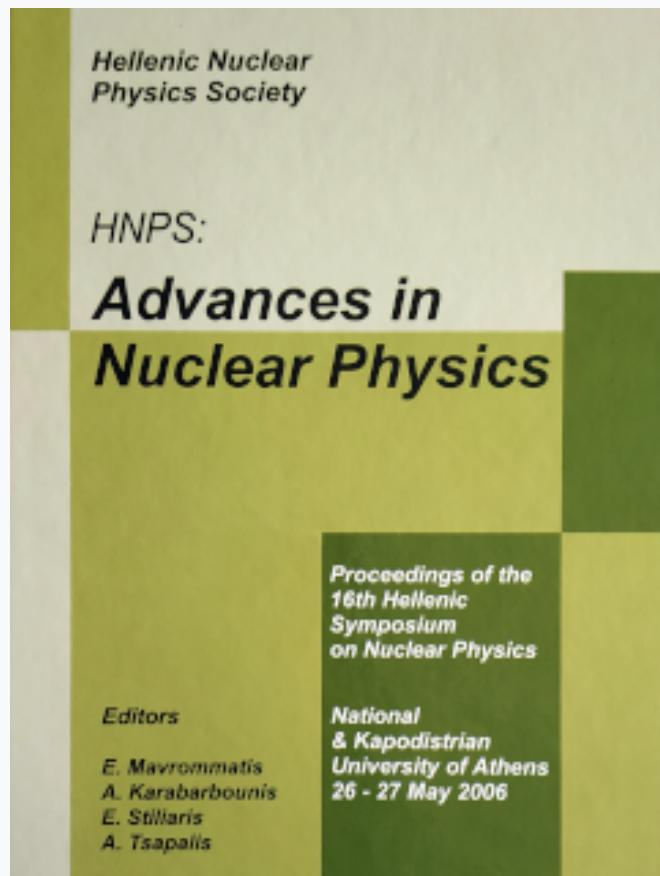


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# Relativistic Hartree Bogoliubov model with Density Dependent meson-nucleon couplings

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## Abstract

A new improved relativistic mean-field effective interaction with explicit density dependence of the meson-nucleon couplings is proposed. The new effective interaction is called DD-ME2 and it is tested in Relativistic Hartree-Bogoliubov (RHB) and RPA calculations of nuclear ground-states and properties of excited states.

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## 1 Introduction

The self-consistent mean-field approach enables a description of the nuclear many-body problem in terms of a universal energy density functional (1). The exact energy functional, which includes all higher-order correlations, is approximated with powers and gradients of ground-state nucleon densities. Although it models the effective interaction between nucleons, a general density functional is not necessarily related to any given NN potential. By employing global effective interactions, adjusted to empirical properties of symmetric and asymmetric nuclear matter, and to bulk properties of few spherical nuclei, self-consistent mean-field models have achieved a high level of accuracy in the description of ground states and properties of excited states in nuclei throughout the periodic table.

An important class of self-consistent mean-field models belongs to the framework of relativistic mean-field theory (RMF) (2; 3). RMF-based models have

been successfully applied in analyses of a variety of nuclear structure phenomena, not only in nuclei along the valley of  $\beta$ -stability, but also in exotic nuclei with extreme isospin values and close to the particle drip lines. The RMF framework has recently been extended to include effective Lagrangians with density-dependent meson-nucleon vertex functions. The functional form of the meson-nucleon vertices can be deduced from in-medium Dirac-Brueckner interactions, obtained from realistic free-space NN interactions, or a phenomenological approach can be adopted, with the density dependence for the  $\sigma$ ,  $\omega$  and  $\rho$  meson-nucleon couplings adjusted to properties of nuclear matter and a set of spherical nuclei. It has been shown that, in comparison with standard non-linear meson self-interactions, relativistic models with an explicit density dependence of the meson-nucleon couplings provide an improved description of asymmetric nuclear matter, neutron matter and nuclei far from stability.

In this work a new effective forces with density-dependent meson-nucleon couplings is introduced to be used in RHB, and RRPA calculations of ground states and excitations of spherical and deformed nuclei. The the new effective interaction DD-ME2 is discussed in Sec. II. In Sec. III the new interaction is employed in a series of calculations of ground-state properties and giant resonances. The main conclusions are summarized in Sec. IV.

## 2 The effective density-dependent interaction DD-ME2

A detailed discussion of the density-dependent nuclear hadron field theory is contained in Refs. (6; 5; 4). The relativistic Hartree-Bogoliubov (RHB) model and the random phase approximation (RPA) based on effective interactions with density dependent meson-nucleon couplings are described in Refs. (7) and (8), respectively. For the sake of completeness we include the essential features of the relativistic Lagrangian density with medium-dependent vertices

$$\begin{aligned}
L = & \bar{\psi} (i\gamma \cdot \partial - m) \psi + \frac{1}{2}(\partial\sigma)^2 - \frac{1}{2}m_\sigma\sigma^2 \\
& - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega^2 - \frac{1}{4}\vec{R}_{\mu\nu}\vec{R}^{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\
& - g_\sigma\bar{\psi}\sigma\psi - g_\omega\bar{\psi}\gamma \cdot \omega\psi - g_\rho\bar{\psi}\gamma \cdot \vec{\rho}\vec{\tau}\psi - e\bar{\psi}\gamma \cdot A\frac{(1-\tau_3)}{2}\psi. \quad (1)
\end{aligned}$$

Vectors in isospin space are denoted by arrows, and bold-faced symbols will indicate vectors in ordinary three-dimensional space. The Dirac spinor  $\psi$  denotes the nucleon with mass  $m$ .  $m_\sigma$ ,  $m_\omega$ , and  $m_\rho$  are the masses of the  $\sigma$ -meson, the  $\omega$ -meson, and the  $\rho$ -meson.  $g_\sigma$ ,  $g_\omega$ , and  $g_\rho$  are the corresponding coupling constants for the mesons to the nucleon.  $e^2/4\pi = 1/137.036$ . The coupling

Table 1  
The parameter set DD-ME2

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$= 939.000$ (MeV)	$\omega = 783.000$ (MeV)
$\rho = 763.000$ (MeV)	$\sigma = 550.124$ (MeV)
$\sigma(\rho_{sat}) = 10.5396$	$\omega(\rho_{sat}) = 13.0189$
$\rho(\rho_{sat}) = 3.6836$	
$\sigma = 1.3854$	$\omega = 1.3879$
$\sigma = 0.9781$	$\omega = 0.8525$
$\sigma = 1.5342$	$\omega = 1.3566$
$\sigma = 0.4661$	$\omega = 0.4957$
$\rho = 0.5008$	

---

constants and unknown meson masses are parameters, adjusted to reproduce nuclear matter properties and ground-state properties of finite nuclei.  $\Omega^{\mu\nu}$ ,  $\vec{R}^{\mu\nu}$ , and  $F^{\mu\nu}$  are the field tensors of the vector fields  $\omega$ ,  $\rho$ , and of the photon while  $g_\sigma$ ,  $g_\omega$ , and  $g_\rho$  are assumed to be vertex functions of Lorentz-scalar bilinear forms of the nucleon operators.

In the phenomenological approach of Refs. (5; 4; 7) the coupling of the  $\sigma$ -meson and  $\omega$ -meson to the nucleon field reads

$$g_i(\rho) = g_i(\rho_{sat}) f_i(x) \quad \text{for } i = \sigma, \omega , \quad (2)$$

where

$$f_i(x) = a_i \frac{1 + b_i(x + d_i)^2}{1 + c_i(x + d_i)^2} \quad (3)$$

is a function of  $x = \rho/\rho_{sat}$ , and  $\rho_{sat}$  denotes the baryon density at saturation in symmetric nuclear matter. For the  $\rho$ -meson coupling the functional form of the density dependence is suggested by Dirac-Brueckner calculations of asymmetric nuclear matter (9)

$$g_\rho(\rho) = g_\rho(\rho_{sat}) \exp [-a_\rho(x - 1)] . \quad (4)$$

The isovector channel is parameterized by  $g_\rho(\rho_{sat})$  and  $a_\rho$ . The eight indepen-

Table 2

Nuclear matter properties at saturation calculated with the density-dependent effective interaction DD-ME2.

$\rho_{\text{sat}} = 0.152 \text{ fm}^{-3}$	$E/A = -16.14 \text{ MeV}$	$K_0 = 250.89 \text{ MeV}$
$m^* = 0.572$	$a_4 = 32.3$	

dent parameters: seven coupling parameters and the mass of the  $\sigma$ -meson, are adjusted to reproduce the properties of symmetric and asymmetric nuclear matter, and the experimental binding energies, charge radii and neutron radii of twelve spherical nuclei (10; 12; 11) :  $^{16}\text{O}$ ,  $^{40}\text{Ca}$ ,  $^{48}\text{Na}$ 48,  $^{72}\text{Ni}$ ,  $^{90}\text{Zr}$ ,  $^{116}\text{Sn}$ ,  $^{124}\text{Sn}$ ,  $^{132}\text{Sn}$ ,  $^{204}\text{Pb}$ ,  $^{208}\text{Pb}$ ,  $^{214}\text{Pb}$ ,  $^{210}\text{Po}$ . For the open shell nuclei pairing correlations were treated in the BCS approximation with empirical pairing gaps (five-point formula).

The parameters of the new interaction, denoted DD-ME2 (13), are listed in Table 1, while in Table 2 are given the corresponding nuclear matter properties at saturation density: binding energy per nucleon, saturation density, nuclear matter compression modulus, Dirac effective mass, and symmetry energy at saturation.

### 3 Applications

We have performed calculations for ground states properties of more than two hundred spherical and deformed nuclei using the new effective interaction DD-ME2. The calculations have been done in the RHB model and in the pairing channel the the Gogny interaction (14) has been used

$$V^{pp}(1, 2) = \sum_{i=1,2} e^{-((\mathbf{r}_1 - \mathbf{r}_2)/\mu_i)^2} (W_i + B_i P^\sigma - H_i P^\tau - M_i P^\sigma P^\tau), \quad (5)$$

with the set D1S (15) for the parameters  $\mu_i$ ,  $W_i$ ,  $B_i$ ,  $H_i$ , and  $M_i$  ( $i = 1, 2$ ).

The calculated binding energies of these nuclei are compared with experimental values in Fig. 1. Except for a few Ni isotopes with  $N \approx Z$  that are notoriously difficult to describe in a pure mean-field approach, and several transitional medium-heavy nuclei, the calculated binding energies are generally in very good agreement with experimental data. Although this illustrative calculation cannot be compared with microscopic mass tables that include more than 9000 nuclei (16; 17; 18; 19), we emphasize that the rms error including all the masses shown in Fig. 1 is less than 900 keV.

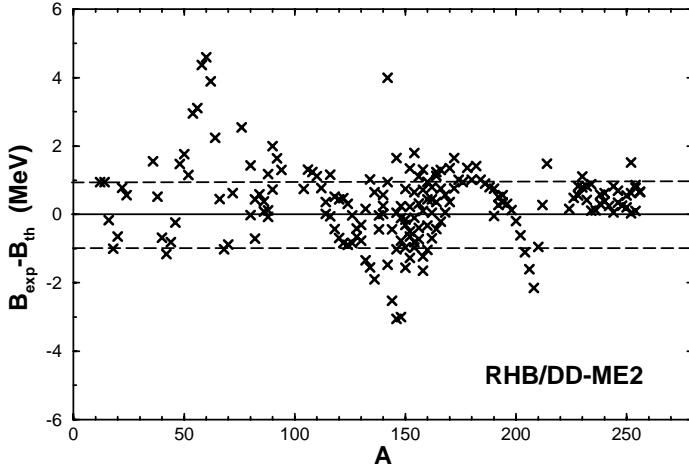


Fig. 1. Absolute deviations of the binding energies calculated with the DD-ME2 interaction from the experimental values (10).

Then, fully self-consistent RRPA (8) have been used to calculate excitation energies of giant resonances in doubly-closed nuclei. The RRPA is formulated in the canonical basis of the RHB model and, both in the *ph* and *pp* channels, the same interactions are used in the RHB equations that determine the canonical quasiparticle basis, and in the matrix equations of the RRPA. For  $^{208}\text{Pb}$  the RRPA results for the monopole and isovector dipole response are displayed in Fig. 2. For the multipole operator  $\hat{Q}_{\lambda\mu}$  the response function  $R(E)$  is defined

$$R(E) = \sum_i B(\lambda_i \rightarrow 0_f) \frac{\Gamma/2\pi}{(E - E_i)^2 + \Gamma^2/4}, \quad (6)$$

where  $\Gamma$  is the width of the Lorentzian distribution, and

$$B(\lambda_i \rightarrow 0_f) = \frac{1}{2J+1} |\langle 0_f || \hat{Q}_\lambda || \lambda_i \rangle|^2. \quad (7)$$

In the examples considered here the continuous strength distributions are obtained by folding the discrete spectrum of RRPA states with a Lorentzian with constant width  $\Gamma = 1$  MeV. The calculated peak energies of the ISGMR:

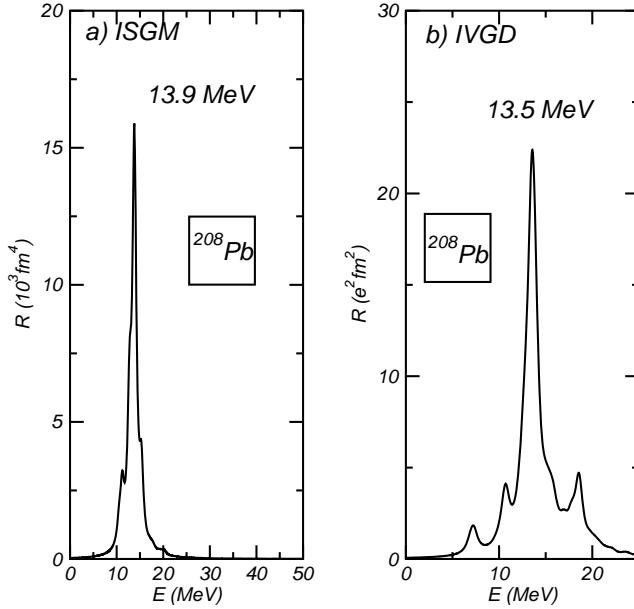


Fig. 2. The isoscalar monopole (a), and the isovector dipole (b) strength distributions in  $^{208}\text{Pb}$  calculated with the effective interaction DD-ME2. The experimental excitation energies are:  $14.1 \pm 0.3$  MeV (20) for the monopole resonance, and  $13.3 \pm 0.1$  MeV (21)

13.9 MeV, and IVGDR: 13.5 MeV should be compared with the experimental excitation energies:  $E = 14.1 \pm 0.3$  MeV (20) for the monopole resonance, and  $E = 13.3 \pm 0.1$  MeV (21) for the dipole resonance, respectively. The agreement of the calculated values with the empirical ones is excellent.

#### 4 Summary and conclusions

A new relativistic mean-field effective interaction with explicit density dependence of the meson-nucleon couplings is proposed. The parameters are adjusted to nuclear matter properties and to bulk properties of twelve spherical nuclei. In order to illustrate the principal features of the new interaction, we have analyzed ground-state properties and excitation energies of giant resonances. Ground states of spherical and deformed nuclei have been calculated in the RHB model with the DD-ME2 effective interaction in the particle-hole channel, and with the Gogny interaction D1S in the pairing channel. The fully self-consistent RRPA has been used to calculate excitation ener-

gies of giant resonances in doubly magic nuclei. We particularly emphasize the very good results for the masses of approximately 200 nuclei and for the isoscalar monopole and isovector dipole giant resonances. DD-ME2 represents a valuable addition to the set of relativistic mean-field interactions. Future applications will include the calculation of a microscopic mass table, mapping the drip lines, and a more extensive study of giant resonances.

## 5 Acknowledgements

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