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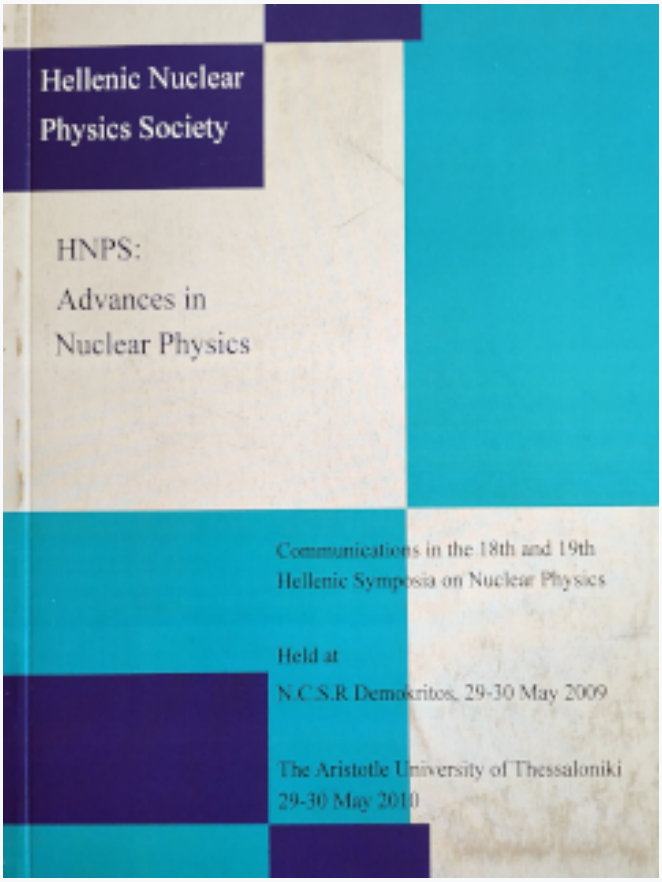

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A CBF calculation of $^1S_0$ Superfluidity in the Inner Crust of Neutron Stars

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Abstract

Singlet S-wave superfluidity of dilute neutron matter in the inner crust of neutron stars is studied within the correlated BCS (Bardeen, Cooper, Schrieffer) method, taking into account both pairing and short-range correlations. First, the equation of state (EOS) of normal neutron matter is calculated within the correlated-basis-function (CBF) method in lowest cluster order using the Argonne V_{18} and $V_{4'}$ potentials and Jastrow-type correlation functions. The $^1S_0$ superfluid gap is then calculated with these potentials and correlation functions. The dependence of our results on the choice of the correlation functions is analyzed and the role of higher-order cluster corrections is considered. The values obtained for the $^1S_0$ gap within this simplified scheme are comparable to those from other, more elaborate, methods.

1. Introduction

The matter in the inner crust matter of neutron stars consists of dilute neutron fluid (with $0.2\ \mathrm{fm}^{-1} \lesssim k_F \lesssim 1.3\ \mathrm{fm}^{-1}$, where k_F is the Fermi wavenumber) interpenetrating a lattice of neutron-rich nuclei that dissolves in the interface with the quantum fluid interior. The neutron system is expected to be in a superfluid phase associated with pairing in the $^1S_0$ channel. The existence of such a phase has direct consequences for post-glitch relaxation and other observed phenomena [1]. The $^1S_0$ superfluid gap has been calculated using various methods of many-body theory. Due to the differences of theoretical methods and assumptions made for the pairing interaction and single-particle energies, there remains considerable ambiguity in the value of the gap as a function of k_F or the density ρ. In this paper we apply the method of CBF in lowest order and evaluate the EOS of normal dilute neutron matter using the Argonne V_{18}

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and the simpler $V_{4'}$ potentials. The $^1S_0$ superfluid gap is then determined by implementing a generalization of the BCS theory within the CBF framework. The calculational strategy is described in Section 2. As discussed in Section 3, the results are found to be comparable to those reported by other authors. Among other aspects of the problem, we investigate the sensitivity of the calculated quantities to the type of short-range correlations assumed and to the contribution of P waves. Further details will be reported in a longer article [2].

2. Calculational Methods

2.1. Equation of State

In calculating the EOS of dilute neutron matter we apply the CBF method [3], in which a correlation operator $F(1,...,N)$ is used to generate a complete set of non-orthogonal correlated wave functions in the N-particle Hilbert space:

$$|m\rangle \equiv |\Psi_m\rangle = I_{mm}^{-1/2} F(1,...,N)|\Phi_m\rangle. \tag{1}$$

$|\Phi_m\rangle$ is an appropriate basis of independent-particle wavefunctions and I_{mm} is a normalization constant. For normal neutron matter, the $|\Phi_m\rangle$ are chosen as Slater determinants of plane waves. The Hamiltonian matrix elements are calculated by using the above non-orthogonal correlated basis with perturbation expansion and cluster approximations. Well-known steps [3] lead to the following variational expression for the energy per neutron (equivalent to first-order perturbation theory in the correlated basis):

$$\frac{E}{N} = \frac{3}{5}E_F + 2\pi\rho \sum_{spin} \int_0^\infty w_2^{spin}(r) G_{spin}(k_F r) r^2 dr. \tag{2}$$

E_F is the ideal Fermi gas energy, $w_2^s(r) = \frac{\hbar^2}{m}\left(\vec{\nabla} f_s(r)\right)^2 + \upsilon_s(r) f_s^2(r)$ are the components of the spin-dependent effective two-neutron interaction in an operatorial decomposition ($s = 1, 2$), the functions $\upsilon_s(r)$ are the corresponding components of the bare spin-dependent two-neutron interaction itself, $f_s(r)$ is a Jastrow two-body correlation function and the quantities G_s are the spin-dependent radial distribution functions. The numerical calculations of E/N are carried out for the Argonne V_{18} [4] potential and the simpler $V_{4'}$ model [5]. Two parametrized correlation functions were studied, (i) the so-called Davé form [6] $f_s(r) = \exp\left\{-\frac{1}{2}\left(\frac{b}{r}\right)^m \exp\left[-\left(\frac{r}{b}\right)^n\right]\right\}$ with parameters b, m and n and (ii) the Benhar form $f_s(r) = [1-\exp(-(r^2/b^2)]^2 + gr\exp(-r^2/c^2)$, with parameters b, c while g is determined by an orthogonality condition for each spin state. In the first set of calculations, we include only contributions from the $S = 0$, $L = 0$ state ("singlet-S only"); in the second, we also include contributions from $S = 1$, $L = 1$ (triplet-P states). The same two functional forms of $f_s(r)$ are considered for the pure $^1S_0$ case and the case including ^3P-state contributions but somewhat different parameter sets are produced when the variational principle is applied.

2.2. $^1S_0$ *Superfluid Gap*

A generalization of BCS theory within the CBF framework [8] is employed to study superfluidity in the dilute neutron-matter system. The dynamically correlated superfluid state is given formally by

$$|CBCS\rangle = \sum_N \sum_m \left(I_{mm}^{(N)}\right)^{-1/2} F_N \left|\Phi_m^{(N)}\right\rangle \left\langle \Phi_m^{(N)} \;\middle|\; BCS\right\rangle \tag{3}$$

where the kets $|\Phi_m^{(N)}\rangle$ are Slater determinants defined for occupied Fermi sea orbitals, $m = m_1, \ldots, m_N$, while $|BCS\rangle$ is the BCS state. Given Eq. (3), steps similar to those of BCS theory, involving evaluation of the expectation value of $\hat{H} - \mu\hat{N}$ in the state (3), where μ is the chemical potential and $\hat{N}$ the number operator, and application of the variational principle, lead [8] ultimately to the familiar gap equation

$$\Delta(k) = -\frac{1}{\pi} \int_0^\infty \frac{V(k,k')}{\sqrt{\left(\varepsilon(k') - \mu\right)^2 + \Delta^2(k')}} \Delta(k') k'^2 dk' \tag{4}$$

but with the bare pairing interaction replaced by its correlation-renormalized version $V_{kk'} = kk'^{-1} \int_0^\infty w_2^{^1S_0}(r) \sin(kr) \sin(k'r) dr$ and the single-particle energies $\varepsilon(k')$ by their CBF counterparts. The same dynamical correlation function $f_s(r)$ is used for both superfluid and normal states. Equation (4) is a singular, nonlinear integral equation. Straightforward solution by an iterative method is possible, if one takes a starting value of the gap from Ref. [6]. A more robust and efficient iteration procedure is based on the separation method [9], in which Equation (4) is first transformed into a system of two coupled equations, namely a "quasi-linear" integral equation for the shape of $\Delta(k)$ and an algebraic equation for its amplitude.

3. Results and Discussion

3.1. Equation of State

In Figure 1 we plot the energy per neutron as a function of k_F for the V_{18} potential and selected dynamical correlation functions. For a given potential, differences (generally small) are seen in the results for the different correlation functions. This is expected since, in particular, the Benhar correlation function overshoots unity whereas the Dáve function does not. Moreover, if we compare E/N results for the singlet-S-only case with those when the P-wave contribution is included, this positive contribution is seen to increase with density and begins to play an important role. Inclusion of the P-wave contribution is necessary for $k_F \gtrsim 0.8 fm^{-1}$. We remark further that the results obtained with (V_{18} and $V_{4'}$) are similar for the same type of correlation function. This implies that in the density range considered, the additional complications present in the highly realistic V_{18} interaction are of little importance. We also infer that the higher-order

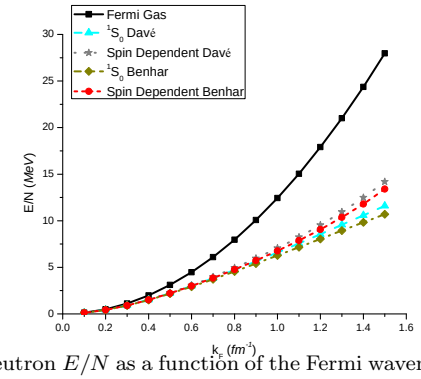

Figure 1: Energy per neutron E/N as a function of the Fermi wavenumber k_F for the potential V_{18} for various cases (see text). We also plot the ideal Fermi gas energy per neutron for comparison.

cluster terms omitted are not essential in this density range, based on the relatively small values of the "wound parameter" $\xi = \rho|\int d^3r[f^2(r)-1]|$ associated with the optimized two-body correlations. Our results for E/N may be compared with those from other methods, including relativistic mean-field methods [10], the variational method [11], the Bethe-Brueckner-Goldstone (BBG) theory [12], and Monte Carlo methods (Green Function Monte Carlo [13] (GFMC) and Auxiliary Field Diffusion Monte Carlo [14] (AFDMC)). Our calculations show reasonably good agreement with these results, the best agreement being achieved when the Benhar correlation function is used and P-wave contributions are included. The main reason that such a simple method is adequate stems from the fact that at the neutron densities of the inner crust of a neutron star, one does not see the large-scale cancellations between kinetic and potential contributions to E/N that make precise calculations more difficult at nuclear density and higher, especially in symmetrical nuclear matter. Finally, three-nucleon interactions are not expected to be important at these densities.

3.2. $^1S_0$ *Superfluid Gap*

We solve the gap equation (4) using the separation method [9], adopting the correlation functions that were determined for the normal state. The resulting energy gap on the Fermi surface, $\Delta(k_F) = \Delta_F$, is plotted in Figure 2 as a function of k_F for the case of the V_{18} interaction (we have also solved the gap equation by straightforward iteration, taking properly into account the small values of the denominator on the Fermi surface with similar results). Similar calculations have been performed for the $V_{4'}$. The density range over which a nonzero gap is found using Davé correlations is larger than that for Benhar correlations. The gap range we found using the Davé type correlations are larger than the ones with Benhar type correlations. Upon comparing the values for the gap Δ_F in the singlet-S-only case with those found including the P-wave contribution to E/N, it is seen that the latter are slightly smaller than the former. Thus, inclusion of the P channel has a small negative effect on

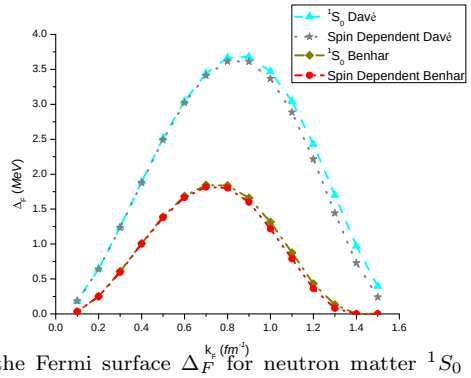

Figure 2: The gap in the Fermi surface Δ_F for neutron matter $^1S_0$ superfluid state as a function of the Fermi wavenumber k_F with the V_{18} potential for various cases (see text).

the gap. This is true for either form of the correlation function. Moreover, when comparing the gap results for the $V_{4'}$ potential with those for the V_{18}, we observe that the former are only slightly different, which is consistent with the expectation that higher-order partial waves have little influence on the value of the $^1S_0$ gap. Comparing our results with those from the simple BCS method with a bare pairing interaction [9] and no corrections for geometrical correlations or polarization effects, we find that our results based on Benhar correlations are somewhat smaller. As previously mentioned, gap calculations have been performed by a number of other methods, including the non-orthogonal CBF approach [6, 15], the orthogonal CBF scheme [16], the renormalization-group technique [17], BBG theory [18], GFMC [13], AFDMC [14], and more [19, 20, 21]. Some of the results are displayed in Figure 3. All of the calculations so represented give results for the gap lower than the pure BCS treatment with a bare pairing interaction. Our results for Benhar correlations are closest to those of Refs. [13, 18, 21]; in addition, the density at which the gap reaches a maximum is similar to the values found in Refs. [15, 18, 19, 21]. In these two respects, the Benhar choice of correlation function is the more realistic for the study of neutron matter in the supefluid state. It is not surprising that there still remains considerable uncertainty in quantitative determination of the behavior of the $^1S_0$ gap Δ_F, in view of the exponential sensitivity of this quantity to the inputs for pairing interaction and density of states.

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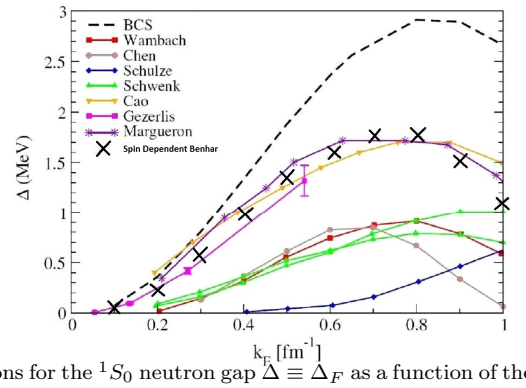

Figure 3: Calculations for the $^1S_0$ neutron gap $\Delta \equiv \Delta_F$ as a function of the Fermi wavenumber k_F with various methods (see text) : BCS [9], Wambach [19], Chen [15], Schulze [20], Schwenk [17], Cao [18], Gezerlis [13], Margueron [21], as seen in Ref.[14], and, with Black X, our results with Benhar type correlation function in the spin-dependent case using the V_{18} potential.

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