Bound state effects in transverse momentum parton distributions

X. N. Maintas, F. K. Diakonos, G. D. Galanopoulos, N. K. Kaplis, C. I. Papoulias, H. E. Tsagkarakis

doi: 10.12681/hnps.2642

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X. N. Maintas,[a]F. K. Diakonos,[a] G. D. Galanopoulos,[a] N. K. Kaplis,[a] C. I. Papoulias,[a] and H. E. Tsagarakis[a]

[a]Department of Physics, University of Athens, GR-15771 Athens, Greece

We use a non-gaussian intrinsic transverse momentum distribution, associated with the wavefunction of the partonic ground state inside the proton, to calculate the production of π^0 with intermediate transverse momentum values in $p - p$ collisions at high energies. It is shown that a perfect description of the experimental data for different experiments is achieved using an almost constant value for the mean transverse momentum of the initial partons, compatible with Heisenberg's uncertainty relation ($\langle k_T \rangle \approx 0.3 \ GeV$). Our analysis suggests the presence of a non-vanishing, non-perturbative contribution from partonic bound states to the $pp \rightarrow \pi^o(\gamma) + X$ and $pp \rightarrow p + X$ cross section.

1. Introduction

Recent tests of the perturbative QCD are focused on the experimental and theoretical study of hard processes like direct photon and π^0 production with large transverse momentum in pp, pA and AA collisions. The main goal is to check the parton distribution functions (PDF) inside the proton as well as the parton fragmentation functions (PFF) determined by Deep Inelastic Scattering or e^+e^- annihilation. Especially the good understanding of the pp processes is crucial for any attempt to extract new physics, related to the formation of a quark-gluon plasma phase, from the pA and AA data. Extensive studies of the π (or γ) production in pp collisions have shown that the transverse momentum distribution $g(k_T)$ of the partons inside the proton, usually assumed to have a Gaussian form, has to be taken into account for a successful description of the observed p_T spectrum [1]. A new nonperturbative parameter is introduced through this approach: the mean instrinsic transverse momentum $< k_T >$ of the partons. However, for a specific class of pp collisions, including the π^0 and single γ production, the mean transverse momentum value needed for the description of the experimental data is too high ($< k_T > \approx 1 - 4 \ GeV$) and cannot be attributed to the internal structure of the proton. On the other hand, as mentioned in [1], the form of $g(k_T)$ can influence significantly the value of $< k_T >$ as well as its p_T dependence. Therefore, in the present work we use a recently derived transverse momentum distribution for the description of π^0 and single γ production within the framework of the improved parton model. This distribution is associated with the ground state wave function of the partons inside the proton and is determined through a potential quark model which has been successfully used to describe the spectra of mesonic and baryonic bound states in the past [2]. It turns out that in this case a relatively low mean transverse momentum $< k_T > (\approx O(300 \ MeV))$, compatible

with intrinsic dynamics inside the proton, for the initial partons is sufficient in order to fit perfectly the experimental data. Our analysis indicates that partonic bound state effects in the transverse direction are important for a successfull description of the experimental data concerning the pion (and gamma) production in pp collisions. The paper is organized as follows: in Section 2 we present the parton model differential cross section for the π^0 production in pp collisions. In section 3 we describe briefly the derivation of the intrinsic transverse momentum disrtibution for the partons inside the proton using the quark potential model of [2]. In section 4 we present our numerical results concerning the description of the data of four different experiments [3–6] as well as the corresponding dependence $< k_T(p_T) >$. Finally in section 5 we give our conclusions.

2. π^0 and direct photon production with large p_T

We consider processes of the type:

$$pp \rightarrow \pi^0(\gamma) + X$$

shown graphically in the figure below:
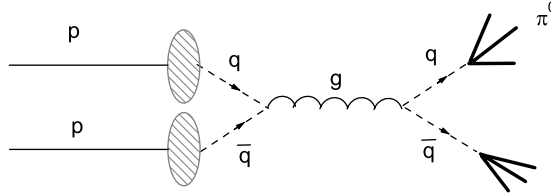
As usual we use the factorization theorem to absorb mass singularities in the parton distribution functions (PDF) and parton fragmentation functions (PFF). Within this scheme (pQCD improved parton model) the differential cross section for π^0 production is [1]:

$$E_\pi \frac{d\sigma}{d^3p}(pp \rightarrow \pi^0 + X) = K \sum_{abcd} \int dx_a dx_b f_{a/p}(x_a, Q^2) \times$$
$$f_{b/p}(x_b, Q^2) \frac{d\sigma}{d\hat{t}}(ab \rightarrow cd) \frac{D_{\pi/c}(z_c, \bar{Q}^2)}{\pi z_c} \tag{1}$$

where $f_{i/p}(x_i, Q^2)$ is the PDF for the involved partons $i = a, b$, x_i is the longitudinal momentum fraction of parton i, Q^2 is the momentum transfer scale, $\frac{d\sigma}{d\hat{t}}$ is the cross section for the partonic subprocess: $a + b \rightarrow c + d$ and $D_{\pi/c}(z_c, \bar{Q}^2)$ is the PFF for the fragmentation of a parton into a π^0 with momentum fraction z_c at momentum transfer scale $\bar{Q}^2$. Using Leading Log Approximation (LLA) we underestimate the experimental data for $\sqrt{s} < 60~GeV$ and $p_T < 7GeV$. The situation can be improved by taking into account the transverse momentum $\vec{k}_T$ of the initial partons $\bar{q}, q, g$ (*asymptotically free* for high momenta – *confined* for low momenta) inside the proton. A consistent way to determine the corresponding transverse momentum distribution is to use suitable projections of the

proton wave function in momentum space. In such a treatment Heisenberg's uncertainty relation implies:

$$\langle k_T \rangle_{parton/proton} \quad \sim \quad 0.2 - 0.5 \; GeV$$

Assuming further a factorization of the partonic momentum distribution into transverse momentum distribution × longitudinal momentum distribution (PDF)

$$dx_a G_{a/A}(x_a, Q^2) \Longrightarrow dx_a d^2 k_{T,a} g(\vec{k}_{T,a}) f_{a/p}(x_a, Q^2)$$

it is straightforward to include partonic transverse momentum effects in the scheme (1). The ususal choice for $g(\vec{k}_T)$ is a Gaussian distribution:

$$g(\vec{k}_T) = \frac{1}{\pi \langle k_T^2 \rangle} e^{-\frac{k_T^2}{\langle k_T^2 \rangle}}$$

In order to fit the experimental data one treats $\langle k_T \rangle$ as a free parameter (for a Gaussian $\langle k_T \rangle = \sqrt{\frac{\pi \langle k_T^2 \rangle}{4}}$)

In particular, using the above Gaussian form one needs, for a good description of the existing experimental data for $pp \to \pi^0(\gamma) + X$ at various energies $\sqrt{s}$ and p_T of the produced pion (photon), mean transverse momentum values in the range:

$$\langle k_T \rangle \quad \sim \quad 1 - 4 \; GeV$$

which are incompatible with proton's internal structure.

The proposal of the present work is to replace the Gaussian distribution by a more realistic form obtained as follows:

- Use a potential quark model to describe qq and $q\bar{q}$ interaction inside the proton

- Solve Schrödinger equation for the three body system qqq to obtain the corresponding wave function

- Use this wave function to extract the partonic transverse momentum distribution inside the proton

3. Bound states and wavefunctions of quarks in proton

To proceed according to the scenario proposed in the previous section we use a phenomenological confining quark-quark potential [2]:

$$V(r) = A_{qq} r^{0.1} + B_{qq}$$

(similar form with $A_{q\bar{q}}$, $B_{q\bar{q}}$ for quark-antiquark interaction) which has been used in the past to describe the meson and baryon spectra (in particular binding energy and size of the proton). Our approach is based on a non-relativistic treatment of the three-body problem

The Hamiltonian operator of the system is given as:

$$\hat{H} = -\frac{\hbar^2}{2m}(\nabla_1^2 + \nabla_2^2 + \nabla_3^2) + V(\vec{r}_{12}) + V(\vec{r}_{23}) + V(\vec{r}_{31})$$

Using Jacobi coordinates one obtains the translation invariant part by solving:

$$\frac{d^2 u_0}{d\xi^2} - \frac{15}{4\xi^2} u_0 + \frac{m}{\hbar^2}(E_G - \tilde{V}_{00}) u_0 = 0$$

for the ground state of the system ($\xi^2 = \frac{2}{3}(r_{12}^2 + r_{23}^2 + r_{31}^2)$)

The wave function in momentum space is:

$$\tilde{\Phi}(k_\xi^2) = N \int_0^\infty d\xi \, \xi^{1/2} u_0(\xi) \frac{J_2(k_\xi \xi) - k_\xi \xi J_3(k_\xi \xi)}{k_\xi^2}$$

where: $k_\xi^2 = k_1^2 + k_2^2 + \vec{k}_1 \cdot \vec{k}_2$ and N is a normalization constant

The one-particle transverse momentum density $g(\vec{k}_T)$ is obtained as:

$$g(\vec{k}_T) = 4\pi \int_0^\infty dk_z \int_{-1}^1 dz \int_0^\infty dk_2 k_2^2 |\tilde{\Phi}(k_T^2 + k_z^2 + k_2^2 + z k_2 \sqrt{k_T^2 + k_z^2})|^2$$

The momentum integrations in the above expression are performed numerically using the VEGAS Monte Carlo algorithm combined wiht the Gauss-Kronrod quadrature leading to the following form of the partonic transverse momentum distribution:
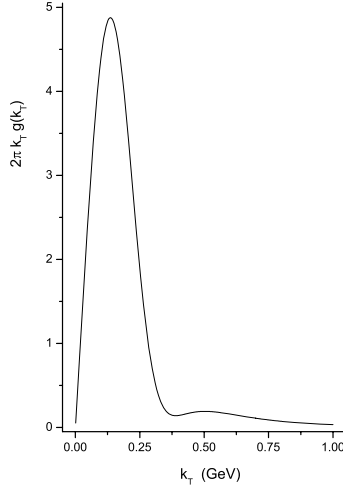
4. Numerical results - fits to the experimental data

Using the distribution $g(\vec{k}_T)$ obtained in the previous section as well as the pQCD improved parton model described in section 2 we attempt a theoretical description of the existing experimental data on: $pp \to \pi^0 + X$. We have considered 4 different sets of: K-factor, PDF, PFF, momentum transfer scales. To obtain an upper limit of $\langle k_T \rangle$ in next-to-leading (NL) order we finally choose:

- $K = 2$ in order to include effectively higher order subprocesses

- NL order MRST2004 PDF [7]

- NL order KKP2000 PFF [8]

- and $Q^2 = \bar{Q}^2 = \frac{2\hat{s}\hat{t}\hat{u}}{\hat{s}^2+\hat{t}^2+\hat{u}^2}$ for the factorization scale [9]

The results for the Chicago-Princeton experiment at Fermilab [3] are presented in Fig. 1. The corresponding $\langle k_T \rangle$ values needed to fit the CP data are given in Fig. 2. For completeness we also include in Figs. 1, 2 both the fit results as well as the required $\langle k_T \rangle$ values for the Gaussian case:
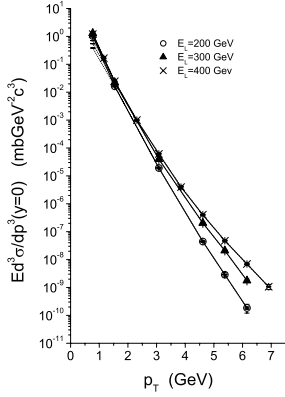
Figure 1. The differential cross section for CP experimental data at three different energies. With solid lines we present the theoretical calculations using a non-Gaussian partonic transverse momentum distribution while with dashed lines we give the corresponding results using a Gaussian $g(\vec{k}_T)$.
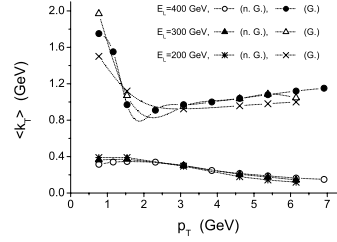
Figure 2. The $\langle k_T \rangle$ values needed for the description of the data shown in Fig. 1.

The analogous analysis for the π^0-production data at the CERN-ISR experiment [4] is shown in Fig. 3:
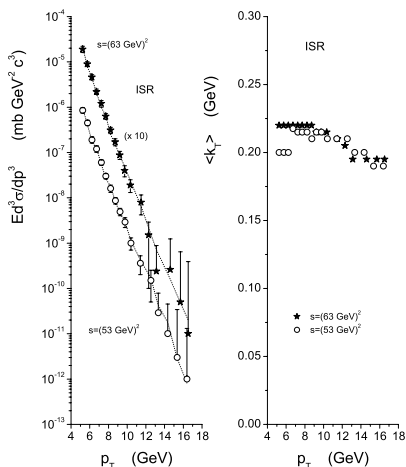
Figure 3. The results for the analysis of the CERN-ISR data using non Gaussian $g(\vec{k}_T)$.

while for the fixed target CERN-SPS experiment WA70 [5] is shown in Figs. 4, 5.
For comparison, in Fig. 5 we give also the $\langle k_T \rangle$ values for the CP experiment at $E_L = 300\ GeV$ which is very close to the WA70 energy. Finally in Figs. 6, 7 we present the results of the analysis of the PHENIX experiment at RHIC ($\sqrt{s} = 200\ GeV$) [6]. It is usefull to summarize all the $\langle k_T \rangle$ results of our analysis for the different experiments in a single plot and compare with the corresponding $\langle k_T \rangle$-values obtained using a Gaussian $g(\vec{k}_T)$. This is done in Fig. 8.

5. Conclusions

Using a quark potential model capable to describe consistently baryonic systems as three quark bound states we have derived an intrinsic transverse momentum distribution $g(\vec{k}_T)$ of partons inside the proton. Describing the k_T-smearing effects in the parton model for pp collisions through this non-Gaussian distribution we calculated the differential cross section for π^0 at midrapidity for different beam energies. Assuming that the corresponding nonperturbative parameter $\langle k_T \rangle$, related to $g(\vec{k}_T)$, depends on p_T of the finally produced hadron as well as the incident energy, we obtain a very good description of the experimental data measured in pp collisions at four different experiments. The corresponding values of $\langle k_T \rangle$ as a function of p_T could originate, according to Heisenberg's uncertainty relation, from the internal partonic structure of the proton and define a narrow band. Similar analysis of single γ-production is in progress. First results indicate that a similar behaviour is valid also in this case ($\langle k_T \rangle \sim [0.2 - 0.4]\ GeV$) [10]. In addition a model including non-Gaussian k_T-smearing effects for the description of pA and AA processes is under development.
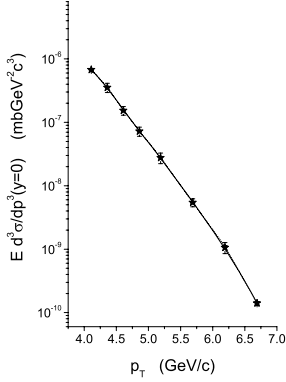
Figure 4. The differential cross section for the WA70 experimental data. With solid line we present the theoretical calculation using a non-Gaussian partonic transverse momentum distribution while with dashed line (not distinguishable in the plot) we give the corresponding result using a Gaussian $g(\vec{k}_T)$.
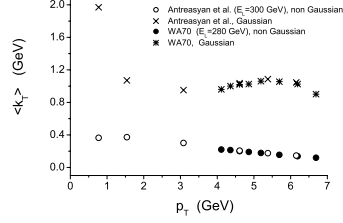
Figure 5. The $\langle k_T \rangle$ values needed for the description of the data shown in Fig. 4.

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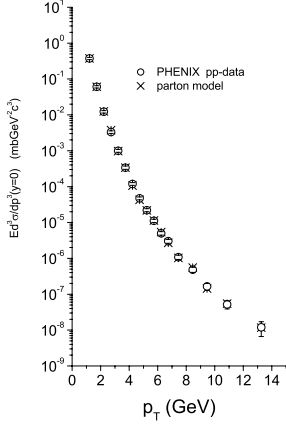
Figure 6. The differential cross section for the PHENIX data. With crosses we present the theoretical calculation using a non-Gaussian partonic transverse momentum distribution.
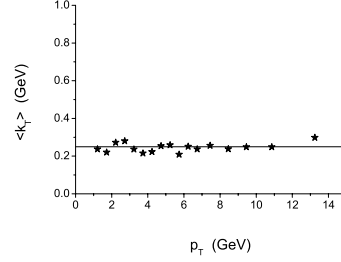
Figure 7. The $\langle k_T \rangle$ values needed for the description of the PHENIX data.
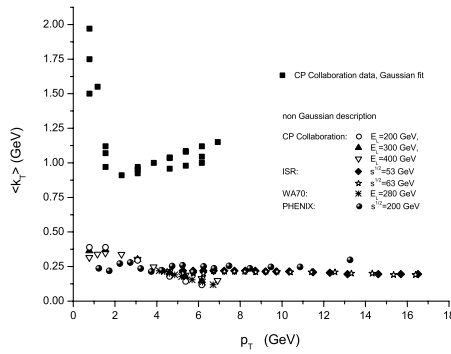

Figure 8. A comparative plot including all the $\langle k_T \rangle$ values needed for the description of the experimental data of the four experiments considered in our analysis both with the non Gaussian as well as the Gaussian $g(\vec{k}_T)$.