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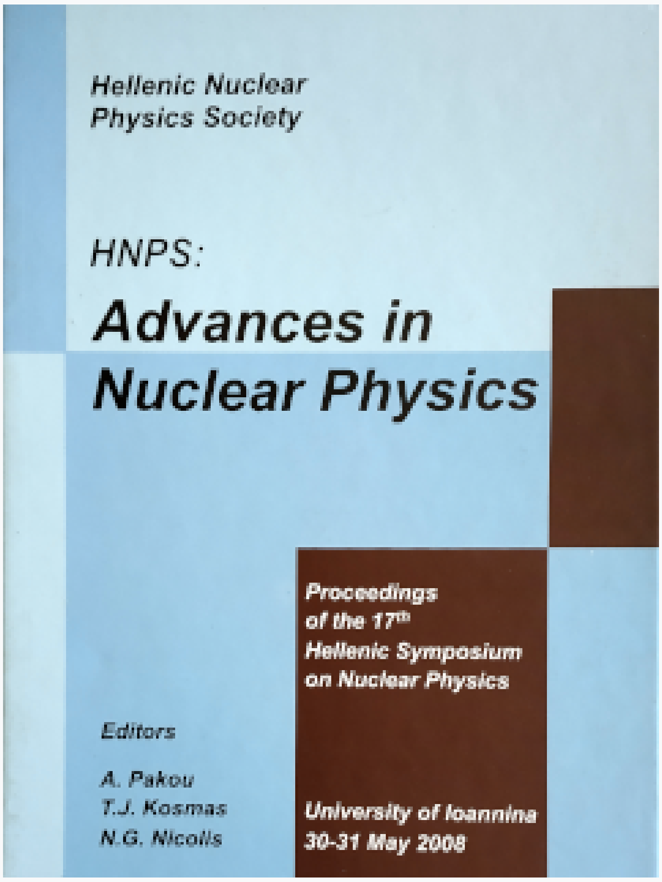

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A NEW Λ- NUCLEUS POTENTIAL

FOR THE Λ- PARTICLE ENERGIES IN HYPERNUCLEI

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Abstract

A new single particle $\Lambda-$nucleus potential is considered for the study of the $\Lambda-$particle energies in hypernuclei. This potential belongs to the class of potentials for which the formalism of the $s-$power series expansions of the Hypervirial Theorems technique is applicable. The numbers $d_k \left(k=0, 1, 2, 3, 4\right)$ related to the derivatives of the potential form factor f, determining the potential shape, are obtained and therefore the approximate analytic expression of the energy level $E_{n\ell}$ (including third -order terms in the small parameter s). Preliminary numerical results are also given and a discussion is made.

1. Introduction

The Quantum Mechanical Hypervirial Theorems (HVT) technique [1] is an efficient one and there are many applications of it in treating various problems, avoiding the use of the wave function [2-4] and thus offering considerable advantages over other typical approximation methods, such as the usual perturbation or variational ones. The method is often used in conjunction with the so called Hellman-Feynmann Theorem [5,6] and thus it is referred sometimes as the HVT-HFT method.

One of its early applications is the treatment of the anharmonic oscillator (see, however, ref. [3] p.339 and refs therein in connection to the treatment of this problem) which led to the well known Swenson and Danforth recurrence relations [7].

One of the basic applications of the method is its use in obtaining approximate expressions in quite a few cases for the energy eigenvalues of the Schrödinger eigenvalue problem when it is not feasible to obtain exact analytic expressions. This is the majority of cases, which one encounters in practice in dealing with various potential models. Among the works studying this sort of problems we mention the papers by Killingbeck, Lai etc [8-11].

More recently, the problem of the approximate determination of the energy eigenvalues of the radial Schrödinger operator was dealt with for the quite general class of the even power series three dimensional central potentials [12] to which also belongs a fairly wide class of potentials which are encountered in practice rather often. The formalism for central potentials belonging to this latter class which leads to so called $s-$power series expansions for the energy eigenvalues and other physically interesting single particle quantities, is briefly reviewed in the next section. For more details the reader is referred to the relevant original papers. [12-16].

We also note that this $s-$power series method was used in recent years in treating a variety of problems in which a small parameter s (see below) can be defined. We mention, in particular, the study of the saturation property of the Bertlmann and Martin [17,18] inequalities [19], the information on the size of $\Lambda-$ orbitals in hypernuclei from the experimental $\Lambda-$energies [20], the approximate analytic evaluation of the Heisenberg uncertainty product [21] etc.

In section 3, a new central potential is considered and the numbers d_k which are expressed in terms of the derivatives of the potential form factor are given. The approximate expression of the level $E_{n\ell}$ in which terms proportional to s^3 are included, follows then immediately in terms of the numbers d_1, d_2 and d_3, the quantum numbers of the state n and ℓ, the potential depth D and the small parameter s.

Finally in the last section, preliminary numerical results for the computed ground – state energies of the Λ are given and compared with the experimental values. A short discussion is also made.

2. The s - power series expansion formalism

We consider the non-relativistic motion of a particle of mass μ moving in a central potential well of the general form

$$V(r) = -D\, f(r/R), \; 0 \le r < \infty \tag{1}$$

In this expression $D > 0$ is the potential depth, $R > 0$ its "radius" and $f\,(f(0)=1)$ the "potential form factor" which determines its shape. The class of the above potentials is further specified by assuming f to be an appropriate analytic function of even powers in $x = r/R$ with $-d^2 f/dx^2\big|_{x=0} > 0$. Therefore, these potentials behave like an harmonic oscillator potential near the origin and thus they are referred sometimes as "oscillator like potentials". It should be noted, however, that apart from the above-mentioned resemblance, their shape is quite different from that of the harmonic oscillator.

Typical potentials of this class are the Gaussian potential $V_G(r)$ and the (reduced) Poeschl-Teller $V_{PT}(r)$ one:

$$V_G(r) = -D\, e^{-(r/R)^2} \quad \text{and} \quad V_{PT}(r) = -D \cosh^{-2}(r/R) \tag{2}$$

Finally, given the mass of the particle and the potential $V(r)$, we can define the dimensionless parameter s:

$$s = \left(\hbar^2 / 2\mu D R^2\right)^{1/2} \tag{3}$$

which is assumed to be a sufficiently small quantity so that the energy eigenvalues (and other physically interesting quantities) can be expanded in a power series in s which can be truncated after the first few terms and obtain a rather reasonable approximation. Fortunately, regarding $\Lambda-$ hypernuclei, this seems to be the case for a range of them.

If we now define the numbers d_k ,$(k=0, 1,2,3,\ldots)$ in terms of the derivatives of the potential form factor f by means of the relation:

$$d_k = \frac{1}{(2k)!}\frac{d^{2k}}{dx^{2k}} f(x)\bigg|_{x=0} \quad \text{with } d_1 < 0 \tag{4}$$

we may write the energy eigenvalues in the form:

$$E_{n\ell} = D\sum_{k=0}^{\infty} e_k\, s^k \tag{5}$$

Then we determine the coefficients e_k in terms of d_k and of the quantities

$$a_{n\ell} = 2n + \ell + \frac{3}{2} \tag{6}$$

and $\ell(\ell+1)$.

It is found (see refs [12-16 and 21]) that :

$$e_0 = -1 \tag{7}$$

$$e_1 = 2\, a_{n\ell} \,|\, d_1 \,|^{1/2} \tag{8}$$

$$e_2 = \frac{d_2}{8\, d_1}\left[12\, a_{n\ell}^2 - 4\,\ell(\ell+1) + 3\right] \tag{9}$$

$$e_3 = \frac{-a_{n\ell}\,(-d_1)^{1/2}}{32 d_1^3} \cdot \left\{4 d_1 d_3 \cdot \left[25 - 12\cdot \ell(\ell+1) + 20\, a^2{}_{n\ell}\right] + d_2^2\left[-67 + 36\cdot \ell(\ell+1) - 68\, a^2{}_{n\ell}\right]\right\} \tag{10}$$

etc.

It is seen that the first few terms of the expansion are fairly simple and can be easily used in practice, as long as the assumptions made are fulfilled.

3. The Λ - nucleus potential considered

The $\Lambda-$ nucleus potential was chosen to be one belonging to the class discussed in the previous section because of the worth-mentioning analytical advantages, which are implied. Furthermore, a third fitting parameter was incorporated which has to be examined whether it offers advantages. The $\Lambda-$nucleus potential considered, is therefore the following:

$$V_{\Lambda-Ac}(r)=-D\,\frac{2^{\lambda}}{\left[1+e^{(r/R)^2}\right]^{\lambda}},\lambda>0,0\leq r<\infty \qquad (11)$$

At the origin $(r=0)$ the potential has the value $-D$, while at very large distances it becomes zero, as one should expect. For R one could take as a first approximation the rigid-core model expression $R=r_0\,A_C^{1/3}$ where A_C is the mass number of the core – nucleus of the hypernucleus, of mass number A, $(A=A_C+1)$. Thus, there are three fitting parameters $(D,r_0$ and $\lambda)$ in the energy expression to be used, in the fitting procedure (see next section).

It is seen from the above expression of the potential that its form factor $f\,(f\,(0)=1)$ is given by the expression

$$f\,(r)=\frac{2^{\lambda}}{\left[1+e^{(r/R)^2}\right]^{\lambda}} \qquad (12)$$

Using this potential form factor the numbers d_k entering the energy levels can be obtained analytically. The calculation is laborious for the higher values of k but it is quite feasible with "Mathematica". The expressions found are given in Table 1.

TABLE 1

The first numbers, d_k of the potential (11)

	d_i
d_0	1
d_1	$-\frac{\lambda}{2}$
d_2	$\frac{1}{8}\lambda(\lambda-1)$
d_3	$-\frac{1}{48}\lambda^2(\lambda-3)$
d_4	$\frac{1}{384}\lambda\left(\lambda^3-6\lambda^2+3\lambda+2\right)$

It is seen that these numbers depend exclusively on the value of λ.

4. Preliminary numerical results and comments

In order that we determine the values of the potential parameters we use the known experimental values of the ground state binding energies of the $\Lambda-$ particle in various hypernuclei. For the mass μ of the particle (appearing in s in the $s-$ power series), the reduced mass of the $\Lambda-$core system was used. Effort was made to use for these experimental values recent results, to a considerable extent, (see refs. 22 and 23 and references therein).

For the theoretical ground state binding energy values $\left(B_{Theor}=-E_{00}\right)$ either the numerical solution of the radial Schrödinger eigenvalue problem (4^{th} column) or the HVT method, the $s-$power series expansion, (5^{th} column) were used. These values are displayed, along with the experimental values and their errors, in Table 2

TABLE 2

A_c	**Symbol**	$B_{EXP} \pm \Delta B_{EXP}$ (MeV)	$B_{THEOR\,(SCHR)}$ (MeV)	$B_{THEOR\,(HVT)}$ (MeV)
15	${}^{16}_{\Lambda}O$	12.42 $\pm$ 0.05	12.83	14.00
27	${}^{28}_{\Lambda}Si$	16.60 $\pm$ 0.2	16.44	16.18
31	${}^{32}_{\Lambda}S$	17.50 $\pm$ 0.5	17.20	16.63
39	${}^{40}_{\Lambda}Ca$	18.70 $\pm$ 1.1	18.39	17.33
50	${}^{51}_{\Lambda}V$	19.97 $\pm$ 0.13	19.54	18.03
55	${}^{56}_{\Lambda}Fe$	21.15 $\pm$ 1.5	20.29	18.28
88	${}^{89}_{\Lambda}Y$	23.11 $\pm$ 0.1	22.00	19.42
138	${}^{139}_{\Lambda}La$	23.80 $\pm$ 1.0	23.63	20.35
207	${}^{208}_{\Lambda}Pb$	26.50 $\pm$ 0.5	24.90	21.07

The best fit values with the solution of the Schrödinger eigenvalue problem found with the CERN program MINUIT were:

$$D=22.99\ MeV,\ r_0=0.990\ fm \text{ and } \lambda=0.681 \qquad (13)$$

while those with the HVT method (and the same program), up to third order terms in s (included) were:

$$D=26.075\ MeV,\ r_0=1.345\ fm \text{ and } \lambda=0.764 \qquad (14)$$

It is seen from the table 2 that the fit is rather satisfactory, mainly for $B_{THEOR(SCHR)}$. It seems, however, that according to our investigations so far (Summer 2008) other simpler potentials lead to a more satisfactory fit. This should be due to the potential form factor and a potential of the form:

$$V_{\Lambda-Ac}(r)=-D\,\frac{1}{\cosh^{2\lambda}(r/R)} \qquad (15)$$

appears worth investigating with regard to its suitability. We should also note that the existing experimental ground – state binding energy data are very poor for $A_C > 90$ and these data would be quite important in reaching a rather firm conclusion concerning this matter.

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