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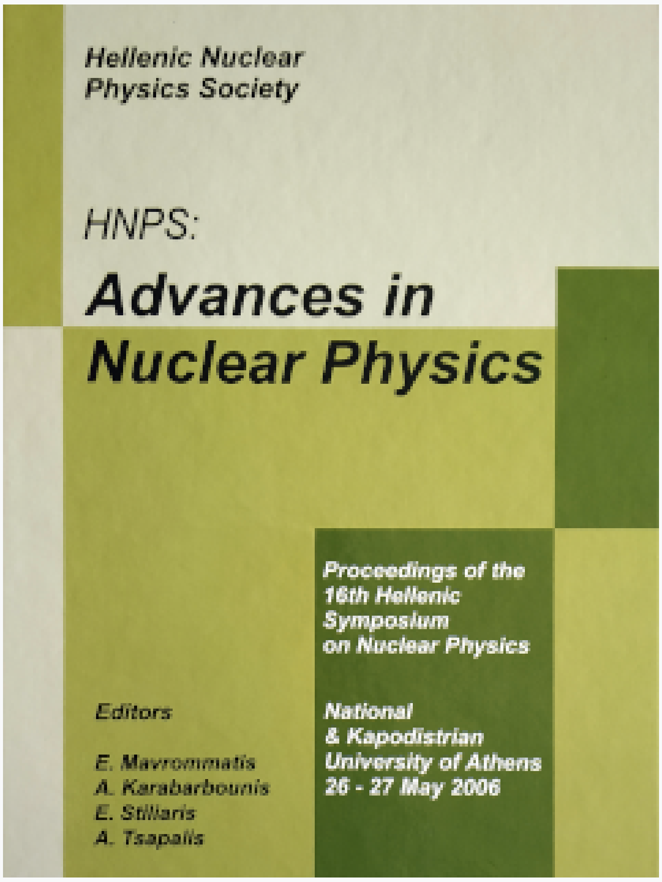

Nuclear phase transitions near the critical points: a study with the Relativistic Hartree-Bogoliubov model, the Interacting Boson Model and the Boson Coherent-State Framework

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Nuclear phase transitions near the critical points: a study with the Relativistic Hartree-Bogoliubov model, the Interacting Boson Model and the Boson Coherent-State Framework

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We present an analysis of the intensity of 2-particle transfer reactions in the Interacting-Boson Model (IBM), and in the Boson Coherent-State framework, as a tool to study nuclear phase transitions. We study transfer reactions between two ground states, and between the ground state and the band head of the beta-vibrational band. We suggest characteristic fingerprints that should allow experimentalists to identify the critical points of the nuclear phase transition. Two analytical solutions, $X(5)$ and $E(5)$, have been proposed recently for two of the critical points. We present a study within the Relativistic Hartree-Bogoliubov model (RHB), using Potential-Energy Surfaces (PES), to test whether the initial approximations made in deriving the analytical solutions are valid.

1. Introduction

Nuclei in certain regions of the nuclear chart can change properties as a function of the number of neutrons or protons they contain. Such structural changes often correspond to shape changes from, say, spherical to ellipsoidal. Experimental evidence suggests that such shape-changing regions can be viewed as the quantum-mechanical equivalent of macroscopic phase transitions. Nuclei can also exhibit phase coexistence (of spherical and ellipsoidal shapes). Several experimental observables are available that allow a detailed study of nuclear phase transitions, e.g. the evolution of the energy spectrum through a series of isotopes, a study of the binding energies, electromagnetic transition probabilities, etc. In this short contribution we will report on an additional experimental observable, two-particle transfer reactions, of which we carried out a detailed analysis in the IBM and in the framework with Boson Coherent States. In this detailed analysis, we concentrated our attention specifically on what happens near the critical points of the nuclear phase transition. Such critical poins have captured the attention of many experimental and theoretical studies in the last few years, especially after the publication of two analytical solutions, $E(5)$ and $X(5)$ [1], for two specific critical points. Several nuclei have been discovered emperically, of which the experimental excitation spectra and the electromagnetic transition rates closely match the predicted theoretical ones. In this short contribution, we also report on a study with the RHB, constructing PES surfaces, to check whether the initial approximations that have been made to derive the analytical solutions $E(5)$ and $X(5)$, are valid.

2. ”Flatness” of PES surfaces in the RHB, and validity of the nuclear infinite square-well potential approximation

The $E(5)$ and $X(5)$ critical point symmetries correspond to special solutions of the Bohr Hamiltonian, in both of which an infinite square-well potential in the quadrupole (β) degree of freedom is assumed. It is of interest to examine if the assumption of an infinite square-well potential in the $E(5)$ and $X(5)$ models [1] is justified through the use of a completely different method, such as a mean-field approach (more specifically, we opted for the Relativistic Hartree Bogoliubov model (RHB)). The calculation of Potential Energy Surfaces (PES) for series of isotopes in which a critical nucleus appears, should result in a relatively flat PES for this particular nucleus [2]. Results of this study have been presented during the Symposium. The example of the $^{96-114}$Pd isotopes is shown in Fig. 1. The ^{100}Pd and ^{108}Pd isotopes had earlier been suggested as candidates for the E(5) symmetry, and the PES calculated in the RHB framework show indeed rather flat surfaces for these specific isotopes. In conclusion, we can say that obtaining a ”flat” potential energy surface in the RHB for a specific nucleus, does not necessarily mean that this nucleus will correspond to the $E(5)$ or $X(5)$ analytical solutions for its excitation spectrum or for its electromagnetic transition rates. On the other hand, for nuclei that are suggested to be good emperical examples for $X(5)$ or $E(5)$, we expect rather ”flat” PES surfaces.

3. Two-particle transfer reactions as a tool to study nuclear phase transtions: a study with the IBM and Boson Coherent-States

Apart from excitation spectra and PES surfaces, important fingerprints for rapidly changing nuclear structure around the critical points can also show up in two-particle transfer reactions. Such transfer reactions can be studied in e.g. the Interacting Boson Model (IBM). The IBM naturally contains several symmetry limits (corresponding to macroscopic vibrational, rotational and γ-unstable excitation modes) between which it is very straightforward to study phase transitions. An alternative approach is offered by using Boson Coherent-States, that allow to study also the geometry of the IBM, and to make a link with the results from mean-field approaches. A study of two-particle transfer reactions is presently being carried out within the IBM model, and also using an approach with Boson Coherent-States [3]. Using the simplest 2-particle creation (1-boson creation) operator, $s^\dagger$, the two-particle transfer of the ground state of a nucleus with mass A (N bosons) towards the ground state of a nucleus with mass $A+2$ ($N+1$ bosons) is taken in consideration, as well as the transfer to the first excited 0^+ (the bandhead of the β-band) of a nucleus with mass $A+2$ ($N+1$ bosons). In the IBM, these calculations in general need to be carried out numerically. In the framework with Boson Coherent-States, on the other hand, using the following expression for a boson condensate with N bosons in the ground state (with a specific quadrupole deformation β) [4],

$$|N;GS(\beta)\rangle = \frac{1}{\sqrt{N!}}\frac{1}{(1+\beta^2)^{N/2}}\left(s^\dagger + \beta d_0^\dagger\right)^N |0\rangle\,, \tag{1}$$

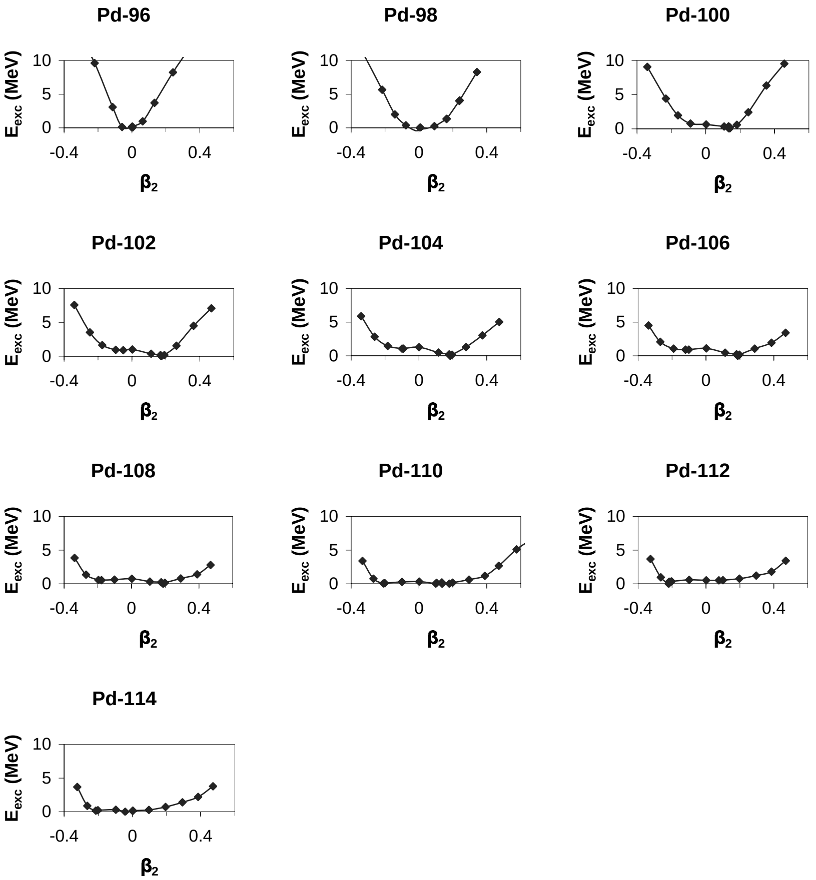

Figure 1. Potential energy surfaces (PES) for $^{96-114}$Pd, calculated using the relativistic Hartree-Bogoliubov theory with the NL3 force. ^{100,102}Pd and ^{108,110}Pd show flat PES. Taken from [2].

and the same boson condensate in a beta-vibrational excitation (generally with the same quadrupole deformation β) [4],

$$|N; BV(\beta)\rangle = \frac{1}{\sqrt{(N-1)!}} \frac{1}{(1+\beta^2)^{N/2}} \left(-\beta s^\dagger + d_0^\dagger\right) \left(s^\dagger + \beta d_0^\dagger\right)^{N-1} |0\rangle \,, \tag{2}$$

simple algebraic expressions can be obtained that relate the matrix elements of transfer reactions between nuclei with N and $N+1$ bosons with their quadrupole deformations, β and β', respectively. For a ground-state (GS) to ground-state (GS) transfer we obtain,

$$\left\langle N+1; GS(\beta') \left| s^\dagger \right| N; GS(\beta) \right\rangle = \sqrt{N+1} \frac{(1+\beta\beta')^N}{\sqrt{(1+\beta'^2)^{N+1}(1+\beta^2)^N}}, \tag{3}$$

whereas for a ground-state (GS) to the band head of the beta-vibrational band (BV), we obtain,

$$\left\langle N+1; BV(\beta') \left| s^\dagger \right| N; GS(\beta) \right\rangle = \frac{(1+\beta\beta')^{N-1}}{\sqrt{(1+\beta'^2)^{N+1}(1+\beta^2)^N}} \Big(-\beta(1+\beta\beta') + N(\beta-\beta') \Big). \tag{4}$$

The quadrupole deformations, β and β', can be obtained as the absolute minima from quadrupole-constrained PES surfaces within the Boson Coherent-State framework, or from mean-field calculations or from experimental data (after an appropriate conversion between the different conventions used within the various models for the quadrupole deformation). The intensities of the 2-particle transfer reactions can be obtained from the above relations, eq. (3) and eq. (4), by taking the square of the matrix elements.

Results on 2-particle transfer reactions for isotope series that show a transition between two of the IBM limits (and that are of possible interest for the X(5) and E(5) symmetries) have been presented during the 16th Hellenic Symposium. In Fig. 2, results for the intensities of two-particle transfer for the $_{44}$Ru isotope series are shown. The $_{44}$Ru isotopes undergo a phase transition from a vibrational structure ($SU(5)$ limit) towards a $\gamma-$unstable structure ($O(6)$ limit). Structure transitions of the $SU(5) \rightarrow O(6)$ type are known to undergo a phase transition of the second order [4]. Indeed, in our results, at the place of the phase transition, a clear discontinuity shows up in the evolution of the intensity of the two-particle transfer (at least in the prediction with the Boson Coherent States). In general, the transfer intensity between two ground states is larger about one order of magnitude than the transfer between the ground state and the band head of the beta-vibrational band.

In Fig. 3, we present results for the 2-particle transfer intensities for some typical series of isotopes that show a transition between a vibrational structure ($SU(5)$ limit) towards a rotational structure ($SU(3)$ limit), namely for the $_{60}$Nd, $_{62}$Sm, $_{64}$Gd and $_{66}$Dy isotope series. Structure transitions of the $SU(5) \rightarrow SU(3)$ type are considered to undergo phase transitions of the first order [4]. We find indeed discontinuities in the behaviour of the transfer intensities (in the predictions of the IBM, as well as in the predictions in the Boson Coherent-State framework). Moreover, for the critical points of the phase transition, for the Gd and Dy isotope series, the intensity for transfer to the band head of the beta-vibrational band grows bigger than the intensity for transfer to the ground state. Within the IBM a third structure transition is possible, namely between the $SU(3)$ and $O(6)$ limits, but it is well known that here no phase transition shows up [4].

Specific groups of experimentalists show a renewed interest to measure two-particle transfer reactions [5]. We hope with our study to offer a tool to interprete experimental results. We would like to suggest the discontinuities in the evolution of the intensities of two-particle transfer reactions, e.g. through a series of isotopes that performs a nuclear phase transition, as a characteristic fingerprint to locate the exact position of the critical points. In the discussed series of isotopes, the intensity for transfer to the ground state is usually about one order of magnitude stronger than the intensity for transfer to the 0^+ band-head of the beta-vibrational band. At and around the critical points, however, important fluctuations in the relative intensities for transfer to the ground state and to the band head of the beta-vibrational band show up. In some series of isotopes (see e.g. the Gd and Dy series in Fig. 3, the intensity for transfer to the band head of the beta-vibrational band even grows stronger than the intensity for transfer to the ground state. In the complete study on 2-particle transfer reactions, see [3], the evolution of the intensities will be studied comprehensively between all of the three IBM symmetry limits of nuclear structure, in a general theoretical way, and through specific isotope series (as presented here already for the $_{44}$Ru, $_{60}$Nd, $_{62}$Sm, $_{64}$Gd and $_{66}$Dy isotope series).

4. Conclusion

Current interests in Nuclear Physics have shifted from the description of a single nucleus to the understanding of structural changes when following long stretches of nuclei, such as a series of isotopes. Often, such structural changes can be interpreted as the quantum-mechanical equivalent of the well-known macroscopical phase transitions. "Fingerprints" of such phase transitions can be traced back in quite a number of experimental observables, such as nuclear excitation spectra, electromagnetic transition rates, etc. Recently, experimentalists have a renewed interest in measuring 2-particle transfer reactions [5]. High-precision data on transfer reactions, through e.g. a series of isotopes, can teach us how swiftly a phase transition evolves. In this short contribution, and in anticipation of future transfer-reaction experiments, we presented a study both in the IBM exact-frame and in the framework with Boson Coherent-States. A more detailed study will be published later [3]. Following a number of specific isotope series that are well known to perform a particular type of phase transition, the Ru isotopes (see Fig. 2 and the Nd, Sm, Gd and Dy isotopes (see Fig. 3), we find that the intensities of 2-particle transfer indeed move from one limit to the other limit as they should. Moreover, and more importantly, at the critical points, we find discontinuities in the behaviour of the transfer intensities. These discontinuities, and the relative intensities of the transfer between two ground states ($0_1^+ \rightarrow 0_1^+$) and the ground state and the band head of the beta-vibrational state ($0_1^+ \rightarrow 0_\beta^+$) are characteristic fingerprints that will allow experimentalists to identify the critical points in a nuclear phase transition, through a specific series of istopes, using two-particle transfer reactions.

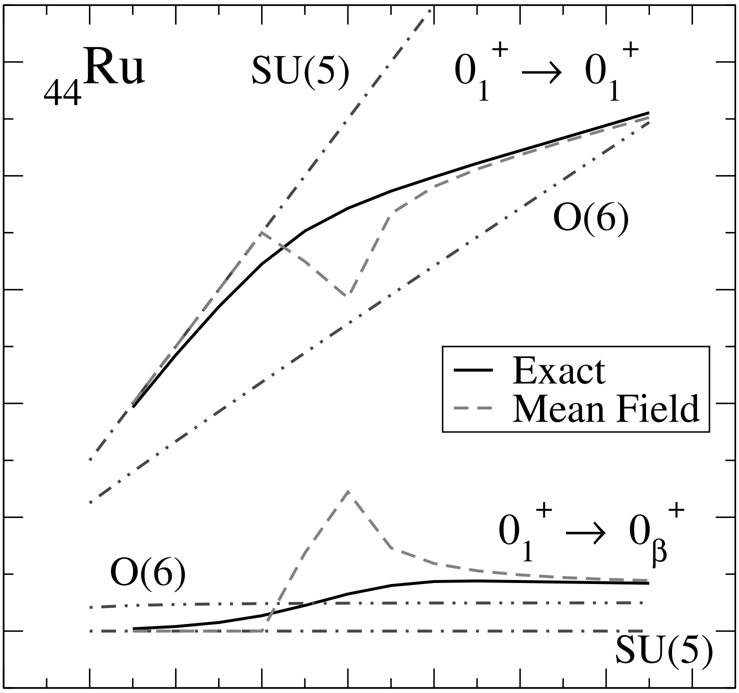

Figure 2. The Ru isotopes are considered to show a structure transition between the $U(5)$ and $O(6)$ IBM limits. We show results for the $^{94-118}_{44}$Ru isotopes (with a number of bosons going from N =3 to 15, taking a closed shell for both protons and neutrons at Z, N=50). Two-particle transfer reactions are considered between nuclei with N and $N+1$ bosons, between their ground states ($0_1^+ \rightarrow 0_1^+$, upper part of the figure), and between the ground state and the band head of the β-vibrational band ($0_1 \rightarrow 0_\beta^+$, bottom part of the figure). Numerical results from the exact IBM lab-frame are shown (full lines). Results with the approximative Boson Coherent-States (broken lines), the so-called "mean field" of the IBM, are shown in comparison (see also eq. (3) and eq. (4)). To guide the eye, the symmetry limits of nuclear structure, SU(5) (vibrational) and O(6) (γ-unstable rotational), are also drawn (dot-dashed lines). Taken from [3].

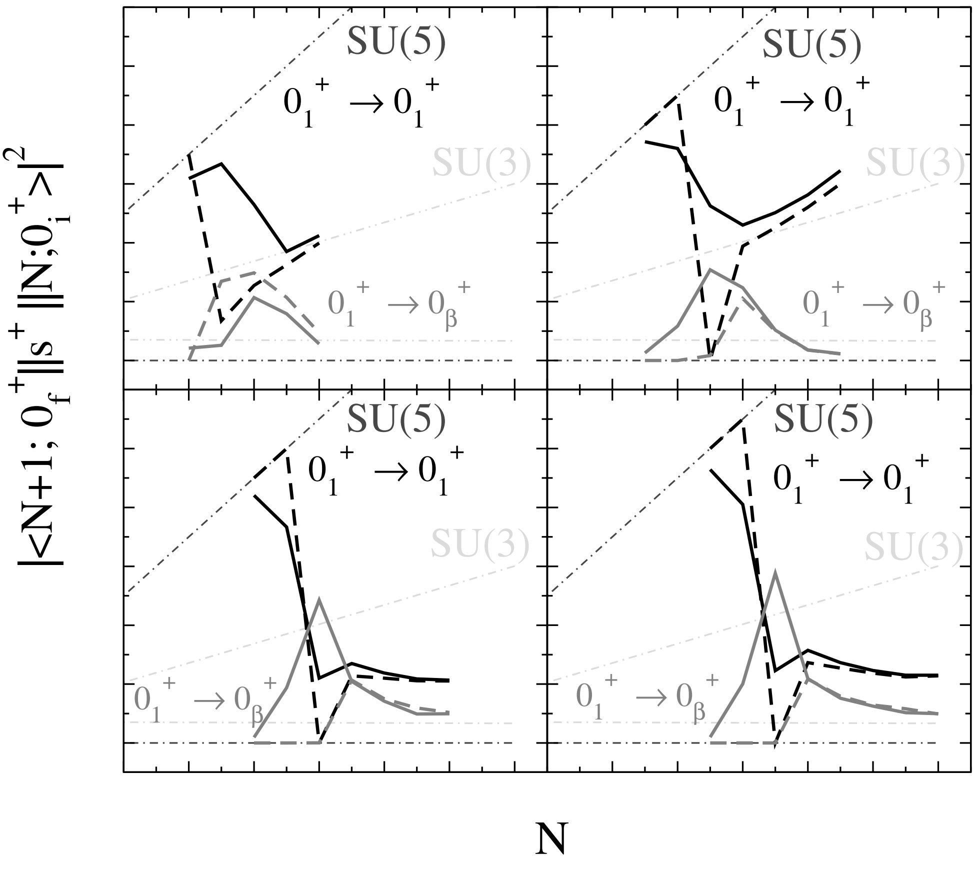

Figure 3. The Nd, Sm, Gd and Dy isotope series are considered to perform a $U(5)$ to $SU(3)$ structure transition. We show results for the ${}^{112-120}_{60}$Nd, ${}^{114-126}_{62}$Sm, ${}^{116-128}_{64}$Gd and ${}^{118-132}_{66}$Dy isotopes (with the number of bosons varying between N =6-10, N =7-13, $N = 8 - 14$ and $N = 9 - 16$, respectively, taking proton and neutron closed shells at $Z, N = 50$). Two-particle transfer reactions are considered between nuclei with N and $N + 1$ bosons, between their ground states ($0_1^+ \rightarrow 0_1^+$, upper part of each figure), and between the ground state and the band head of the β-vibrational band ($0_1 \rightarrow 0_\beta^+$, bottom part of each figure). Numerical results from the exact IBM lab-frame are shown (full lines). Results with the approximative Boson Coherent-States (broken lines), the so-called "mean field" of the IBM, are shown in comparison (see also eq. (3) and eq. (4)). To guide the eye, the symmetry limits of nuclear structure, SU(5) (vibrational) and SU(3) (rotational), are also drawn (dot-dashed lines). Taken from [3].

In the last few years, the critical points of nuclear phase transitions have been a focal point of both experimental and theoretical interest, especially after the publication of the approximative but analytical solutions X(5) and E(5) [1]. To obtain the analytical solutions, the Bohr collective Hamiltonian has been solved, approximating the real $V(\beta)$ potential by an infinite square well in the quadrupole (β) degree of freedom. Since then, several nuclei have been identified with excitation spectra that closely match the predicted ones, and also the electromagnetic transition rates seem to confirm the validity of the $X(5)$ and $E(5)$ models. In this contribution, we presented another test of the critical-point models, namely PES surfaces calculated in the independent relativistic RHB mean-field model [2], that indeed show relatively flat surfaces for the specific isotopes that have been suggested as critical-point candidates.

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