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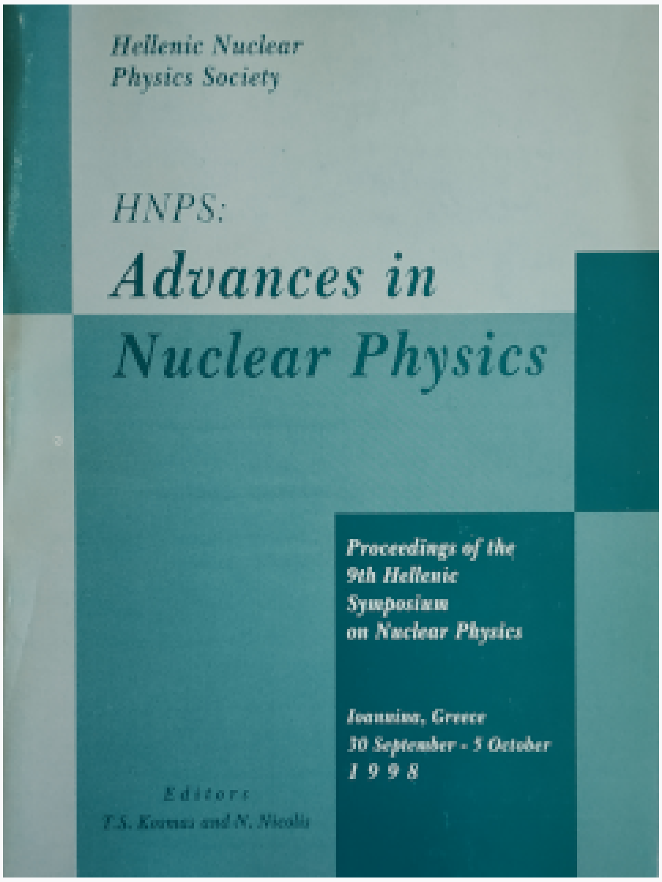

Super-Kamiokande neutrino oscillations and the supersymmetric model

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Super-Kamiokande neutrino oscillations and the supersymmetric model

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Abstract

The standard model predicts a ratio of 2 for the number of atmospheric muon to electron neutrinos, while super-Kamiokande and others measure a much smaller value (1.30 ± 0.02 for super-Kamiokande). Super-Kamiokande is also able to measure roughly the direction and the energy of the neutrinos. The zenith-angle dependence for the muon neutrinos suggests that the muon neutrinos oscillate into a third neutrino species, either into the τ neutrino or a sterile neutrino. This finding is investigated within the supersymmetric model. The neutrinos mix with the neutralinos, this meaning the wino, the bino and the two higgsinos. The 7×7 mass matrix is calculated on the tree level. One finds that the mass matrix has three linearly dependent rows, which means that two masses are zero. They are identified with the two lightest neutrino masses. The fit of the super-Kamiokande data to oscillations between three neutrinos yields, together with the result of supersymmetry, that the third neutrino mass lies between 2×10^{-2} and 10^{-1} eV. The two lightest neutrino masses are in supersymmetry on the tree level zero. The averaged electron neutrino mass which is the essential parameter in the neutrinoless double-beta decay is given by $\langle m_{\nu e} \rangle \simeq m_{\nu_3} P_{3e} \leq 0.8 \times 10^{-2}$ eV (95% confidence limit). It is derived from the super-Kamiokande data in this supersymmetric model to be two orders smaller than the best value (1 eV) from the neutrinoless double-beta decay.

1 Introduction

Five hundred and fifty years before Christ, Empedokles from Agragas in Sicily (today Agrigento) came up with the idea that there are four elements: earth, water, air and fire. In addition he had two forces, an attractive force $\phi\iota\lambda\acute{\iota}\alpha$ (friendship) and a repulsive force $\nu\varepsilon\tilde{\iota}\kappa o\varsigma$ (hate). Two hundred years later Aristoteles supplemented the four elements by a fifth one, the ether, which according to him fills the spheres beyond the Moon. According to his understanding there could be no vacuum and thus he needed a 'quinta essentia'. Qualitatively this picture explained many things and gave a rational unified view of our world, although we would say today that this hypothesis was not correct. This model of our universe and the forces in it survived 2000 years and was the official standard model in medieval times.

Newton made a decisive step forward in 1687 in his *Principia*. He was the first to assume that the forces in Heaven are the same as those on Earth. The famous story with the Moon and the apple illustrates this fact. Even if it might not be true, it is well invented: while an apple was falling down in Newton's garden he had the idea that the same force which keeps the Moon in orbit around the Earth is also responsible for the apple falling from the tree.

Maxwell unified the electric and magnetic forces into a single force. He showed that it depends only on the reference frame if we call something an electric or a magnetic field. A charge at rest has only an electric field, while the same charge in another reference frame, in which it is moving, has also a magnetic field. Since Maxwell we speak of electromagnetic forces.

In 1968 Glashow, Salam and Weinberg unified the electromagnetic forces of Maxwell and the weak interaction of Fermi into the electroweak forces.

Since the 1970s one formulates grand unified theories which treat the electroweak and the strong forces of quantum chromodynamics (QCD) as a single force. There exists a large variety of grand unified theories which describe the known data. High-precision experiments at low energies like double-beta decay or extremely high energy experiments are needed to distinguish between the different theories.

The last force, gravitation, might be also included in a unification in supersymmetric models. At least there are steps in this direction. A generalization of supersymmetric models from a global gauge theory into a local gauge theory is called supergravity. This might be a way to quantize general relativity and include all forces.

The neutrino plays an important role in testing grand unified theories and supersymmetric models. Indeed the neutrino was always an interesting particle, since it was proposed by Wolfgang Pauli in a famous letter from 4th December 1930 which he wrote from Zürich to a conference of the German Physical Society on radioactivity at Tübingen. This well-known letter starts with: "Sehr geehrte radioaktiven Damen und Herren" (Dear radioactive ladies and gentlemen). In this letter Pauli proposes the existence of a 'neutron' to conserve energy and angular momentum in beta decay.

2 Neutrino Oscillations

If the neutrinos are massive, the 'mass eigenstates' must not be identical with the 'weak eigenstates', in which the different neutrinos are created. In the standard model we have for each of the three families a neutrino: ν_e, ν_μ and ν_τ. The mass eigenstates ν_1, ν_2 and ν_3 are connected with the weak eigenstates ν_e, ν_μ, ν_τ by a unitary transformation

$$\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = \begin{pmatrix} U_{1e} & U_{1\mu} & U_{1\tau} \\ U_{2e} & U_{2\mu} & U_{2\tau} \\ U_{3e} & U_{3\mu} & U_{3\tau} \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} . \tag{1}$$

The neutrinos are created in a weak eigenstate ν_e, ν_μ or ν_τ, but they propagate in the mass eigenstates. If we consider for a moment only two neutrinos and invert (1) using unitarity, we obtain

$$\begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \cos\vartheta & -\sin\vartheta \\ \sin\vartheta & \cos\vartheta \end{pmatrix} \begin{pmatrix} \nu_2 \\ \nu_3 \end{pmatrix} ,$$

$$\nu_\mu(t) = \cos\vartheta |\hat{\nu}_2\rangle \exp(\mathrm{i}pr - \mathrm{i}E_2 t) - \sin\vartheta |\hat{\nu}_3\rangle \exp(\mathrm{i}pr - \mathrm{i}E_3 t) , \tag{2}$$

$$E_2 = \sqrt{p^2 + m_{\nu_2}^2} \approx p + \tfrac{m_{\nu_2}^2}{2p} ,$$

$$E_3 = \sqrt{p^2 + m_{\nu_3}^2} \approx p + \tfrac{m_{\nu_3}^2}{2p} .$$

The probability to find a τ neutrino after the distance L, if a μ neutrino is created, is given by the overlap of the τ neutrino wave function at time $t = 0$ and the μ neutrino wave function at $t = L/c$

$$\begin{aligned} P_{\mu\to\tau}(t = L/c) &= |\langle \nu_\tau(t=0) | \nu_\mu(t = L/c)\rangle|^2 \\ &= \sin^2(2\vartheta) \sin^2\left[\frac{1.27\,\Delta m^2\,(\mathrm{eV})\,L\,(\mathrm{km})}{E_\nu\,(\mathrm{GeV})}\right] , \end{aligned} \tag{3}$$

with $\Delta m_{32}^2 = m_3^2 - m_2^2$.

3 Atmospheric Neutrinos and the Super-Kamiokande Experiment

Cosmic rays bombard the atmosphere of the Earth with protons and nuclei with energies up to 10^{20} eV. These particles interact strongly and produce specifically very many pions in the atmosphere

$$\begin{aligned} \pi &\longrightarrow \mu + \nu_\mu \;, \\ &\mu \rightarrow e + \nu_\mu + \nu_e \;, \\ &N_{\nu\mu}/N_{\nu e} = 2 \;. \end{aligned} \tag{4}$$

The standard model predicts therefore for the ratio of muon to electron neutrinos a value of 2. Experimentally this ratio is found to be much lower. The best value of 1.30 ± 0.02 is obtained from the super-Kamiokande experiment [1]. But super-Kamiokande cannot only measure the ratio of μ to e neutrinos; it can also determine roughly the direction and the energy of the different neutrinos. The detector consists of a cylindrical cavity in a mine with 60 000 tons of very pure water, 35 meters high and 35 meters in diameter. An inner cylinder with 25 000 tons of water is used as the real detector. The inside of this inner cylinder is lined with more than 11 000 photomultiplier tubes (PMTs), each 50 cm in diameter. The outside of the inner cylinder is optically shielded and is used to eliminate background produced by beta-decay of neutrons, emitted from nuclei in the wall of the cavity by high-energy muons from the cosmic radiation, which penetrate through the mountain. The detection of the electron and muon neutrinos and antineutrinos is done using the Čerenkov radiation of the relativistic electrons, positrons and muons from the inverse beta decay

$$\begin{aligned} \overline{\nu}_e + p &\longrightarrow n + e^+ \;, \\ \nu_e + n &\longrightarrow p + e^- \;, \\ \overline{\nu}_\mu + p &\longrightarrow n + \mu^+ \;, \\ \nu_\mu + n &\longrightarrow p + \mu^- \;. \end{aligned} \tag{5}$$

Electrons and muons can be distinguished by the muon and the electron Čerenkov rings. The direction of the electrons and muons is within 10 to 20 degrees identical with the direction of the neutrinos. The size of the rings and the total light output allow us to determine the energy of the neutrinos. The results are shown in Fig. 1 as a function of the zenith angle θ, where $\theta = 0$ corresponds to neutrinos from above and $\theta = \pm 180$ degrees to neutrinos from below, which were formed somewhere over the South Atlantic and traveled through the Earth to Japan. Figure 1 shows for the electron neutri-

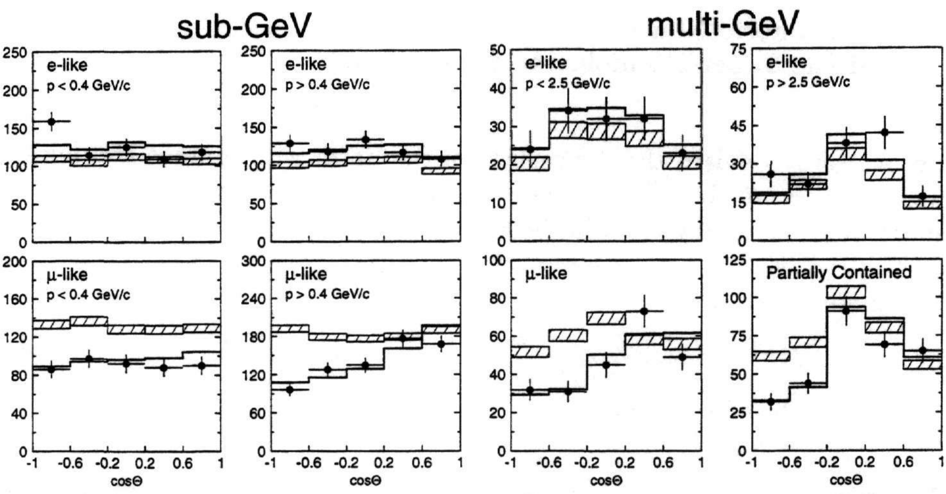

Fig. 1. Zenith-angle distribution of μ and e neutrino events for sub-GeV and multi-GeV data sets. The zenith-angle is given as $\cos\theta$. $\cos\theta = 1$ means neutrinos from above with a short path from about 10 to 20 km created in the atmosphere. Kamiokande neutrinos produced by cosmic radiation over the South Atlantic, which travel through the Earth to Japan, have $\cos\theta = -1$. The sub-GeV events are divided into the ones below 400 MeV/c and the ones above 400 MeV/c. The electron neutrinos for multi-GeV events are divided into the ones below and the ones above 2.5 GeV/c. All muon neutrino-like events fully contained in the inner counter are shown in the third position in the lower row. The last figure in the lower row shows muon-like events ('μ neutrinos') which are not fully contained in the inner counter volume. Monte Carlo expectation for no oscillations with statistical errors (*hatched region*). Best-fit (*bold line*) for μ neutrino to τ neutrino oscillations with an overall flux normalization fitted as a free parameter

nos practically no deviation from a Monte Carlo simulation without neutrino oscillations. For the muon neutrinos ('μ-like' events), already one sees below 0.4 GeV/c a depletion, which may indicate that at this low energy muon neutrinos from all directions oscillate to τ neutrinos (or to a sterile neutrino). Between 0.4 and 1.0 GeV/c one sees clearly that there are more μ neutrino events from above than from below, especially if one compares with Monte Carlo expectations. The same is true for the fully contained events above 1 GeV/c and also for the partially contained μ neutrino events. This behavior is made clear in Fig. 2. A two-neutrino fit to the data as indicated in (2) and (3) yields the following results

$$\sin^2 2\vartheta \geq 0.82 \, ,$$
$$5 \times 10^{-4} < \Delta m^2 = m^2_{\nu_3} - m^2_{\nu_2} < 6 \times 10^{-3}\,(\mathrm{eV}^2) \, . \tag{6}$$

The results are graphically shown in Fig. 3. Barger, Weiler and Whishnant did a three-neutrino analysis of the super-Kamiokande data [2]. The unitarity of the transformation matrix (1) requires

$$P_{3e} = |U_{3e}|^2; \quad P_{3\mu} = |U_{3\mu}|^2; \quad P_{3\tau} = |U_{3\tau}|^2 \; ; \qquad\qquad (7)$$

$$P_{3e} + P_{3\mu} + P_{3\tau} = 1 \; .$$

Here $P_{3\mu}$ signifies the probability that the third mass eigenstate is a μ neutrino. The analysis of [2] is shown in Fig. 4. The 95% confidence limit indicated in

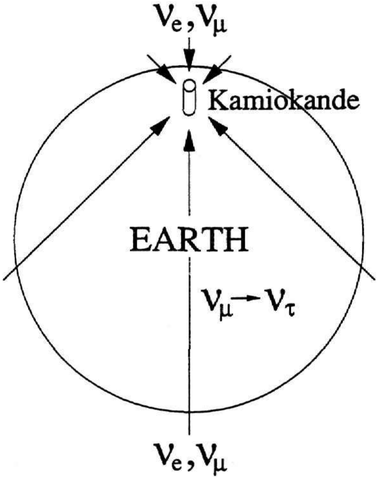

Fig. 2. Schematic drawing of the Earth with the super-Kamiokande detector in the upper part in a mine about 2 700 m of water equivalent below the mountains. The counter is a cylinder with 60 000 tons of purest water, 35 meters high and 35 meters in diameter. An inner cylinder with 25 000 tons of water is optically shielded from an outer area. This inner cylinder is lined with more than 11 000 photomultiplier tubes (PMTs) with a diameter of 50 cm. The PMT's measure the Čerenkov radiation from the relativistic electrons and muons. The light output and the opening angle of the Čerenkov cone give the energy. The Čerenkov ring determines the direction. Čerenkov rings from muons are more clearly defined, while the ones from electrons are more smeared out. Due to the long flight path of the neutrinos transversing the Earth, the muon neutrino can oscillate away. Since they do not appear as electron neutrinos, they must either oscillate to τ neutrinos, or to a sterile neutrino

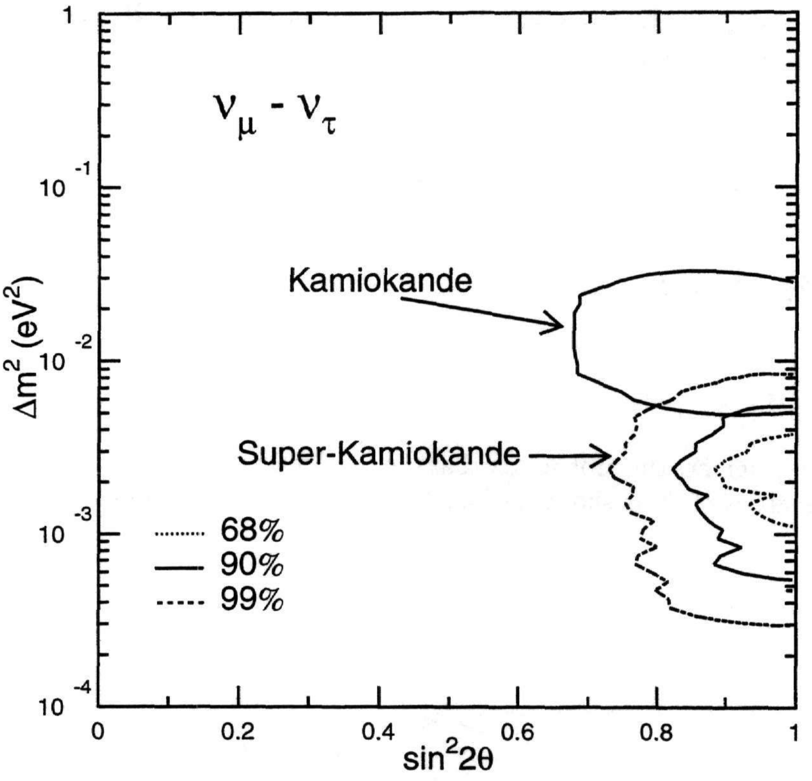

Fig. 3. The 68%, 90% and 99% confidence limits are shown for a Δm^2 and $\sin^2(2\theta)$ plot for μ to τ two-neutrino oscillations, based on the super-Kamiokande data. The lines are inclusion plots. The limits are given in (6). The super-Kamiokande data are more trustworthy

Fig. 4 can also be given as numbers

$$
\begin{aligned}
&0.5 \times 10^{-3} \le \delta m_{32}^2 \le 10 \times 10^{-3}\,(\mathrm{eV}^2)\,, \\
&0 \le P_{3e} \le 0.08\,, \\
&0.25 \le P_{3\mu} \le 0.75\,, \\
&0.25 \le P_{3\tau} \le 0.75 \\
&(95\%\ \text{confidence limit})\,.
\end{aligned}
\tag{8}
$$

The probability that the third neutrino mass eigenstate m_{ν_3} is the electron neutrino is most likely zero with an upper limit of 0.08. Due to the unitarity (7) $P_{3\tau}$ must be 0.75 if $P_{3\mu}$ is 0.25. In general the sum of all three probabilities must be unity.

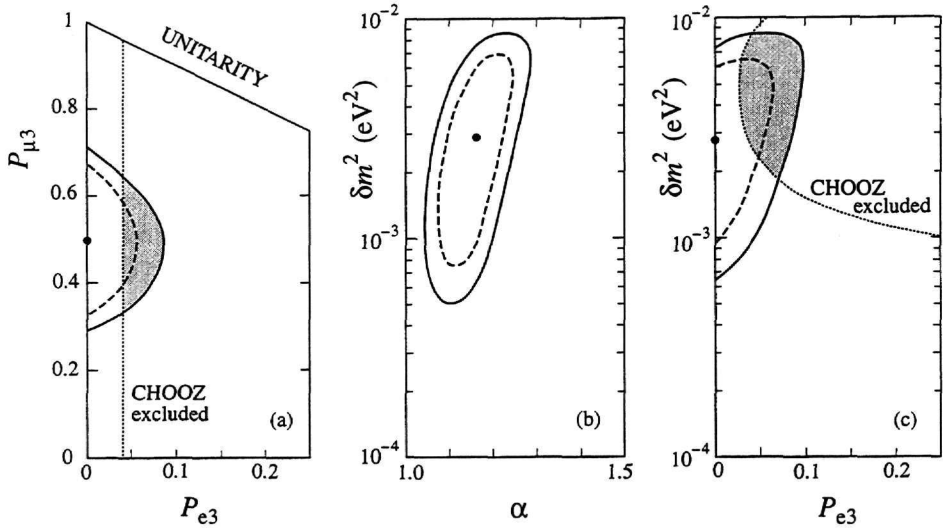

Fig. 4. Three-oscillation fit [2] to the super-Kamiokande data. The plot shows the 95% confidence limit for the probability that (**a**) the third mass eigenstate is an electron neutrino P_{3e}, that the third mass eigenstate is a muon neutrino $P_{3\mu}$, (**b**) the overall flux normalization for $\alpha_0 = 1.16$ and (**c**) the difference of masses squared $\delta m_{32}^2 = (m_3^2 - m_2^2)$ (eV^2). Additional exclusions (*dashed area*) by the CHOOZ neutrino-oscillation experiment [3]

4 Supersymmetry and Neutrino Oscillations

Supersymmetry (SUSY) was invented to reduce the divergence of the mass of scalar bosons from a quadratic dependence on the cut-off mass to a logarithmic dependence. It also helps to solve the hierarchy problem: it fills the gap between the electroweak mass scale of about 100 GeV and the Planck mass of 10^{19} GeV. Supersymmetry puts bosons and fermions into one multiplet (see Table 1). Several books [4,7] and review articles [5,6] exist about supersymmetry.

In the minimal supersymmetric model each particle of the standard model has its SUSY partner. For example the weak doublet of the electron neutrino and the electron in the first lepton family gets as partners a sneutrino (s-neutrino) and a selectron (s-electron). For scalar particles like the Higgs doublet (h_d^0, h_d^-) one can get the antiparticles by Hermitian conjugation. For the supersymmetric multiplet including higgsinos, this is not possible, since the higgsinos are fermions. Thus we have a second multiplet and both are

Table 1
Properties of bosons and fermions, which supersymmetry puts into one multiplet. Since the properties of these two objects are so different, it took a long time to find out that there exist generators of groups which allow bosons and fermions to be put into one multiplet

Bosons $\|B\rangle$	*Fermions* $\|F\rangle$
Integer spin	Half-Integer spin
Tensor	Spinor
Commutator	Anti-commutator
$B_1B_2 - B_2B_1 = \delta(1,2)$	$F_1F_2 + F_2F_1 = \delta(1,2)$
Bose statistics	Pauli principle
Carrier of forces	Matter particles
Have classical limit	Quantum object
$Q_+\|B\rangle\alpha\|F\rangle$	$Q_-\|F\rangle\alpha\|B\rangle$

extended to supermultiplets (9)

$$L_1 = \begin{pmatrix} \nu_e \\ e^- \\ \tilde{\nu}_e \\ \tilde{e}^- \end{pmatrix} ; \; L_2 = \begin{pmatrix} \nu_\mu \\ \mu^- \\ \tilde{\nu}_\mu \\ \tilde{\mu}^- \end{pmatrix} ; \; L_3 = \begin{pmatrix} \nu_\tau \\ \tau^- \\ \tilde{\nu}_\tau \\ \tilde{\tau}^- \end{pmatrix} ;$$

$$Q_1 = \begin{pmatrix} u \\ d \\ \tilde{u} \\ \tilde{d} \end{pmatrix} ; \quad Q_2 = \begin{pmatrix} c \\ s \\ \tilde{c} \\ \tilde{s} \end{pmatrix} ; \quad Q_3 = \begin{pmatrix} t \\ b \\ \tilde{t} \\ \tilde{b} \end{pmatrix} ;$$

$$H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix} = \begin{pmatrix} h_u^+ \\ h_u^0 \\ \tilde{h}_u^+ \\ \tilde{h}_u^0 \end{pmatrix} ; \quad H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix} = \begin{pmatrix} h_d^0 \\ h_d^- \\ \tilde{h}_d^0 \\ \tilde{h}_d^- \end{pmatrix} . \tag{9}$$

In addition to the lepton and quark doublets, which are extended to quartet superfields for the leptons $L_i (i = 1, 2, 3)$ and for quarks $q_i (i = 1, 2, 3)$ one has also singlets, which in case of massless fields would be right handed. In the case of the minimal supersymmetric model (MSSM) they contain only e^-, μ^- and τ^- and the quarks d, s and b

$$E_{1s} = \begin{pmatrix} e^- \\ \tilde{e}^- \end{pmatrix}_s ; \quad E_{2s} = \begin{pmatrix} \mu^- \\ \tilde{\mu}^- \end{pmatrix}_s ; \quad E_{3s} = \begin{pmatrix} \tau^- \\ \tilde{\tau}^- \end{pmatrix}_s \tag{10}$$

and corresponding superfields for the d, s, b quarks, which we write as $D_{i\text{S}}$. The S indicates that it is a singlet with respect to particles and with respect to SUSY particles separately. SUSY particles are, in the above superfields, always indicated by a tilde. The superfields are written as capital letters.

The potential part of the minimal supersymmetric Lagrange density must always contain two superfields and a standard-model field, so that the R-parity is conserved (the R-parity is given as +1 for an even number of SUSY particles or as −1 for an odd number of SUSY particles)

$$W_{\text{MSSM}} = h^e_{ij} L_i H_d \overline{E}_{js} + h^d_{ij} Q_i H_d D_{js} + h^u_{ij} Q_i H_u \overline{U}_{js} + \mu H_d \cdot H_u \, . \tag{11}$$

The bar indicates the conjugate superfields and the indices i, j run over the three families, while s indicates the singlet. The two Higgs superfields H_u and H_d are defined in (9). The gauge bosons and their SUSY partners are contained in the kinetic energy in the covariant derivative. The SUSY partners of the gauge bosons are called *photinos* ($\tilde{\gamma}$), *zinos* ($\tilde{Z}^0$), *winos* ($\tilde{W}^{+/-}, \tilde{W}^0$) and *binos* ($\tilde{B}^0$). The terms are constructed so that the Lorentz structure is guaranteed and that hypercharge is conserved. The lepton superfields L_i have for example hypercharge $Y = -1$ while the singlets E_{js} have hypercharge $Y = -2$. The conjugate fields have naturally opposite hypercharge. The quark superfields Q_i have hypercharge $Y = 1/3$ and the singlet quark fields D_{js} have $Y = -2/3$. The hypercharge of the Higgs superfield H_u is $Y = 1$ and for H_d it is $Y = -1$. R-parity conservation in (11) guarantees that the number of SUSY particles stays either always even or odd.

R-parity-violating terms violate supersymmetry. One divides the R-parity-violating potential part of the Lagrange density into those terms which contain one SUSY field only ($W_{\not{R}}$) and into those which contain three SUSY fields ($V_{\not{R}}$)

$$\begin{aligned} W_{\not{R}_P} &= \frac{1}{2}\lambda_{ijk} L_i L_j \overline{E}_{ks} + \lambda'_{ijk} L_i Q_j \overline{D}_{ks} \\ &\quad + \frac{1}{2}\lambda''_{ijk} \overline{U}_{is} \overline{D}_{js} \overline{D}_{ks} + x_i L_i \cdot H_u \, , \\ V_{\not{R}_P} &= \Lambda_{ijk} \tilde{L}_i \tilde{L}_j \tilde{E}^c_{ks} \\ &\quad + \Lambda'_{ijk} \tilde{L}_i \tilde{Q}_j \tilde{D}^c_{ks} + \Lambda''_{ijk} \tilde{U}^c_{is} \tilde{D}^c_{js} \tilde{D}^c_k \end{aligned}$$

$$+\mu_{uj}^2 \tilde{L}_j H_u + \mu_{dj}^2 \tilde{L}_j H_d^+ \ . \tag{12}$$

The upper index c at SUSY fields indicates the conjugate field. The standard-model Higgs field h_d has a finite vacuum expectation value, by which the vector bosons of the weak interaction get a mass (Higgs mechanism). In supersymmetry we now have two Higgs doublets with finite vacuum expectation values which might be different

$$\begin{aligned} \langle h_u \rangle &= \left\langle \begin{matrix} h_u^+ \\ h_u^0 \end{matrix} \right\rangle = \begin{pmatrix} 0 \\ c \end{pmatrix} , \\ \langle h_d \rangle &= \left\langle \begin{matrix} h_d^0 \\ h_d^- \end{matrix} \right\rangle = \begin{pmatrix} b \\ 0 \end{pmatrix} . \end{aligned} \tag{13}$$

The parameters of the Lagrange density (11) and (12) can now be such that this finite expectation value of a Higgs field leads to a finite vacuum expectation value of the sneutrino fields

$$\begin{aligned} V_{\tilde{\nu}} &= -d\tilde{\nu} + f\tilde{\nu}^2 , \\ \langle V_{\tilde{\nu}} \rangle &\approx -d\langle \tilde{\nu} \rangle + f\langle \tilde{\nu} \rangle^2 . \end{aligned} \tag{14}$$

This finite sneutrino expectation value now allows that all neutral particles with spin-1/2 can mix with each other. These particles are the three neutrinos and the neutralinos (wino, bino and the two higgsinos $\tilde{h}_u^0$, $\tilde{h}_d^0$)

$$(\nu_e,\ \nu_\mu,\ \nu_\tau,\ \tilde{W}^0,\ \tilde{B}^0,\ \tilde{h}_d^0,\ \tilde{h}_u^0) \ . \tag{15}$$

Figure 5 shows the tree diagrams for this mixing. The loop with the finite vacuum expectation value of the sneutrino $\langle \tilde{\nu} \rangle$ is counted as a tree diagram, since the sneutrino finite expectation value acts like an outside potential.

The structure of the mass matrix from the above neutrino and neutralino mixing is the following

$$M_0 = \begin{pmatrix} 0 & m \\ m^T & M_\chi \end{pmatrix} ,$$

$$
M = \begin{pmatrix} M_z S_w C_\beta v_e & M_z C_w C_\beta v_e & 0 & -\mu u_e \\ -M_z S_w C_\beta v_\mu & M_z C_w C_\beta v_\mu & 0 & -\mu u_\mu \\ -M_z S_w C_\beta v_\tau & M_z C_w C_\beta v_\tau & 0 & -\mu u_\tau \end{pmatrix} ,
$$

$$
M_\chi = \begin{pmatrix} M_d & 0 & -M_z S_w C_\beta & M_z S_w S_\beta \\ 0 & M_u & M_z C_w C_\beta & -M_z C_w S_\beta \\ -M_z S_w C_\beta & -M_z C_w S_\beta & -\mu & 0 \\ M_z S_w S_\beta & -M_z C_w S_\beta & -\mu & 0 \end{pmatrix} , \tag{16}
$$

where

$$
\begin{aligned}
&\tan\beta = \langle h_u^\circ \rangle / \langle h_d^\circ \rangle , \quad S_w = \sin\vartheta_w , \\
&c_w = \cos\vartheta_w , \quad S_\beta = \sin\beta , \quad C_\beta = \cos\beta , \\
&v_i = \langle \tilde{\nu}_i \rangle / \langle h_d^\circ \rangle = \langle \tilde{\nu}_i \rangle / b \ll 1 , \\
&u_i = \mu_i / \mu \ll 1 .
\end{aligned}
$$

M_0 of (16) is the mass matrix. Its upper left element is the neutrino-mass matrix, which is identically zero, and in the off-diagonal position we have

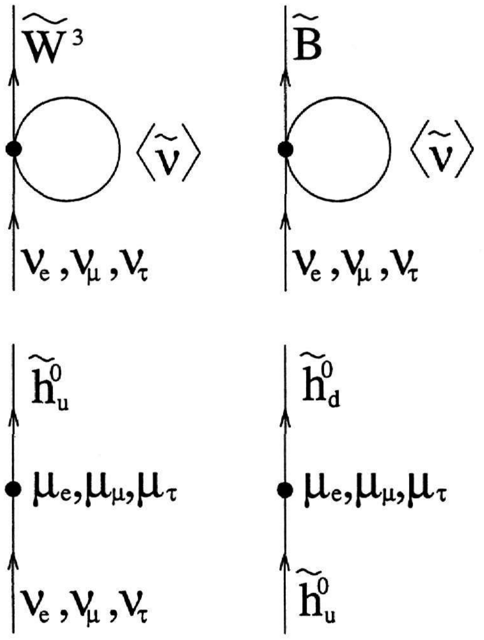

Fig. 5. Tree diagrams for the mixing of the three neutrinos with the neutralinos (wino, bino and the two higgsinos). The loop with the finite vacuum expectation value of the sneutrino is counted as a tree diagram, since it acts like an outside potential. The selection rules can partially be understood by hypercharge conservation

the mixing between the neutrinos and the neutralinos in the sequence (wino, bino, down-higgsino, up-higgsino) = $(\tilde{W}^0, \tilde{B}^0, \tilde{h}_d^0, \tilde{h}_u^0)$. The quantities in the mass matrix are the Z^0 mass M_z and the higgsino masses M_d, M_d, the Weinberg angle ϑ_W and the angle β defined as the tangent of the ratio of the finite vacuum expectation values of the neutral up- and neutral down-Higgs particles.

In our work [8] we realized that the mass matrix M_0 has three linearly dependent rows and therefore two mass eigenvalues are zero. This is independent of the details of the parameters apart from using a finite vacuum sneutrino expectation value $\langle\tilde{\nu}_e\rangle$ and an expansion in the quantities which are assumed to be small in (16). The coupling constants μ_i and μ are defined in (11) and (12). The linear dependence of three rows of the mass matrix yields two masses to be identically zero. We identify these masses with the two lowest mass eigenstates of the neutrinos. So on the tree level we have the masses

$$m_{\nu_1} = 0; \;\; m_{\nu_2} = 0; \;\; m_{\nu_3} \neq 0 \,. \tag{17}$$

Naturally the zero masses for the first two neutrinos are modified if the loop corrections are included.

If we now use the result of the super-Kamiokande Collaboration (8), we can immediately extract from δm_{32}^2 the limits for the mass of the third neutrino. One can now also derive the parameter for the 'averaged' electron neutrino mass, which is the quantity relevant for the zero neutrino double-beta decay

$$2 \times 10^{-2}\,\mathrm{eV} \leq m_{\nu_3} \leq \; 10^{-1}\,\mathrm{eV}\,, \tag{18}$$

$$\langle m_{\nu e}\rangle = \sum_{i=1}^{3} m_{\nu i}\xi|U_{ie}|^2 \,. \tag{19}$$

The sum runs over the three neutrino families. If one has a left–right symmetric model from grand unification, the sum goes over three light and three heavy right-handed neutrinos. Here we assume that the mixing coefficients between the light and the heavy neutrinos are so small that they can be neglected or that we have only left-handed neutrinos. The quantity ξ_i is the CP (C = charge conjugation; P = parity) relative phase. If only one mass of a neutrino is different from zero, this relative phase can be chosen to be $\xi_3 = 1$. Thus we get from (19) an expression for the 'averaged' electron neutrino mass

$$\langle m_{\nu e}\rangle = m_{\nu_3}|U_{3e}|^2 = m_{\nu_3}P_{3e} \leq 0.8 \times 10^{-2}\,\mathrm{eV} \quad (95\%\ \text{confidence limit})\,. \tag{20}$$

This limit is by two orders of magnitude more stringent than the limit derived from zero neutrino double-beta decay which is presently at around 1 eV

$$0\nu\beta\beta \text{ decay}: \ \langle m_{\nu e}\rangle \leq 1\,\text{eV} . \tag{21}$$

It is interesting to see in (20) that the averaged electron neutrino mass is essentially determined by the third mass eigenstate. It is only so small due to the fact that the probability of the third mass eigenstate being an electron neutrino is less than 0.08 according to the super-Kamiokande data (8).

5 Summary

We started out by discussing atmospheric neutrinos produced by cosmic radiation. They are created by the decay of pions from the interaction of the cosmic rays with the atmosphere. The pions decay in about 10^{-8} s to a muon and a μ neutrino and the muon again into an electron, a μ neutrino and an e neutrino, where the particle or antiparticle character depends on whether one starts with a positive or a negative pion. The ratio of μ neutrinos to e neutrinos should be 2. But experiments show that the ratio lies somewhere between 1 and 1.4. super-Kamiokande finds a ratio $N_{\nu\mu}/N_{\nu e} = 1.30 \pm 0.02$. But super-Kamiokande can also measure roughly the energy and the direction of the neutrinos. One finds that the muon to electron neutrino ratio is larger for neutrinos coming from above (zenith angle = 0) than the ratio for neutrinos coming through the Earth from below. This is explained by the fact that due to the long path of the neutrinos coming from below, the μ neutrinos with $L \approx 13\,000$ km can oscillate away into τ neutrinos (or into a sterile neutrino). The fit to the super-Kamiokande data allowing oscillations between two neutrino species yields the result (8). The third neutrino mass state has practically no overlap with the electron neutrino, but large overlaps of about 50% each with μ and τ neutrinos.

Supersymmetry allows us to mix the three neutrino states with the neutralinos (wino, bino, down-higgsino, up-higgsino). This 7×7 mass matrix has three linearly dependent rows and therefore yields two neutrinos with masses zero. Only the third mass eigenstate of the neutrinos is, on the tree level, different from zero. The third mass eigenstate lies between 2×10^{-2} eV and 10^{-1} eV. The 'averaged' electron neutrino mass (19) which plays a decisive role in the neutrinoless double-beta decay, is less than 0.8×10^{-2} eV (see (20)). The limit from the neutrinoless double-beta decay is today around 1 eV, which is two orders of magnitude less stringent.

One could ask if the super-Kamiokande result is not affected by the Mikeyev–

Smirnov–Wolfenstein effect of neutrino oscillations in matter. First, the density of the Earth is much less than in the Sun. The smaller density would probably not be able to make the electron neutrino mass degenerate with the μ neutrino mass. But secondly the MSW effect does not apply: the μ neutrino and the τ neutrino masses which are important here, are shifted in the same way due to the universality of the weak interaction. (The Sun or the Earth does not contain muons or tauons. It contains only electrons. In this way only the mass of the electron neutrino is shifted differently, while the μ and the τ neutrinos experience the same shift and thus the mixing between μ and τ neutrinos is not modified by the MSW effect).

One should stress that the neutrino masses are also affected by loop diagrams which are not calculated here and which are assumed to be smaller. The solar-neutrino problem is attributed to the mass difference (δm_{12}^2). Oscillations between the electron and the μ eigenstates would be allowed due to loop corrections, but the mass differences would be smaller and one would then need a larger path L to see the oscillations.

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