geq 1\,. \tag{30}$$

Thus, the larger the value of the parameter L/E, the more sensitive will be an experiment to the value of Δm^2. Typical values of the parameter $L\,(\mathrm{m})/E\,(\mathrm{MeV})$ for SBL and LBL accelerator experiments, for SBL and LBL reactor experiments, for atmospheric-neutrino experiments and for solar-neutrino experiments are

$$1; \quad 10^2\text{–}10^3; \quad 10^2; \quad 10^3; \quad 10^2\text{–}10^3; \quad 10^{11}\,,$$

respectively.

4 The Status of Neutrino Oscillations

Evidence and indications in favor of neutrino oscillations were found in many neutrino-oscillation experiments. We will discuss here briefly the results that have been obtained.

4.1 Atmospheric Neutrinos

Atmospheric neutrinos are mainly produced in the decay of pions and muons

$$\pi^{\mp} \to \mu^{\mp} \nu_\mu (\overline{\nu}_\mu)\,, \qquad \mu^{\mp} \to e^{\mp} \overline{\nu}_e \nu_\mu (\nu_e \overline{\nu}_\mu)\,, \tag{31}$$

pions being produced in the interaction of cosmic rays with nuclei in the Earth's atmosphere. At energies ≤ 3 GeV the ratio of fluxes of $\nu_\mu, \overline{\nu}_\mu$ and $\nu_e, \overline{\nu}_e$ is equal to two, while at higher energies it is larger than two (since not all muons have time to decay in the atmosphere). The ratio can be predicted, however, with accuracy better than 5% (the absolute fluxes of muon and electron neutrinos are calculated with accuracy not better than 20%).

The results of atmospheric-neutrino experiments are usually presented in the form of a double ratio

$$R = \left(\frac{N_\mu}{N_e}\right)_{\rm exp} / \left(\frac{N_\mu}{N_e}\right)_{\rm MC} , \tag{32}$$

where $N_\mu(N_e)$ is the total number of muon (electron) events (in modern detectors neutrino and antineutrino events cannot be distinguished) and $(N_\mu/N_e)_{\rm MC}$ is the ratio predicted by Monte Carlo simulations.

We shall discuss mainly the results of the super-Kamiokande experiment [1]. In this experiment a large 50 kton water Čerenkov detector is used. The detector consists of two parts: the inner one (22.5 kton fiducial volume) is covered with 11 146 photomultipliers; the outer part, 2.75 m thick, is covered with 1885 photomultipliers. The electrons and muons are detected by observing their Čerenkov radiation. The effectiveness of particle identification is larger than 98%.

The observed events are divided into fully contained events (FC), for which all Čerenkov light is deposited in the inner detector, and partially contained events (PC), in which the muon track deposits part of its Čerenkov radiation in the outer detector. FC events are further divided into sub–GeV events ($E_{\rm vis} \leq 1.33$ GeV) and multi-GeV events ($E_{\rm vis} \geq 1.33$ GeV).

In the super-Kamiokande experiment for the double ratio R, from FC events (736 days) and PC events (685 days), the following values were found

$$R = 0.67 \pm 0.02 \pm 0.05 , \quad \text{(sub-GeV)} , \tag{33}$$
$$R = 0.66 \pm 0.04 \pm 0.08 , \quad \text{(multi-GeV)} . \tag{34}$$

Analogous results were obtained in the previous water Čerenkov Kamiokande [5] and IMB [3] experiments and in the iron calorimeter Soudan-2 [4] experiment

$$\begin{aligned} R &= 0.65 \pm 0.05 \pm 0.08 , && \text{(Kamiokande)} , \\ R &= 0.54 \pm 0.05 \pm 0.11 , && \text{(IMB)} , \\ R &= 0.64 \pm 0.11 \pm 0.06 , && \text{(Soudan−2)} . \end{aligned} \tag{35}$$

The fact that R is significantly less than one could imply disappearance of atmospheric ν_μ or appearance of ν_e (or both). In the super-Kamiokande experiment there was found compelling evidence in favor of the disappearance of ν_μ due to neutrino oscillations: in this experiment a significant up–down asymmetry of the multi-GeV muon events was discovered.

For atmospheric neutrinos the distances between production points and detector can differ from $L \simeq 10$ km (down-going neutrinos, $\theta = 0$, θ being the zenith angle) to $L \simeq 10^4$ km (up-going neutrinos, $\theta = \pi$). At high energies the effect of the Earth's magnetic field is small and the expected number of neutrino events can not depend on the zenith angle θ. However the Super–Kamiokande Collaboration found a significant θ dependence of the multi-GeV muon neutrino events. For the integral up–down asymmetry

$$\mathcal{A} = \frac{\mathcal{U} - \mathcal{D}}{\mathcal{U} + \mathcal{D}}, \tag{36}$$

there was obtained the value

$$\mathcal{A}_\mu = -0.311 \pm 0.043 \pm 0.010 \,. \tag{37}$$

Here $\mathcal{U}$ is the number of up-going events ($-1 \leq \cos\theta \leq -0.2$) and $\mathcal{D}$ is the number of down-going events ($0.2 \leq \cos\theta \leq 1$). No significant asymmetry of the electron neutrino events was found

$$\mathcal{A}_e = -0.036 \pm 0.067 \pm 0.02 \,. \tag{38}$$

The super–Kamiokande data can be described if we assume that $\nu_\mu \to \nu_\tau$ or $\nu_\mu \to \nu_s$ oscillations take place. In the $\nu_\mu \to \nu_\tau$ case the following best-fit values for the oscillation parameters were found [26]

$$\sin^2 2\theta = 1 \,, \qquad \Delta m^2 = 3.5 \times 10^{-3}\,\mathrm{eV}^2 \,, \tag{39}$$

(with $\chi^2_{\min} = 6.21$ for 67 degrees of freedom). In the case of $\nu_\mu \to \nu_s$ oscillations the best-fit values of the parameters are

$$\sin^2 2\theta = 1 \,, \qquad \Delta m^2 = 4.5 \times 10^{-3}\,\mathrm{eV}^2 \,, \tag{40}$$

(with $\chi^2_{\min} = 64.3$ for 67 degrees of freedom).

Table 1
Main sources of solar neutrinos

Reaction	Neutrino energy	Expected flux (cm^{-2} s^{-1}) BP 98 [28]
$pp \to d\, e^+ \,\nu_e$	≤ 0.42 MeV	6×10^{10}
$e^- \,{}^7\mathrm{Be} \to \nu_e \,{}^7\mathrm{Li}$	0.86 MeV	4.8×10^9
${}^8\mathrm{B} \to {}^8\mathrm{Be}^* \,e^+ \,\nu_e$	$\leq$ 15 MeV	5×10^6

4.2 Solar Neutrinos

The energy of the Sun is produced in the reactions of thermonuclear pp and CNO cycles. From the thermodynamical point of view the energy of the Sun is produced in the transition

$$2e^- + 4p \longrightarrow {}^4\mathrm{He} + 2\nu_e \,. \tag{41}$$

Thus the production of energy in the Sun is accompanied by the emission of electron neutrinos.

The main sources of solar neutrinos are the reactions of the pp cycle listed in Table 1. As is seen from the table, solar neutrinos are mainly low energy pp neutrinos and intermediate energy, monochromatic ^{7}Be neutrinos, while the high-energy part of the solar-neutrino spectrum is due to the ^{8}B decay.

The total flux of solar neutrinos is connected with the luminosity of the Sun, $L_\odot$, by the relation

$$Q \sum_{i=pp,\ldots} \left(1 - 2\frac{\overline{E}_i}{Q}\right) \Phi_i = \frac{L_\odot}{2\pi R^2} \,, \tag{42}$$

where $Q = 4m_p + 2m_e - m_{^4\mathrm{He}} \simeq 26.7$ MeV is the energy release in the transition (41), Φ_i is the total flux of neutrinos from the source i ($i = pp$, ^{7}Be, ^{8}B,...), R is the Sun–Earth distance and $\overline{E}_i$ is the average energy of neutrinos from the source i. The above relation was derived under the assumption that $P(\nu_e \to \nu_e) = 1$. Notice that for pp and ^{7}Be neutrinos the term $\overline{E}_i/Q$ can be neglected, while $\Phi_{^8\mathrm{B}}$ gives a very small contribution to the left-hand side of (42).

The results of five solar-neutrino experiments are available at present: they are reported in Table 2.

Table 2
Results of solar-neutrino experiments (1 SNU=10^{-36} events/(atom sec))

Experiment	Observed rate	Predicted rate BP 98	$\frac{\text{Data}}{\text{Prediction}}$
Homestake [7]	$2.56 \pm 0.16 \pm 0.11$ SNU	$7.7^{+1.2}_{-1.0}$ SNU	0.33 ± 0.06
GALLEX [8]	$77.5 \pm 6.2^{+4.3}_{-4.7}$ SNU	129^{+8}_{-6} SNU	0.60 ± 0.07
SAGE [9]	$66.6^{+6.8+3.8}_{-7.1-4.0}$ SNU	129^{+8}_{-6} SNU	0.52 ± 0.07
Kamiokande [5]	$(2.80 \pm 0.19 \pm 0.33)\times 10^6$ $(\text{cm}^{-2}\,\text{s}^{-1})$	$(5.15^{+1.0}_{-0.7})\times 10^6$ $(\text{cm}^{-2}\,\text{s}^{-1})$	0.54 ± 0.07
Super-Kamiokande [11]	$(2.44 \pm 0.05^{+0.09}_{-0.07})\times 10^6$ $(\text{cm}^{-2}\,\text{s}^{-1})$	$(5.15^{+1.0}_{-0.7})\times 10^6$ $(\text{cm}^{-2}\,\text{s}^{-1})$	$0.47^{+0.07}_{-0.09}$

In the radiochemical Homestake experiment [7] solar ν_e's are detected by observing the Pontecorvo–Davis reaction

$$\nu_e + {}^{37}\text{Cl} \longrightarrow e^- + {}^{37}\text{Ar}\,. \tag{43}$$

The threshold of this process is $E_{\text{th}} = 0.81$ MeV. Thus mainly ^{8}B and ^{7}Be neutrinos are detected in this experiment (according to the standard solar model (SSM) the contributions of ^{8}B and ^{7}Be neutrinos to the total rate are 77% and 14%, respectively).

In the radiochemical GALLEX [8] and SAGE [9] experiments solar ν_e's are detected through the observation of the reaction

$$\nu_e + {}^{71}\text{Ga} \longrightarrow e^- + {}^{71}\text{Ge}\,, \tag{44}$$

whose threshold is $E_{\text{th}} = 0.23$ MeV. Hence neutrinos from all sources can be detected by these experiments. According to the SSM, the contributions of *pp*, ^{7}Be and ^{8}B neutrinos to the total rate are, respectively, 54%, 27%, 10%.

Finally, in the direct-counting Kamiokande [10] and super-Kamiokande [11] experiments, solar neutrinos are detected by the observation of recoil electrons from the process

$$\nu_e + e \longrightarrow \nu_e + e\,. \tag{45}$$

Due to the high energy threshold ($E_{\text{th}} = 7$ MeV in the Kamiokande experiment; in the super-Kamiokande one $E_{\text{th}} = 6.5$ MeV and in the most recent runs $E_{\text{th}} = 5.5$ MeV) mainly ^{8}B neutrinos are detected in these experiments.

As is seen from Table 2, the observed rates in all solar-neutrino experiments are significantly smaller than the predicted rates. The existing data can not be described if we assume that $P(\nu_e \to \nu_e) = 1$, even when the total fluxes Φ_i are left as free fitting parameters (there are no acceptable fits at 99.99% confidence level [27]).

Instead, the data presented in Table 2 can be accounted for by assuming that there is a two-neutrino mixing (if the SSM values for the neutrino fluxes Φ_i are used). In the case of $\nu_e \to \nu_\mu$ (or ν_τ) transitions the following best-fit values of the oscillation parameters were obtained [26]

- small mixing angle MSW solution

$$\begin{aligned} &\sin^2 2\theta = 5 \times 10^{-3}, \quad \Delta m^2 = 7.1 \times 10^{-6}\ \text{eV}^2; \\ &\text{confidence level} = 1.6\%\ . \end{aligned} \tag{46}$$

- large mixing angle MSW solution

$$\begin{aligned} &\sin^2 2\theta = 0.7, \quad \Delta m^2 = 2.8 \times 10^{-5}\ \text{eV}^2; \\ &\text{confidence level} = 1.2\%\ . \end{aligned} \tag{47}$$

- vacuum oscillation solution

$$\begin{aligned} &\sin^2 2\theta = 0.89, \quad \Delta m^2 = 4.3 \times 10^{-10}\ \text{eV}^2; \\ &\text{confidence level} = 9.9\%\ . \end{aligned} \tag{48}$$

These solutions were obtained by fitting the data, presented in Table 2, together with the super-Kamiokande data on the measurement of the spectrum of recoil electrons in $\nu_e \to \nu_e$ scattering.

In the near future two new solar-neutrino experiments, SNO [16] and Borexino [17], will be started. In the SNO experiment (heavy water Čerenkov detector, 1 kton of D_2O) solar neutrinos will be detected by measuring the CC process

$$\nu_e\, d \longrightarrow e^-\, pp\ , \tag{49}$$

as well as the NC one

$$\nu\, d \longrightarrow \nu\, n\, p\ , \tag{50}$$

and the process

$$\nu\, e \longrightarrow \nu_e\, e\ . \tag{51}$$

Owing to the high energy threshold ($E_{\text{th}} = 5$ MeV for the processes (49) and (51) and $E_{\text{th}} = 2.2$ MeV for the NC process (50)), mainly ^{8}B neutrinos will be detected in this experiment.

By measuring the electron energy in the CC process (49) it will be possible to determine the spectrum of the solar ν_e's on Earth. Moreover the detection of solar neutrinos through the observation of the NC process (50) (the neutron will be detected) will allow us to obtain information on the flux of all active neutrinos, ν_e, ν_μ, ν_τ on Earth. From the comparison of these data a model-independent conclusion on the transitions of solar ν_e's into other states can be drawn.

In the Borexino experiment [17] (300 tons of liquid scintillator) the observation of the process $\nu e \to \nu e$ will allow us to detect monochromatic ^{8}B neutrinos, with energy $E = 0.86$ MeV. The threshold for the detection of the recoil electrons is $E_{\text{th}} = 250$ keV. The SSM predicts for this experiment $\simeq 50$ events/day. In the case of vacuum oscillations, a significant seasonal variation of the number of events would be observed.

4.3 LSND Experiment

Only in one accelerator-neutrino experiment (LSND [12]) indications in favor of neutrino oscillations were found. This experiment was done at the Los Alamos linear accelerator (the proton energy being 800 MeV). Neutrinos were produced in the decays of π^+ and μ^+ at rest

$$\begin{aligned} \pi^+ &\longrightarrow \mu^+ \nu_\mu \,, \\ \mu^+ &\longrightarrow e^+ \nu_e \overline{\nu}_\mu \,. \end{aligned} \tag{52}$$

The large scintillator neutrino detector (LSND) was located at a distance of about 30 m from the neutrino source. The LSND Collaboration searched for $\overline{\nu}_e$ through the observation of the process

$$\overline{\nu}_e + p \longrightarrow e^+ + n \,. \tag{53}$$

Both e^+ and delayed γ's from the capture $np \to d\gamma$ were detected.

In the LSND experiment 33.9 ± 8.0 events were observed in the interval of e^+ energies $30 \le E \le 60$ MeV. If these events are due to $\overline{\nu}_\mu \to \overline{\nu}_e$ oscillations, for the transition probability it was found

$$P(\overline{\nu}_\mu \to \overline{\nu}_e) = (0.31 \pm 0.09 \pm 0.06) \times 10^{-2} \,. \tag{54}$$

It corresponds to the following allowed ranges of the oscillation parameters

$$\begin{aligned} &0.3 < \Delta m^2 \leq 1\,\mathrm{eV}^2 \,, \\ &2 \times 10^{-3} \leq \sin^2 2\theta \leq 4 \times 10^{-2} \,. \end{aligned} \tag{55}$$

These values do not contradict the data of other-neutrino oscillation experiments, including those of the KARMEN Collaboration [29], searching for the $\bar{\nu}_\mu \to \bar{\nu}_e$ transition at the spallation neutron facility of the Rutherford Laboratory. The LSND region of the oscillation parameters will be thoroughly investigated by the future BooNE [19] experiment at Fermilab.

5 Neutrino Masses and Mixing from Oscillation Data

From all existing neutrino-oscillation data, it follows that there are three different scales of Δm^2: hence in order to describe these data we must assume the existence of at least four massive neutrinos. If the data of the LSND experiment should not be confirmed by future experiments, then it would be enough to assume the existence of three massive neutrinos only. We will consider both these possibilities.

5.1 Mixing of Three Massive Neutrinos

For the mixing of three massive neutrinos it is natural to assume the following mass hierarchy

$$m_1 \ll m_2 \ll m_3 \,,$$

with Δm^2_{21} and Δm^2_{31} relevant for the oscillations of solar and atmospheric neutrinos, respectively.

It is easy to see that for atmospheric and LBL neutrino-oscillation experiments the inequality

$$\frac{\Delta m^2_{21} L}{2p} \ll 1 \tag{56}$$

holds. Hence, from (21), the $\nu_\alpha \to \nu_\beta$ transition probability becomes [30]

$$P\left(\nu_\alpha \to \nu_\beta\right) = \frac{1}{2} A_{\alpha;\beta} \left(1 - \cos \frac{\Delta m^2_{31} L}{2p}\right) \,, \qquad (\alpha \neq \beta) \,; \tag{57}$$

and the ν_α survival probability is given by

$$P(\nu_\alpha \to \nu_\alpha) = 1 - \sum_{\beta \neq \alpha} P(\nu_\alpha \to \nu_\beta) = 1 - \frac{1}{2} B_{\alpha;\alpha} \left(1 - \cos\frac{\Delta m_{31}^2 L}{2p}\right) , \quad (58)$$

the oscillation amplitudes being

$$A_{\alpha;\beta} = 4|U_{\alpha 3}|^2 \, |U_{\beta 3}|^2 , \qquad B_{\alpha;\alpha} = 4|U_{\alpha 3}|^2 (1 - |U_{\alpha 3}|^2) . \quad (59)$$

The expressions (57), (58) have the same form as the ones for two-neutrino transition probabilities, (26), (27). They describe, however, all possible transitions between three flavor neutrinos.

Notice that, due to the unitarity of the mixing matrix, $\sum_{\alpha=e,\mu,\tau} |U_{\alpha 3}|^2 = 1$. Thus, in the framework of three-neutrino mixing, the transition probabilities for the atmospheric and LBL neutrino experiments are described by three parameters: $|U_{e3}|^2$, $|U_{\mu 3}|^2$ and Δm_{31}^2.

Information on the parameter $|U_{e3}|^2$ can be obtained from the LBL experiment CHOOZ [31]. In this first reactor LBL experiment (the distance between the reactor and the detector being about 1 km) there were found no indications in favor of neutrino oscillations. The ratio R between the number of observed events and the number of predicted events (under the assumption that there are no oscillations) was found to be

$$R = 0.98 \pm 0.04 \pm 0.04 . \quad (60)$$

From the results of the experiment the CHOOZ Collaboration obtained the exclusion plot in the plane of the parameters $(\sin^2 2\theta, \Delta m^2)$. It was shown that the region

$$0.2 \leq \sin^2 2\theta_{\mathrm{CHOOZ}} \leq 1 , \qquad \Delta m_{31}^2 \geq 2 \times 10^{-3}\,\mathrm{eV}^2 , \quad (61)$$

is excluded.

From (59) it is easy to see that the parameter $|U_{e3}|^2$ is connected with the amplitude $B_{e;e}$ by the relation

$$|U_{e3}|^2 = \frac{1}{2}\left(1 \pm \sqrt{1 - B_{e;e}}\right) . \tag{62}$$

From the exclusion curve of the CHOOZ experiment in the region $\Delta m^2_{31} \geq 2 \times 10^{-3}$ eV^2 the following upper bound can be obtained

$$B_{e;e} \leq B^0_{e;e}, \tag{63}$$

the values $B^0_{e;e} \equiv (\sin^2 2\theta)_{\text{CHOOZ}}$ depending on Δm^2_{31}. From (62) and (63) the parameter $|U_{e3}|^2$ turns out to be subject to the conditions [32]

$$|U_{e3}|^2 \leq a_0 \qquad \text{or} \qquad |U_{e3}|^2 \geq 1 - a^0_e , \tag{64}$$

where

$$a^0_e = \frac{1}{2}\left(1 - \sqrt{1 - B^0_{e;e}}\right) . \tag{65}$$

At $\Delta m^2_{31} \geq 2 \times 10^{-3}$ eV^2 the CHOOZ exclusion curve entails $B^0_{e;e} < 0.18$ and hence

$$|U_{e3}|^2 \leq 5 \times 10^{-2} \qquad \text{or} \qquad |U_{e3}|^2 \geq 0.95 . \tag{66}$$

Large values of $|U_{e3}|^2$ are excluded by the results of solar-neutrino experiments. Indeed for the probability of solar neutrinos to survive we have [33]

$$P^{\text{solar}}(\nu_e \to \nu_e) = \left(1 - |U_{e3}|^2\right)^2 P^{(1,2)}(\nu_e \to \nu_e) + |U_{e3}|^4 , \tag{67}$$

where $P^{(1,2)}(\nu_e \to \nu_e)$ is the transition probability due to the coupling of ν_e to ν_1, ν_2. If $|U_{e3}|^2 \geq 0.95$ then $P^{\text{solar}}(\nu_e \to \nu_e) \geq 0.90$, a result which is not compatible with the outcome of solar-neutrino experiments. Hence we come to the conclusion that

$$|U_{e3}|^2 \leq 5 \times 10^{-2} . \tag{68}$$

From the data of atmospheric-neutrino experiments it also follows that the element $|U_{e3}|^2$ is small in the region $10^{-3} \leq \Delta m^2_{31} \leq 8 \times 10^{-3}$ eV^2: in these experiments there is no indication in favor of $\nu_\mu \to \nu_e$ oscillations; moreover the amplitude of $\nu_\mu \to \nu_\tau$ oscillations is close to the maximum allowed value, which means that $|U_{\mu 3}|^2 \simeq |U_{\tau 3}|^2 \simeq 1/2$ and $|U_{e3}|^2 \simeq 0$.

The unitary 3×3 matrix U is characterized by three mixing angles, $\theta_{12}, \theta_{13}, \theta_{23}$, and one phase, ϕ. The condition $U_{e3} \simeq 0$ is equivalent to $\theta_{13} \simeq 0$; in this case the phase is irrelevant and the remaining angles θ_{12} and θ_{23} can be determined from solar- and atmospheric-neutrino data, respectively (the oscillations of solar and atmospheric neutrinos are decoupled [32]).

5.2 Mixing of Four Massive Neutrinos

Two types of mass spectra are possible in the presence of three scales for Δm^2 ($\Delta m^2_{\text{atm}} \simeq 10^{-3}$ eV^2, $\Delta m^2_{\text{solar}} \simeq 10^{-5}$ eV^2 (10^{-10} eV^2), $\Delta m^2_{\text{LSND}} \simeq 1$ eV^2). In the spectra of the first type a group of three close masses is separated from the fourth one by the 'LSND gap' of about 1 eV. In the spectra of the second type two pairs of close masses are separated by a $\simeq 1$ eV gap.

Neutrino mass spectra of the first type are not compatible with the data of neutrino-oscillation experiments [34,35]. In fact let us consider the case of a mass hierarchy of four neutrinos. In SBL experiments $\Delta m^2_{21} L/2p \ll 1$ and $\Delta m^2_{31} L/2p \ll 1$; in this situation, from (21) for the $\nu_\alpha \to \nu_\beta$ transition probability, we can obtain expressions similar to (57)–(59), providing the following replacements are performed: $U_{\alpha 3} \to U_{\alpha 4}$, $\Delta m^2_{31} \to \Delta m^2_{41}$. Furthermore, taking into account solar- and atmospheric-neutrino data, we have, in analogy with (68)

$$|U_{\alpha 4}|^2 \leq a^0_\alpha \,, \qquad (\alpha = e, \mu) \,. \tag{69}$$

Here

$$a^0_\alpha = \frac{1}{2}\left(1 - \sqrt{1 - B^0_{\alpha;\alpha}}\right) \tag{70}$$

and $B^0_{\alpha;\alpha}$ can be found from the exclusion curves of SBL reactor and accelerator disappearance experiments. We get

$$\begin{aligned} a^0_e &\leq 4 \times 10^{-2} \qquad \text{for} \qquad \Delta m^2_{41} \geq 0.1\,\text{eV}^2 \,, \\ a^0_\mu &\leq 2 \times 10^{-1} \qquad \text{for} \qquad \Delta m^2_{41} \geq 0.3\,\text{eV}^2 \,. \end{aligned} \tag{71}$$

For the amplitude of SBL $\nu_\mu \to \nu_e$ transitions we have the following upper bound

$$A_{\mu;e} \equiv 4|U_{e4}|^2|U_{\mu 4}|^2 \leq 4a^0_e a^0_\mu \,. \tag{72}$$

$$\text{(A)} \quad \overbrace{m_1 < m_2}^{\text{atm}} \ll \overbrace{m_3 < m_4}^{\text{solar}} \atop \underbrace{\hphantom{m_1 < m_2 \ll m_3 < m_4}}_{\text{LSND}}$$

$$\text{(B)} \quad \overbrace{m_1 < m_2}^{\text{solar}} \ll \overbrace{m_3 < m_4}^{\text{atm}} \atop \underbrace{\hphantom{m_1 < m_2 \ll m_3 < m_4}}_{\text{LSND}}$$

Fig. 1. The schemes (**A**) and (**B**) for the neutrino mass spectrum discussed in the text

However it can be shown [34,36] that the quantity $4a_e^0 a_\mu^0$ is too small to be compatible with the allowed region in the plot obtained by the LSND Collaboration. The same considerations apply to all other spectra of the first type.

Only two schemes of mixing of four massive neutrinos (A and B), with the mass spectra shown in Fig. 1, can describe all existing neutrino-oscillation data. In these schemes, in place of the inequality (69), we have

$$\text{(A)} \quad \begin{array}{l} \sum_{i=1,2} |U_{ei}|^2 \leq a_e^0 \\ \sum_{k=3,4} |U_{\mu k}|^2 \leq a_\mu^0 \end{array} , \qquad \text{(B)} \quad \begin{array}{l} \sum_{k=3,4} |U_{ek}|^2 \leq a_e^0 \\ \sum_{i=1,2} |U_{\mu i}|^2 \leq a_\mu^0 \end{array} . \tag{73}$$

In both schemes, for the amplitude of the SBL $\nu_\mu \to \nu_e$ transition we have the following upper bound

$$A_{\mu;e} \leq 4 \sum_{i=1,2} |U_{ei}|^2 \times \sum_{i=1,2} |U_{\mu i}|^2 \leq 4 \min\left(a_e^0, a_\mu^0\right) . \tag{74}$$

This bound is linear in the (small) quantities a_e^0 and a_μ^0 and is compatible with the LSND results.

The above schemes of mixing of four neutrinos suggests the existence of a sterile neutrino. Taking into account the big bang nucleosynthesis constraint on the effective number of neutrinos, it can be shown [35,37] that in both schemes A and B the dominant transition of solar neutrinos is the $\nu_e \to \nu_s$ one and the dominant transition of atmospheric neutrinos is $\nu_\mu \to \nu_\tau$. These predictions will be tested by future solar, atmospheric and LBL experiments.

6 Conclusions

The latest discoveries opened a new era in neutrino physics: massive and mixed neutrinos became real physical objects. Many new experiments must be implemented to investigate further neutrino properties and to reveal the physics which governs them. Undoubtedly the investigation of neutrino properties is one of the most important directions in the search for new physics.

Neutrino physics could also greatly help in solving other problems. One example is the so-called problem of the spin of the nucleon, which is connected with the strange content of the nucleon. The detailed investigation of NC neutrino-induced processes and, specifically, elastic neutrino (antineutrino)–proton scattering could allow us to obtain model-independent information on the strange form factors of the nucleon [38]. Here the neutrino plays the role of a probe to test the hadronic structure at a high level of precision: for this a deep knowledge of the structure of the neutrino currents is required.

Acknowledgments

We would like to thank warmly C. Giunti and W. Grimus for numerous fruitful discussions on the problem of neutrinos and for useful advice in the preparation of the manuscript. S.M.B. would like to thank Prof. H. Pietschmann, Director of the Institute for Theoretical Physics of the University of Vienna, for hospitality and support.

References

[1] T. Kajita, Super-Kamiokande Collab., Talk presented at *Neutrino '98*, Takayama, Japan, June 1998;
Y. Fukuda *et al.*, Super-Kamiokande Collab., Phys. Rev. Lett. **81**, 1562 (1998).

[2] J.D. Vergados, Phys. Reports 133, 1 (1986).

[3] R. Becker-Szendy *et al.*, IMB Collab., Nucl. Phys. (Proc. Suppl.) B **38**, 331 (1995).

[4] W.W.M. Allison *et al.*, Soudan-2 Collab., Phys. Lett. B **391**, 491 (1997).

[5] Y. Fukuda *et al.*, Kamiokande Collab., Phys. Lett. B **335**, 237 (1994).

[6] M. Ambrosio *et al.*, MACRO Collab., Phys. Lett. B **434**, 451 (1998).

[7] R. Davis, D.S. Harmer and K.C. Hoffman, Phys. Rev. Lett. **21**, 1205 (1968);
R. Davis, Prog. Part. Nucl. Phys. **32**, 13 (1994);
B.T. Cleveland *et al.*, Astrophys. J. **496**, 505 (1998).

[8] P. Anselmann *et al.*, GALLEX Collab., Phys. Lett. B **285**, 376 (1992);
W. Hampel *et al.*, GALLEX Collab., Phys. Lett. B **388**, 384 (1996).

[9] A.I. Abazov *et al.*, SAGE Collab., Phys. Rev. Lett. **67**, 3332 (1991);
D.N. Abdurashitov *et al.*, SAGE Collab., Phys. Rev. Lett. **77**, 4708 (1996).

[10] K.S. Hirata *et al.*, Kamiokande Collab., Phys. Rev. D **38**, 448 (1988);
K.S. Hirata *et al.*, Kamiokande Collab., Phys. Rev. Lett. **77**, 1683 (1996).

[11] K. Inoue, Super-Kamiokande Collab., Talk presented at the 5th International Workshop on *Topics in Astroparticle and Underground Physics*, Gran Sasso, Italy, September 1997;
Y. Fukuda *et al.*, Super-Kamiokande Collab., Phys. Rev. Lett. **81**, 1158 (1998);
Y. Suzuki, Super-Kamiokande Collab., Talk presented at *Neutrino '98*, 1998.

[12] C. Athanassopoulos *et al.*, LSND Collab., Phys. Rev. Lett. **77**, 3082 (1996);
C. Athanassopoulos *et al.*, LSND Collab., Phys. Rev. Lett. **81**, 1774 (1998);
C. Athanassopoulos *et al.*, LSND Collab., Phys. Rev. Lett. **75**, 2650 (1995);
C. Athanassopoulos *et al.*, LSND Collab., Phys. Rev. C **54**, 2685 (1996).

[13] D. Ayres *et al.*, MINOS Collab., report NUMI-L-63 (1995).

[14] P. Cennini *et al.*, ICARUS Collab., report LNGS-94/99-I (1994).

[15] H. Shibuya *et al.*, OPERA Collab., report CERN-SPSC-97-24 (1997);
P. Picchi and F. Pietropaolo, Talk presented at *Neutrino '98*, 1998 [e-print `hep-ph/9812222`].

[16] A. McDonald, SNO Collab., Talk presented at *Neutrino '98*, 1998;
SNO WWW page: `http://www.sno.queensu.ca`.

[17] L. Oberauer, Borexino Collab., Talk presented at *Neutrino '98*, 1998;
Borexino WWW pages: `http://almime.mi.infn.it/`;
`http://pupgg.princeton.edu/~borexino/welcome.html`.

[18] F. Suekane, preprint TOHOKU-HEP-97-02 (1997);
A. Suzuki, Talk presented at *NOW' 98*, Amsterdam, September 1998.

[19] Booster Neutrino Experiment (BooNE), `http://nu1.lampf.lanl.gov/BooNE`.

[20] F. Wilczek, Talk presented at *Neutrino '98*, 1998, e-print `hep-ph/9809509`.

[21] C. Caso *et al.*, Particle Data Group, Eur. Phys. J. C **3**, 1 (1998).

[22] S.M. Bilenky, C. Giunti and W. Grimus, e-print `hep-ph/9812360` [to be published in Progress in Particle and Nuclear Physics].

[23] M. Gell-Mann, P. Ramond and R. Slansky, in *Supergravity*, , edited by F. van Nieuwenhuizen and D. Freedman, North Holland, Amsterdam (1979), p. 315;
T. Yanagida, Procceedings of the *Workshop on Unified Theory and the Baryon Number of the Universe*, KEK, Japan, 1979;
R.N. Mohapatra and G. Senjanović, Phys. Rev. Lett. **44**, 912 (1980).

[24] B. Pontecorvo, J. Exptl. Theoret. Phys. **34**, 247 (1958) [Sov. Phys. JETP **7**, 172 (1958)];
B. Pontecorvo, Zh. Eksp. Teor. Fiz. **53**, 1717 (1967) [Sov. Phys. JETP **26**, 984

(1968)];
V. Gribov and B. Pontecorvo, Phys. Lett. B **28**, 493 (1969);
S.M. Bilenky and B. Pontecorvo, Phys. Rep. **41**, 225 (1978).

[25] W. Grimus, P. Stockinger and S. Mohanty, Phys. Rev. D **59**, 013011 (1998).

[26] Y. Suzuki, in Procceedings of *WIN 99*, Cape Town, 24–30 January, 1999.

[27] J.N. Bahcall, P.I. Krastev and A.Yu. Smirnov, Phys. Rev. D **58**, 096016 (1998).

[28] J.N. Bahcall, S. Basu and M.H. Pinsonneault, Phys. Lett. B **433**, 1 (1998).

[29] B. Zeitnitz, KARMEN Collab., Talk presented at *Neutrino '98*, 1998.

[30] A. De Rujula *et al.*, Nucl. Phys. B **168**, 54 (1980);
V. Barger and K. Whisnant, Phys. Lett. B **209**, 365 (1988);
S.M. Bilenky, M. Fabbrichesi and S.T. Petcov, Phys. Lett. B **276**, 223 (1992);
S.M. Bilenky, C. Giunti and C.W. Kim, Astropart. Phys. **4**, 241 (1996).

[31] M. Apollonio *et al.*, CHOOZ Collab., Phys. Lett. B **420**, 397;
C. Bemporad, Talk presented at the Ringberg Euroconference *New Trends in Neutrino Physics*, Ringberg Castle, Tegernsee, Germany, May 1998.

[32] S.M. Bilenky and C. Giunti, Phys. Lett. B **444**, 379 (1998).

[33] X. Shi and D.N. Schramm, Phys. Lett. B **283**, 305 (1992).

[34] S.M. Bilenky, C. Giunti and W. Grimus, Eur. Phys. J. C **1**, 247 (1998).

[35] N. Okada and O. Yasuda, Int. J. Mod. Phys. A **12**, 3669 (1997).

[36] S.M. Bilenky, C. Giunti, W. Grimus and T. Schwetz, e-print hep-ph/9903454.

[37] S.M. Bilenky, C. Giunti, W. Grimus and T. Schwetz, e-print hep-ph/9804421 [to be published in Astropart. Phys.].

[38] W.M. Alberico, S.M. Bilenky, C. Giunti and C. Maieron, Z. für Physik C **70**, 463 (1996) and references therein.