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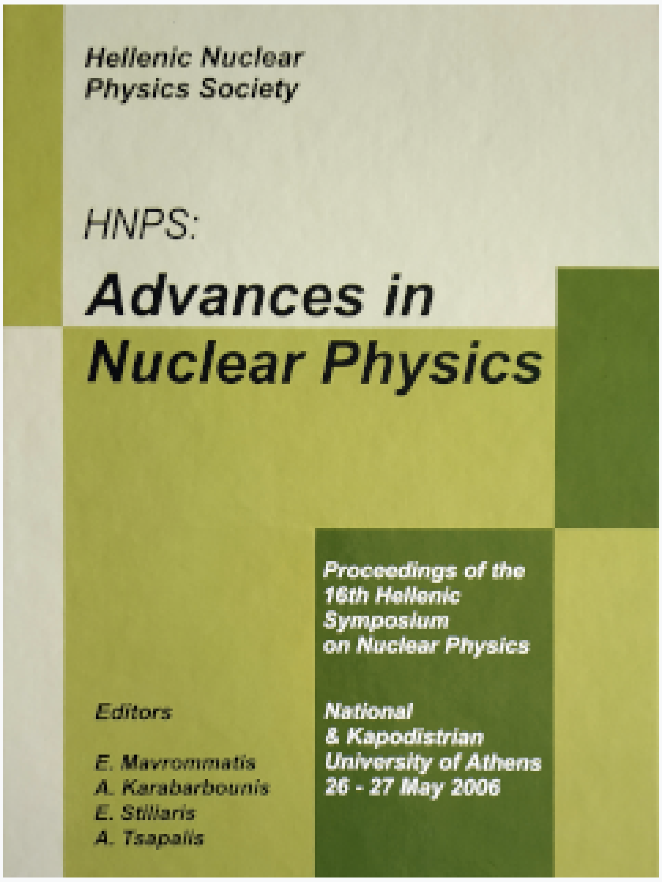


## Nuclear symmetry energy effects on neutron stars properties

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# Nuclear symmetry energy effects on neutron stars properties

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We construct a class of nuclear equations of state based on a schematic potential model, that originates from the work of Prakash et. al. [1], which reproduce the results of most microscopic calculations. The equations of state are used as input for solving the Tolman-Oppenheimer-Volkov equations for corresponding neutron stars. The potential part contribution of the symmetry energy to the total energy is parameterized in a generalized form both for low and high values of the baryon density. The obtained nuclear equations of state are applied for the systematic study of the global properties of a neutron star (masses, radii and composition). We also address on the problem of the existence of correlation between the pressure near the saturation density and the radius.

## 1. Introduction

Neutron stars (NS) are some of the densest manifestations of massive objects in the universe which provide very rich information for testing theories of dense matter physics and also provide a connection among nuclear physics, particle physics, statistical physics and astrophysics [2–8]. The global aspects of neutron stars, such as the masses, radii and composition are determined by solving the so-called Tolman-Oppenheimer-Volkov (TOV) equations [9]. However there are large variations in predicted radii and maximum masses because of the uncertainties in the nuclear equation of state (EOS) near and mainly above the saturation density $n_s$ [6,10–15]. The total energy of neutron rich matter (the case of a neutron star) can be written as a sum of two parts. The first one is the contribution of the symmetric nuclear matter (which is well known) and the second is the symmetry energy (SE) which still is uncertain although several constraints exist from ground state masses (binding energies) and giant dipole resonances of laboratory nuclei.

In general, the value of the SE at nuclear saturation density and mainly the density dependence of the SE are both difficult to be determined in the laboratory. The motivation of the present work is to propose a new parameterization for the potential part of the symmetry energy $E_{sym}(n)$ in order to be able to reproduce the results of a variety of microscopic models both in low and high values of the baryon density. Especially the trend of the symmetry energy just above the equilibrium density $n_s$ is a critical factor in determining the neutron star radius.

Special effort has been devoted to find analytical relations between the radius $R$ and the pressure $P$ which correspond to a special density $n$ for a fixed value of the mass $M$ of the neutron star. So an accurate determination of a neutron star radius will permit evaluation of the pressure of neutron star matter. All the above will provide a direct determination of the density dependence of the nuclear SE at these densities [16].

## 2. The model

In general, the energy per baryon of neutron-rich matter may be written to a very good approximation as

$$\frac{E(n,x)}{A} = \frac{E(n,\frac{1}{2})}{A} + (1-2x)^2 E_{sym}(n) \ , \tag{1}$$

where $n$ is the baryon density ($n = n_n + n_p$) and $x$ is the proton fraction($x = \frac{n_p}{n}$). The symmetry energy $E_{sym}(n)$ can be expressed in terms of the difference of the energy per baryon between neutron ($x = 0$) and symmetry ($x = 1/2$) matter

$$E_{sym}(n) = \frac{E(n,0)}{A} - \frac{E(n,\frac{1}{2})}{A} \ . \tag{2}$$

In the present work we consider a schematic equation for symmetric nuclear matter energy (energy per baryon $E/A$ or equivalently the energy density per nuclear density $\epsilon/n$) which is given by the expression [1]

$$\frac{E(n,1/2)}{A} = \frac{\epsilon_{sym}}{n} = m_N c^2 + \frac{3}{5}E_F^0 u^{2/3} + V(u), \qquad u = n/n_s \tag{3}$$

where $E_F^0 = (3/5)(\hbar k_F^0)^2/2m_N$ is the mean kinetic energy per baryon in equilibrium state and $n_s$ is the saturation density.

The density dependent potential $V(u)$ of the symmetric nuclear matter is parameterized, based on the previous work of Prakash et. al. [1,7] as follows

$$V(u) = \frac{1}{2}Au + \frac{Bu^\sigma}{1+B'u^{\sigma-1}} + 3\sum_{i=1,2} C_i \left(\frac{\Lambda_i}{p_F^0}\right)^3 \left(\frac{p_F}{\Lambda_i} - \arctan\frac{p_F}{\Lambda_i}\right), \tag{4}$$

where $p_F$ is the Fermi momentum, related to $p_F^0$ by $p_F = p_F^0 u^{1/3}$. The parameters $\Lambda_1$ and $\Lambda_2$ parameterize the finite forces between nucleons. The values used here are $\Lambda_1 = 1.5p_F^0$ and $\Lambda_2 = 3p_F^0$. The parameters $A$, $B$, $B'$, $\sigma$, $C_1$ and $C_2$ are determined with the constraints provided by the properties of nuclear matter saturation. In the present work the values of the above parameters are determined in order that $E(n = n_s)/A - m_N c^2 = -16$ MeV, $n_s = 0.16$ fm$^{-3}$ and $K_0 = 240$ MeV. In general the parameter values for three possible values of the compression modulus $K_0$ $\left(K_0 = 9n_0^2 \frac{d^2(E/A)}{dn^2}|_{n_0}\right)$ are displayed in table I, on Ref. [1].

To a very good approximation, the nuclear symmetry energy $E_{sym}$ can be parameterized as follows [5]

$$E_{sym}(u) = \left(2^{2/3}-1\right)\frac{3}{5}E_F^0\left(u^{2/3} - F(u)\right) + S_0 F(u), \tag{5}$$

where $S_0$ is the SE at the saturation point, $S_0 = E_{sym}(u = 1)$. In general, theoretical predictions give $S_0 = 25 - 35$ MeV. In the present work we consider $S_0 = 30$ MeV. The function $F(u)$ parameterizes the potential contribution of the nuclear SE and has to satisfy the constraints $F(u=0)=0$ and $F(u=1)=1$. Equation (5) can be written in a more instructive form by separating the kinetic and the potential contribution of the SE.

$$E_{sym}(u) \simeq \underbrace{13u^{2/3}}_{Kinetic} + \underbrace{17F(u)}_{Potential} . \tag{6}$$

The information gained from microscopic theoretical calculations shows that SE exhibits different trends in low and high densities. So, one should try to find a formula for the function $F(u)$ which satisfies the above restrictions. In the spirit of the previous statement we propose a new parameterization of the function $F(u)$. The new function reproduces the SE for most realistic calculations and has the following form

$$F(u) \;=\; \begin{cases} u^{c_1} & u \leq 1 \\ \\ u^{c_2} e^{1-u} + (u-1)(c_1+1-c_2) & u \geq 1\ . \end{cases} \tag{7}$$

The function $F(u)$ satisfies the constraints $F(u \to 1^+) = F(u \to 1^-)$ and $F'(u \to 1^+) = F'(u \to 1^-)$. The derivative of the function is determined by the parameters $c_1$ and $c_2$ (hereafter called potential parameters).

In order to construct the nuclear equation of state, the expression of the pressure is needed. In general, the pressure, at temperature $T=0$, is given by the expression

$$P = n^2 \frac{d(\epsilon/n)}{dn} = n\frac{d\epsilon}{dn} - \epsilon. \tag{8}$$

From equations (1), (3) and (8) we found that the contribution of the baryon to the total pressure is given by the relation

$$P_b = \left[\frac{2}{5}E_F^0 n_0 u^{5/3} + u^2 n_0 \frac{dV(u)}{du}\right] + n_0(1-2x)^2 u^2 \frac{dE_{sym}(u)}{du}. \tag{9}$$

The electrons originating for the condition of the beta stable matter contribute also to the total energy and total pressure [5]. The electrons which are the ingredients of the neutron star are considered as non-interacting Fermi gas. In that case their contribution to the total energy and pressure is given by

$$\epsilon_{e^-} = \frac{m_e^4 c^5}{8\pi^2\hbar^3}\left[(2z^3+z)(1+z^2)^{1/2} - \sinh^{-1}(z)\right], \tag{10}$$

$$P_{e^-} = \frac{m_e^4 c^5}{24\pi^2\hbar^3}\left[(2z^3-3z)(1+z^2)^{1/2} + 3\sinh^{-1}(z)\right], \tag{11}$$

where $z = k_F/m_e c$.

Now the total energy and pressure of charge neutral and chemically equilibrium nuclear matter is

$$\epsilon_{tot} = \epsilon_b + \epsilon_{e^-}, \qquad\qquad P_{tot} = P_b + P_{e^-}\ . \tag{12}$$

From equations (12) we can construct the equation of state in the form $\epsilon = \epsilon(P)$. What remains is the determination of the proton fraction $x$ in $\beta$-stable matter. It is easy to show that $x$ is given as the solution of the following equation [5]

$$4(1-2x)E_{sym}(n) = \hbar c(3\pi^2 n_e)^{1/3} = \hbar c(3\pi^2 n x)^{1/3}. \tag{13}$$

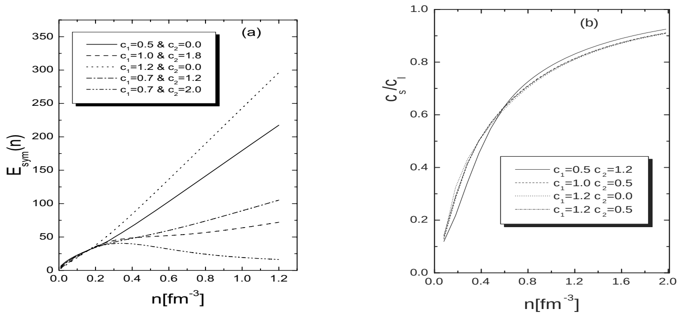


Figure 1. (a) $E_{sym}(n)$ for various values of the potential parameters $c_1$ and $c_2$ of the function $F(u)$, given by the equation (7), versus the baryon density $n$. (b) The ratio $c_s/c_l$ versus the baryon density $n$ for various values of the potential parameters $c_1$ and $c_2$.

It is worthwhile to notice that the present model satisfies the relativistic causality. That means the speed of sound which was defined from the relation,

$$\left(\frac{c_s}{c_l}\right)^2 = \frac{dP}{d\epsilon} = \frac{dP/dn}{d\epsilon/dn}, \tag{14}$$

does not exceed the speed of light for any value of the baryon density. This is a basic treat for any realistic EOS, regardless the details of the interactions among matter constituents or the many body approach [13].

In order to calculate the gross properties of a NS we assume that a NS has a spherically symmetric distribution of mass in hydrostatic equilibrium and is extremely cold ($T = 0$). Effects of rotations and magnetic fields are neglected and the equilibrium configurations are obtained by solving the Tolman-Oppenheimer-Volkoff equations [9]

$$\begin{aligned} \frac{dP(r)}{dr} &= -\frac{Gm(r)\rho(r)}{r^2}\left(1+\frac{P(r)}{c^2\rho(r)}\right)\left(1+\frac{4\pi r^3 P(r)}{c^2 m(r)}\right)\left(1-\frac{2Gm(r)}{c^2 r}\right)^{-1}, \\ \frac{dM(r)}{dr} &= 4\pi r^2\rho(r) = \frac{4\pi r^2\epsilon(r)}{c^2}. \end{aligned} \tag{15}$$

To solve the set of equations (15) for $P(r)$ and $M(r)$ one can integrate outwards from the origin ($r = 0$) to the point $r = R$ where the pressure becomes zero. This point defines R as the radius of the star.

## 3. Results and discussion

In figure 1a we display $E_{sym}(n)$ as a function of the density $n$ for various values of the potential parameters $c_1$ and $c_2$. The potential parameters $c_1$ and $c_2$ varied between $0.5 \leq c_1 \leq 1.2$ and $0 \leq c_2 \leq 2$ in order to get a reliable density dependent SE. In general the case is as follows, for fixed values of the parameter $c_2$, the SE is an increasing function of $c_1$. In addition, for fixed values of the parameter $c_1$ the increase of the parameter $c_2$ leads to a decrease of the SE. It is seen that, within the present model, the stiff or soft behavior of $E_{sym}(n)$ found in various macroscopic calculations, is reproduced. In figure

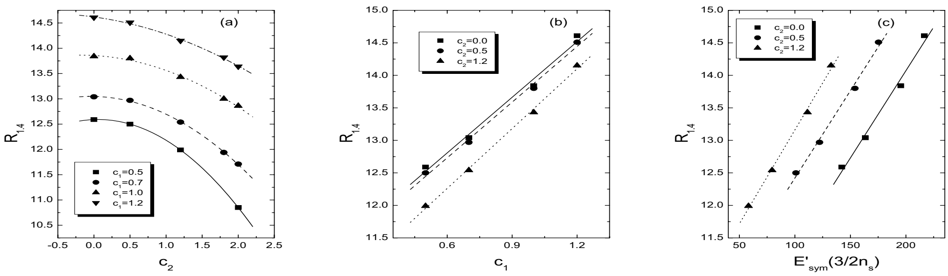


Figure 2. (a) The radius $R_{1.4}$ as a function of the second potential parameter $c_2$ for various values of the first potential parameter $c_1$. The lines correspond to the least-squares fit expressions (16) . (b) The radius $R_{1.4}$ as a function of the second potential parameter $c_2$ for various values of the first potential parameter $c_1$. The lines correspond to the least-squares fit expressions (17). (c) The radius $R_{1.4}$ versus the derivative of the symmetry energy $E'_{sym}(3n_s/2)$ for various values of the potential parameter $c_2$. The lines correspond to the least-squares fit expressions (19), (20) and (21) respectively.

1b we display the ratio $c_s/c_l$ as a function of the density for various cases. It is obvious that the relativistic causality is satisfied.

In order to calculate the global properties of the neutron star, radius and mass we solved numerically the TOV equations (15) with the given equations of state constructed with the present model. For very low densities ($n < 0.08$ fm$^{-3}$) we used the equation of state taken from Feynman, Metropolis and Teller [17] and also from Baym, Bethe and Sutherland [18].

Figure 2a illustrates the behavior of the radius $R_{1.4}$ as a function of the second potential parameter $c_2$ for various values of the first potential parameter $c_1$. The calculated points for various values of $c_1$ can be reproduced by a second order polynomial.

$$
\begin{aligned}
R_{1.4} &= 12.58956 + 0.05378c_2 - 0.46172c_2^2, \qquad c_1 = 0.5 \\
R_{1.4} &= 13.04786 - 0.02752c_2 - 0.32340c_2^2, \qquad c_1 = 0.7 \\
R_{1.4} &= 13.85705 - 0.08087c_2 - 0.21393c_2^2, \qquad c_1 = 1.0 \\
R_{1.4} &= 14.61946 - 0.19441c_2 - 0.14536c_2^2, \qquad c_1 = 1.2 \ . \qquad (16)
\end{aligned}
$$

In all examined cases, the radius $R_{1.4}$ is a decreasing function of the potential parameter $c_2$. This is a direct consequence of the softening of the equation of state due to increase of the parameter $c_2$.

In addition, in figure 2b the behavior of the radius $R_{1.4}$ as a function of the second potential parameter $c_1$ is reproduced for various values of the first potential parameter $c_2$. The least-squares fit values are given for the following linear equations

$$
\begin{aligned}
R_{1.4} &= 11.09603 + 2.85172c_1, \qquad c_2 = 0.0 \\
R_{1.4} &= 11.01810 + 2.85517c_1, \qquad c_2 = 0.5 \\
R_{1.4} &= 10.42034 + 3.06724c_1, \qquad c_2 = 1.2 \ . \qquad (17)
\end{aligned}
$$

$R_{1.4}$ is an increasing function of the potential parameter $c_1$. The increase of the parameter $c_1$ leads to the stiffness of the SE as indicated in figure 1a. It is worthwhile to note

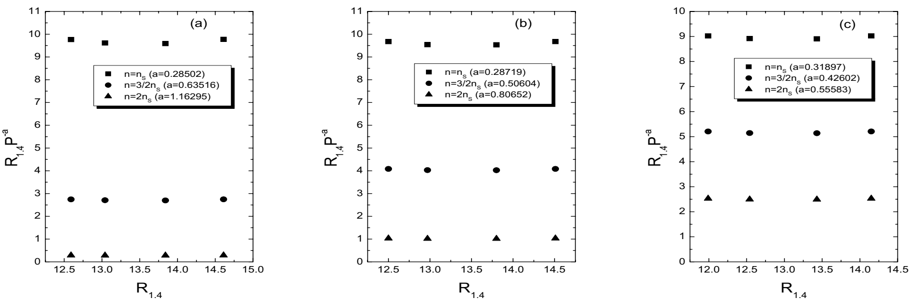


Figure 3. (a) The quantity $RP^{-a}$ as a function of the radius $R_{1.4}$ for pressure determined at $n = n_s$, $n = 3n_s/2$ and $n = 2n_s$ for $c_2 = 0$. (b) The same as before for $c_2 = 0.5$. (c) The same as before for $c_2 = 1.2$. For each density, the least-squares fit value for the exponent a is indicated.

that the slopes of the best fit lines are almost the same and there is just a shift of the lines depending on the values of the parameter $c_2$.

Also, from figure 2a and 2b we conclude that the radius $R_{1.4}$ depends mainly on the parameter $c_1$ which determines the derivative of the $E_{sym}(n)$ and also the pressure $P_{sat}$ at the saturation density $n_s$. However, there is a small dependence on the parameter $c_2$ which is connected with the trend of $E_{sym}(n)$ at higher values of the density $n_s$.

To illustrate further this point, we studied the correlations between the derivative of the symmetry energy $E'_{sym}$ which is given by

$$E'_{sym}(n) = \begin{cases} \frac{1}{n_s}\left[\frac{26}{3}u^{-1/3} + 17c_1 u^{c_1-1}\right] & u \le 1 \\ \frac{1}{n_s}\left[\frac{26}{3}u^{-1/3} + 17\left(c_1 + 1 - c_2 + e^{1-u}u^{c_2}(\frac{c_2}{u} - 1)\right)\right] & u \ge 1 \end{cases} \quad (18)$$

and the radius $R_{1.4}$ close to the saturation point $n = 3n_s/2$. In figure 2c we plot the radius $R_{1.4}$ versus the derivative of the symmetry energy $E'_{sym}(3n_s/2)$ for fixed values of the potential parameter $c_2$. One can see that there is a linear relation between $R_{1.4}$ and $E'_{sym}(3n_s/2)$. The effect of the parameter $c_2$ is to induce a parallel shift of the best fit lines. The least-squares fit values for various values of the parameter $c_2$ are given for the following equations

$$R_{1.4} = 8.70394 + 0.02684E'_{sym}(3n_s/2) \qquad c_2 = 0.0 \quad (19)$$

$$R_{1.4} = 9.73291 + 0.02687E'_{sym}(3n_s/2) \qquad c_2 = 0.5 \quad (20)$$

$$R_{1.4} = 10.27305 + 0.02887E'_{sym}(3n_s/2) \qquad c_2 = 1.2 \quad (21)$$

We also tried to find the correlation between the pressure $P$ (and consequently the radius R) and the SE for other values of the density $n$. In order to clarify the problem of the expected relation between the radius and the pressure we present a more simplified model of a non-relativistic equation with a polytrope type of EOS. Thus the EOS has the form

$$P = K\rho^{\gamma}, \qquad \gamma = 1 + \frac{1}{\lambda}. \quad (22)$$

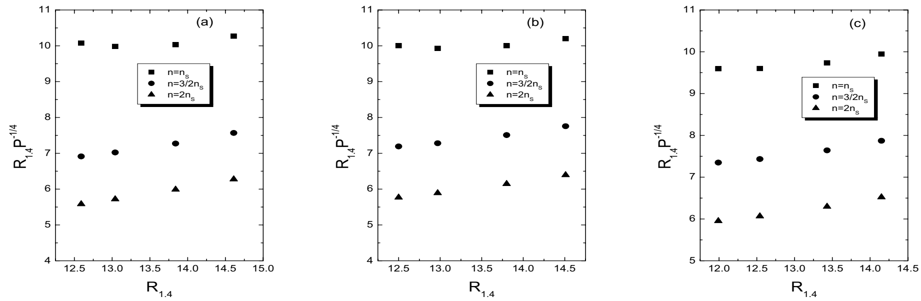


Figure 4. (a) The quantity $RP^{-1/4}$ as a function of the radius $R_{1.4}$ for pressure determined at $n = n_s$, $n = 3n_s/2$ and $n = 2n_s$ for $c_2 = 0$. (b) The same as before for $c_2 = 0.5$. (c) The same as before for $c_2 = 1.2$.

It is easy to show that in the case where $\lambda = 1$ (or $\gamma = 2$) we get [3,16]

$$\frac{R}{P^{1/2}} = \left[\frac{2G}{\pi}\right]^{1/2} \frac{1}{\rho} \ . \tag{23}$$

Thus, from equation (23) we concluded that in the case of a polytrope with $\gamma = 2$ there is a universal relation of the ratio $R/P^{1/2}$ calculated for a specific value of the density $\rho$. However if general relativity effects are included in the above analysis the exponent 1/2 of the pressure is found to be smaller [16].

Figure 3 illustrates the behavior of the quantity $R_{1.4}P^{-a}$ as a function of the radii $R_{1.4}$ for pressure determined at $n = n_s$, $3n_s/2$, $2n_s$, and also for $c_2 = 0$ (figure 3a), $c_2 = 0.5$ (figure 3b) and $c_2 = 1.2$ (figure 3c).

In figure 4 we plot the quantity $R_{1.4}P^{-1/4}$ as a function of $R_{1.4}$ for the pressure determined at $n = n_s$, $3n_s/2$, $2n_s$ and for $c_2 = 0$ (figure 4a), $c_2 = 0.5$ (figure 4b) and $c_2 = 1.2$ (figure 4c). It is obvious once again that the quantity $R_{1.4}P^{-1/4}$ is almost constant only when the pressure is calculated at the saturation point $n_s$. When the pressure is calculated at densities $n = 3n_s/2$ and $n = 2n_s$ the quantity $R_{1.4}P^{-1/4}$ is an increasing function of the radius $R_{1.4}$. Thus, in our proposed parameterization of the SE, it is concluded that there is a dependence of the quantity $R_{1.4}P^{-1/4}$ from the first potential parameter $c_1$ as well as from the second potential parameter $c_2$ and consequently from the trend of the SE both for low and high values of the baryon density.

## 4. Summary

In the present work we performed a systematic study of the effect of the potential part of the SE on the global properties of neutron stars (masses, radii and composition). The potential part of the SE was parameterized in a generalized form both for low and high values of the baryon density in order to be efficient in reproducing the results of most microscopic calculations of dense nuclear matter.

As a result it is found that $R_{1.4}$ is a function of both potential parameters $c_1$ and $c_2$. This means that the value of $R_{1.4}$ is affected from the density dependent trend of the SE, both in low and high densities. However, we showed that for fixed values of the parameter

$c_2$, close to the saturation point ($n = 3n_s/2$), a linear relation between the $R_{1.4}$ and the $E'_{sym}(3n_s/2)$ stands. Finally, the quantity $R_{1.4}P^{-a} = C(n)$ appears to be constant after a suitable parameterization of the parameters $a$ and $C(n)$ but still remains dependent from the second potential parameter $c_2$. The quantity $R_{1.4}P^{-1/4}$, exhibits an increasing behavior as a function of the $R_{1.4}$ for density values above the saturation point.

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