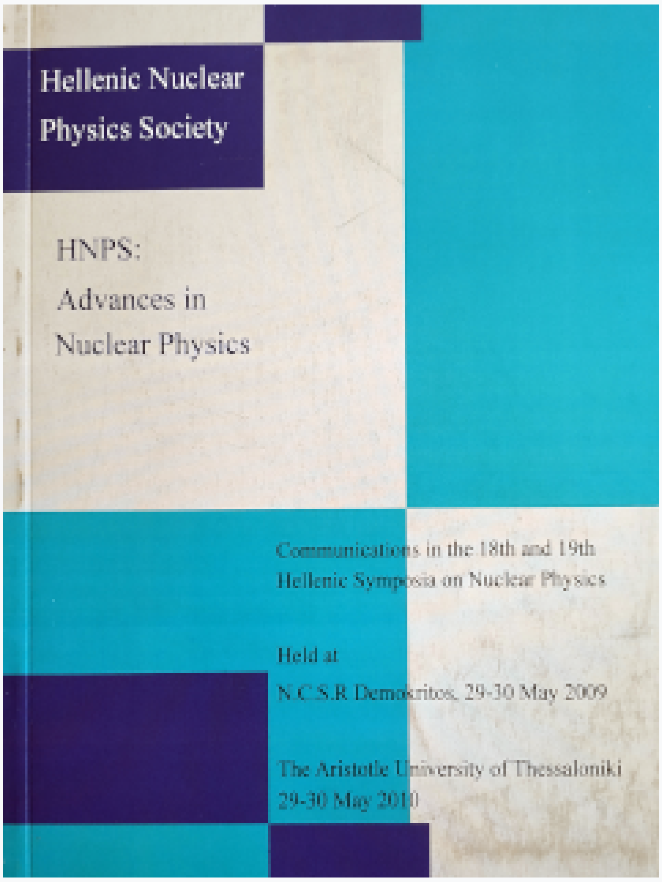
HNPS Advances in Nuclear Physics

Vol 18 (2010)

HNPS2010



Electron Capture in Nuclear Structure and Astrophysics

P. Giannaka, T. S. Kosmas, V. Tsakstara

doi: [10.12681/hnps.2556](https://doi.org/10.12681/hnps.2556)

Electron Capture in Nuclear Structure and Astrophysics

P. Giannaka [a] T.S. Kosmas [a] and V. Tsaktsara [a]

[a] *Theoretical Physics Section, University of Ioannina, GR 45110 Ioannina, Greece*

Abstract

Electron capture on nucleons and nuclei, is one of the most important weak interaction processes in the dynamics and evolution of massive stars. Especially on nuclei of the Fe mass region the role of e^- capture is crucial in the phase of stellar core collapse. Furthermore, a realistic treatment of electron capture on heavy nuclei provides significant information in the hydrodynamics of core collapse and bounce. In this work, we exploit the advantages of a recently published numerical approach to perform nuclear structure calculations of the electron capture in Fe group nuclear isotopes. As a first concrete example, which is simultaneously o ering a good test of our method, we choose the reaction $^{56}Fe(e^-, \nu_e)^{56}Mn^*$ that plays a decisive role in core collapse supernovae. We also improve the previous formalism by constructing compact analytical expressions for the required reduced matrix elements of all basic multipole operators in isospin representation. Such a compact formalism o ers the advantage of performing state-by-state calculations of the transition rates for semi-leptonic nuclear processes through advantageous computer codes written in isospin representation.

1 Introduction

In the unified description of semi-leptonic electro-weak processes in nuclei developed by Walecka and Donnelly [1 3]), the calculation of the transition rates relies on a multipole decomposition of the hadronic current-density matrix elements leading to a set of eight independent irreducible tensor operators. These operators contain spherical Bessel functions, j_l, combined with spherical harmonics, Y^L_M, or vector spherical harmonics, $\mathbf{Y}^{(L,1)J}_M$ as

$$M^J_M(q\mathbf{r}) = \delta_{LJ} j_L(qr) Y^L_M(r), \tag{1}$$

$$\mathbf{M}^{(L1)J}_M(q\mathbf{r}) = j_L(qr)\mathbf{Y}^{(L1)J}_M(r)\,. \tag{2}$$

Preprint submitted to Elsevier *2 May 2011*

where

$$\mathbf{Y}_{M}^{(L,1)J}(r) \; = \; \sum_{M_L,q} \langle L m_L 1 q | J M \rangle Y^{L}_{M_L}(r) e_q \tag{3}$$

The parameters $q^{\mu} = (q_0, \mathbf{q})$, $q = |\mathbf{q}|$ are determined from the kinematics of the process in question. The standard multipole expansion of the polar-vector (hadronic) current (J_{λ}) density matrix elements leads to the operators: Coulomb (M^{coul}_{JM}), longitudinal (L_{JM}), transverse electric (T^{el}_{JM}) and transverse magnetic (T^{mag}_{JM}) [1]. Correspondingly, the axial-vector current component, J^5_{λ}, leads to the operators M^{Coul5}_{JM}, L^5_{JM}, T^{el5}_{JM} and T^{mag5}_{JM}. In the context of the conserved vector current (CVC) theory assumed by many authors, the longitudinal component is linearly dependent on the Coulomb one, $L_{JM}(q) = (q_0/q)M^{Coul}_{JM}(q)$, and then the number of independent operators emerging out of the decomposition procedure is reduced to seven [1 3]. Then, the matrix elements of these seven basic operators involve momentum dependent form factors, $F_X(q^2_{\mu})$ where X=1, 2, A, P,and in the Walecka-Donnelly method [1] seven new operators are defined.

In Ref. [3] we suppressed isospin labels from the multipole operators and we concentrated on the proton-neutron representation applied in the QRPA method.

2 The Donnelly–Walecka decomposition method

2.1 Proton-Neutron representation formalism

The latter operators are denoted as $T_i^{JM}(q\mathbf{r})$, $i = 1, 2, ..., 7$ and are given by the expressions

$$T_1^{JM} \; \equiv \; M^J_M(q\mathbf{r}) \; = \; \delta_{LJ}\, j_L(\rho) Y^L_M(r), \tag{4}$$

$$T_2^{JM} \; \equiv \; \mathbf{\Sigma}^J_M(q\mathbf{r}) \; = \; \mathbf{M}^{JJ}_M \cdot \boldsymbol{\sigma}, \tag{5}$$

$$T_3^{JM} \; \equiv \; \mathbf{\Sigma}'^J_M(q\mathbf{r}) \; = \; -i \left[\frac{1}{q} \nabla \times \mathbf{M}^{JJ}_M(q\mathbf{r}) \right] \cdot \boldsymbol{\sigma}, \tag{6}$$

$$T_4^{JM} \; \equiv \; \mathbf{\Sigma}''^J_M(q\mathbf{r}) \; = \; \left[\frac{1}{q} \nabla M^J_M(q\mathbf{r}) \right] \cdot \boldsymbol{\sigma}, \tag{7}$$

$$T_5^{JM} \; \equiv \; \mathbf{\Delta}^J_M(q\mathbf{r}) \; = \; \mathbf{M}^{JJ}_M(q\mathbf{r}) \cdot \frac{1}{q} \nabla, \tag{8}$$

$$T_6^{JM} \equiv \boldsymbol{\Delta}'^{J}_{M}(q\mathbf{r}) = -i\Big[\frac{1}{q}\nabla \times \mathbf{M}_M^{JJ}(q\mathbf{r})\Big] \cdot \frac{1}{q}\nabla, \tag{9}$$

$$T_7^{JM} \equiv \boldsymbol{\Omega}_M^J(q\mathbf{r}) = M_M^J(q\mathbf{r})\boldsymbol{\sigma} \cdot \frac{1}{q}\nabla. \tag{10}$$

(in addition to the unified notation T_i^{JM}, we keep also the common notation [1,3]).

In the present work, we first construct explicit analytic expressions for the reduced matrix elements of the basic tensor operators by using harmonic oscillator wave functions [4] in the isospin representation.

These expressions allow the systematic calculation of all basic multipole matrix elements by separating out the geometrical coe cients from the kinematical parameters (energy, momentum, scattering angle) of the studied reaction.

One of the main goals of this e ort is to construct an advantageous code for calculating electron-capture cross sections in various currently interesting nuclear isotopes on the basis of the present formalism and within the context of the quasi-particle random phase approximation (QRPA) [3]

2.2 Tensor Multipole operators

Most physical observables in semi-leptonic electro-weak processes in nuclei, are reliably expressed in terms of reduced matrix elements of the above basic one-body operators between two single particle orbits $|n(l1/2)j\rangle \equiv |j\rangle$, i.e. matrix elements of the form

$$\langle n_1(l_1 1/2)j_1||T_i^J||n_2(l_2 1/2)j_2\rangle \equiv \langle j_1||T_i^J||j_2\rangle, \quad i = 1, 2, ...7. \tag{11}$$

Due to the fundamental importance of such reduced nuclear matrix elements in Ref. [3] we constructed compact analytic expressions for their evaluation.

The above eight types of irreducible tensor multipole operators [Eqs. (14)-(21)] are acting on the nuclear Hilbert space and have rank J. The components of the polar vector $(\rho(\mathbf{r}), \mathbf{J}(\mathbf{r}))$ and axial vector $(\rho(\mathbf{r})^5, \mathbf{J}(\mathbf{r})^5)$ currents are defined e.g. in Ref. [2].

The multipole operators are written in terms of the seven basic operators, $T_i^{JM}(q\mathbf{r})$, $i = 1, 2, ..., 7$ (the exact expressions are given in the Appendix of Ref. [5]) For the definition of the form factors $F(q_\mu^2)$ (assuming CVC theory) see Ref. [3]

Using Eqs. (**??**), (**??**) and (**??**), one can straightforwardly deduce a general closed analytic formula for the reduced matrix elements $\langle j_1||T_i^J||j_2\rangle$ of the seven basic operators Eqs. (4)-(2.1) as

$$\langle j_1||T^J||j_2\rangle \;=\; e^{-y}y^{\beta/2}\sum_{\mu=0}^{n_{max}}\mathcal{P}^J_\mu\, y^\mu\,, \qquad y=(qb/2)^2 \tag{12}$$

In the summation of Eq. (12) the upper index n_{max} represent the maximum h.o. quanta included in the model space chosen.

$$n_{max}=(N_1+N_2-\beta)/2\,. \tag{13}$$

2.3 Formalism in Isospin representation

In the isospin representation the eight basic multipole operators are written as follows. The four components of the polar vector current are

$$M^{coul}_{JM;TM_T} \;=\; \int d\mathbf{r} M^J_M(q\mathbf{r})\rho(\mathbf{r})_{TM_T}, \tag{14}$$

$$L_{JM;TM_T} \;=\; i\int d\mathbf{r}\left(\frac{1}{q}\nabla M^J_M(q\mathbf{r})\right)\cdot\mathbf{J}(\mathbf{r})_{TM_T}, \tag{15}$$

$$T^{el}_{JM;TM_T} \;=\; \int d\mathbf{r}\left(\frac{1}{q}\nabla\times\mathbf{M}^{JJ}_M(q\mathbf{r})\right)\cdot\mathbf{J}(\mathbf{r})_{TM_T}, \tag{16}$$

$$T^{mag}_{JM;TM_T} \;=\; \int d\mathbf{r}\mathbf{M}^{JJ}_M(q\mathbf{r})\cdot\mathbf{J}(\mathbf{r})_{TM_T}. \tag{17}$$

The four components of the axial vector current are

$$M^5_{JM;TM_T} \;=\; \int d\mathbf{r} M^J_M(q\mathbf{r})\rho(\mathbf{r})^5_{TM_T}, \tag{18}$$

$$L^5_{JM;TM_T} \;=\; i\int d\mathbf{r}\left(\frac{1}{q}\nabla M^J_M(q\mathbf{r})\right)\cdot\mathbf{J}(\mathbf{r})^5_{TM_T}, \tag{19}$$

$$T^{el5}_{JM;TM_T} \;=\; \int d\mathbf{r}\left(\frac{1}{q}\nabla\times\mathbf{M}^{JJ}_M(q\mathbf{r})\right)\cdot\mathbf{J}(\mathbf{r})^5_{TM_T}, \tag{20}$$

$$T^{mag5}_{JM;TM_T} \;=\; \int d\mathbf{r}\mathbf{M}^{JJ}_M(q\mathbf{r})\cdot\mathbf{J}(\mathbf{r})^5_{TM_T}. \tag{21}$$

2.4 Nuclear level multipole operators

At nuclear level the multipole operators which contain the nucleon form factors $F_X(q_\mu^2)$ are given by the expressions

$$M^{coul}_{JM;TM_T} = \sum_{i=1,A} F_V^\alpha M_J(\mathbf{q}_i) \tag{22}$$

$$\begin{aligned} L_{JM;TM_T} = \sum_{i=1,A} \Bigg(& \frac{F_V^\alpha}{2M}\frac{1}{[J]}\Big((J+1)^{1/2}\mathbf{M}_{J+1,J}(\mathbf{q}_i) + J^{1/2}\mathbf{M}_{J-1,J}(\mathbf{q}_i)\Big)\cdot(\overrightarrow{\boldsymbol\nabla}_i - \overleftarrow{\boldsymbol\nabla}_i) \\ & - \frac{F_V^\alpha + F_M^\alpha}{2M}\frac{i}{[J]}\Big((J+1)^{1/2}\mathbf{M}_{J+1,J}(\mathbf{q}_i) + J^{1/2}\mathbf{M}_{J-1,J}(\mathbf{q}_i)\Big)\cdot\boldsymbol\sigma\times\overrightarrow{\boldsymbol\nabla}_i\Bigg) \end{aligned} \tag{23}$$

$$\begin{aligned} T^{el}_{JM;TM_T} = \sum_{i=1,A} \Bigg(& \frac{F_V^\alpha}{2M}\frac{1}{[J]}\Big((J+1)^{1/2}\mathbf{M}_{J-1,J}(\mathbf{q}_i) - J^{1/2}\mathbf{M}_{J+1,J}(\mathbf{q}_i)\Big)\cdot(\overrightarrow{\boldsymbol\nabla}_i - \overleftarrow{\boldsymbol\nabla}_i) \\ & - \frac{F_V^\alpha + F_M^\alpha}{2M}\frac{i}{[J]}\Big(J+1)^{1/2}\mathbf{M}_{J-1,J}(\mathbf{q}_i) - J^{1/2}\mathbf{M}_{J+1,J}(\mathbf{q}_i)\Big)\cdot\boldsymbol\sigma\times\overrightarrow{\boldsymbol\nabla}_i\Bigg) \end{aligned} \tag{24}$$

$$T^{mag}_{JM;TM_T} = \sum_{i=1,A}\Bigg(-\frac{iF_V^\alpha}{2M}\mathbf{M}_{J,J}(\mathbf{q}_i)\cdot(\overrightarrow{\boldsymbol\nabla}_i - \overleftarrow{\boldsymbol\nabla}_i) - \frac{F_V^\alpha + F_M^\alpha}{2M}\mathbf{M}_{J,J}(\mathbf{q}_i)\boldsymbol\sigma\times\overrightarrow{\boldsymbol\nabla}_i\Bigg) \tag{25}$$

$$M^5_{JM;TM_T} = \sum_{i=1,A}\Bigg(-\frac{iF_A^\alpha}{2M}M_J(\mathbf{q}_i)\boldsymbol\sigma\cdot(\overrightarrow{\boldsymbol\nabla}_i - \overleftarrow{\boldsymbol\nabla}_i) - \frac{i\omega}{2M}F_P^\alpha M_J(\mathbf{q}_i)\boldsymbol\sigma\cdot\overrightarrow{\boldsymbol\nabla}_i\Bigg) \tag{26}$$

$$L^5_{JM;TM_T} = \sum_{i=1,A} F_A^\alpha\frac{i}{[J]}\Big((J+1)^{1/2}\mathbf{M}_{J+1,J}(\mathbf{q}_i) + J^{1/2}\mathbf{M}_{J-1,J}(\mathbf{q}_i)\Big)\cdot\boldsymbol\sigma \tag{27}$$

$$T^{el5}_{JM;TM_T} = \sum_{i=1,A} F_V^\alpha\frac{i}{[J]}\Big(J+1)^{1/2}\mathbf{M}_{J-1,J}(\mathbf{q}_i) - J^{1/2}\mathbf{M}_{J+1,J}(\mathbf{q}_i)\Big)\cdot\boldsymbol\sigma \tag{28}$$

$$T^{mag5}_{JM;TM_T} = \sum_{i=1,A} F_A^\alpha\mathbf{M}_{J,J}(\mathbf{q}_i)\boldsymbol\sigma \tag{29}$$

where A is the mass number of the studied nucleus and nabla operators are acting to the right and left respectively. The isospin dependence of the operators (14)-(21) is included in the operator $I_T^{M_T}$ which is written as [1]

$$I_T^{M_T} = \begin{cases} 1, & T=0,\ M_T=0 \\ \tau_0=\tau_3, & T=1,\ M_T=0 \\ \tau_\pm = \mp\frac{1}{\sqrt{2}}(\tau_1 \pm \tau_2), & T=1,\ M_T=\pm 1 \end{cases} \tag{30}$$

The exact form of $I_T^{M_T}$, i.e. the values of the quantum numbers of T and M_T, is determined from the specific reaction studied (charged or neutral current type reaction).

3 e^-–Capture in stellar evolution

3.1 Electron Capture in ^{56}Fe

In the present work we perform detailed calculations of the electron capture process in the iron group nuclei. As a first concrete example we study the reaction

$$e^- +^{56} Fe \rightarrow^{56} Mn + v_e \tag{31}$$

The required nuclear matrix elements are calculated in the context of the quasi-particle RPA. In Tables 1 and 2 we list the values of the model parameters through which we construct the wave functions for the initial (ground) and final $|J^\pi\rangle$ states.

b (fm)	g^n_{pair}	g^p_{pair}	S_n	S_p	Δ_p^{exp}	Δ_p^{th}	Δ_n^{exp}	Δ_n^{th}
1.996	0.945	0.890	11.197	10.183	1.568	1.579	1.362	1.359

Table 1. Parameters for the interaction of proton pairs, g^p_{pair}, and neutrons pairs, g^n_{pair}. They are fixed in such a way that the corresponding experimental

gaps, Δ_p^{exp} and Δ_n^{exp}, to be reproduced.

State	Strength parameters		*Low-lying Energies (MeV)*	
J^π	g_{ph}	g_{pp}	E^{exp}	E^{theor}
0^+	0.442	0.853	0.004	0.000
1^+	1.183	1.191	3.120	3.120
2^+	0.550	1.171	0.847	0.847
4^+	0.801	1.195	2.085	2.085
0^-	1.119	1.138	3.610	3.610
3^-	0.800	0.918	3.077	3.076
5^-	1.000	0.807	5.122	5.122

Table 2. Strength parameters for the particle-particle (g_{pp}) and particle-hole (g_{ph}) interaction for various multipolarities and the corresponding low-lying energies of the ^{56}Fe spectrum.

A detailed description for the adjustment of the QRPA pamameters

$$g_{pair}^{p,n}, g_{ph}^{p,n} and g_{pp}^{p,n}$$

is done in Ref. [4,5].

3.2 QRPA matrix elements calculations

This is an extension of the numerical approach, which, by using harmonic oscillator basis in the proton-neutron representation, analytical expressions for all basic multipole RME have been obtained. They are expressed in terms of elementary functions, i.e. products of an exponential times a simple polynomial with constant coe cients.

The model space chosen in our QRPA includes the harmonic oscillator levels $0d_{5/2}, 1s_{1/2}$, $0d_{3/2}$, $0f_{7/2}$, $1p_{1/2}$,$1p_{3/2}$, $0f_{5/2}$, $0g_{9/2}$, $1d_{5/2}$, $0g_{7/2}$, $1d_{3/2}$, $2s_{1/2}$, $0h_{9/2}$, $0h_{11/2}$. Our method has been checked in the reproducibility of the low-lying (up to about 5 MeV) spectrum of the ^{56}Fe isotope by using: (i) at the BCS level the pairing parameters for proton-pairs, g_{pair}^p and neutron-pairs, g_{pair}^n [4], and (ii) at the QRPA level the fitting parameters for the strength of the residual interaction, i.e. the g_{ph}, for the particle-hole, and the g_{pp}, for the particle-particle channel, respectively [4,3].

The reliability of the present calculations, could be acquired by the comparison of our electron capture cross sections with those evaluated in other processes, e.g the charged current neutrino nucleus reaction, $^{56}Fe(\nu_e, e^-)^{56}Co$ which is the particle conjugate process reaction of the electron capture.

4 Summary

During the presupernova and collapse phase, electron captures on nuclei, and in the late stage also on free protons, plays an important role, as does nuclear β-decay during silicon burning. Electron captures are made possible by the increasing density in the star's center, accompanied by an increase of the chemical potential (Fermi energy) of the degenerate electron gas. Electron captures reduce the electron-to-baryon ratio Y_e of the matter composition which has important consequences for the subsequent evolution.

In this work, we are going to use a numerical approach constructed recently based on analytical expressions for all basic multipole reduced matrix elements, to perform systematic studies electron capture on nuclear isotopes in the mass range of Fe. The nuclear wave functions would be obtained by using the quasi-particle RPA.

References

[1] T.W. Donnelly and R.D. Peccei, Phys. Rep. **50** (1979) 1.

[2] N.T. Zinner, K. Langanke, P. Vogel, Phys. Rev. **C 74** 74 (2006) 024326.

[3] V.C. Chasioti, T.S. Kosmas, Nucl. Phys. **A 829** (2009) 234.

[4] T.S. Kosmas, Nucl. Phys. **A 683** (2001) 443.

[5] V. Tsakstara and T.S. Kosmas, Phys. Rev. C, in press