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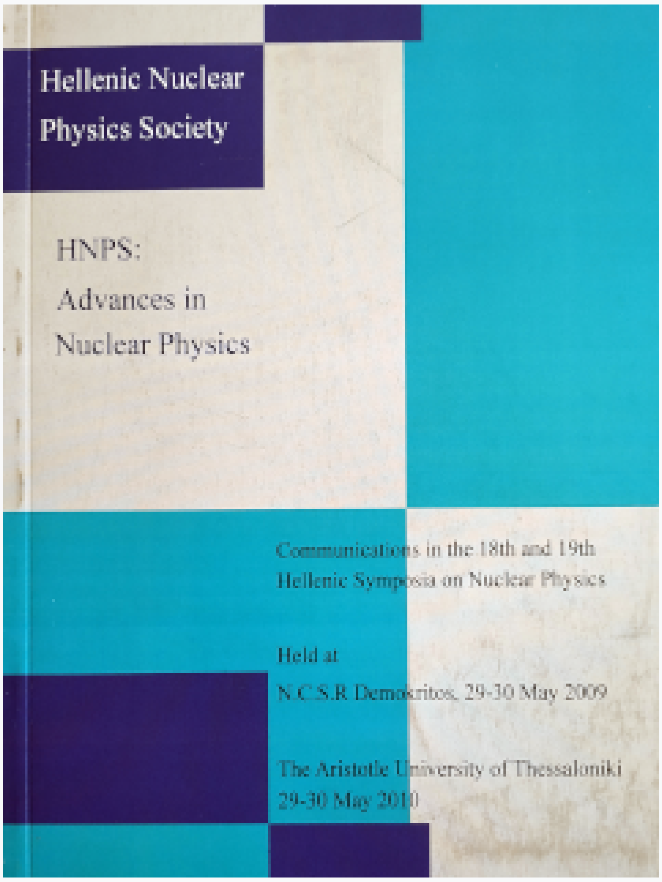

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Towards applications of graded Paraparticle algebras

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Abstract

An outline is sketched, of applications of the ideas and the mathematical methods presented at the 19^{th} symposium of the HNPS in Thessaloniki, May 2010

Keywords: Relative Paraparticle algebras, general linear superalgebra, multiple-level system

1. Introduction

In [2, 4] we have studied algebraic properties of the Relative Parabose algebra P_{bf} and the Relative Parafermi algebra P_{fb} such as their gradings, braided group structures, θ-colored Lie structures, their subalgebras etc. These algebras, constitute paraparticle systems defined in terms of (parabosonic and parafermionic) generators [1] and (trilinear) relations. We have then proceeded in building realizations of an arbitrary Lie superalgebra $L = L_0 \oplus L_1$ (of either fin or infin dimension) in terms of these mixed paraparticle algebras. Utilizing a given (graded), fin. dim., matrix representation of L, we have actually constructed maps of the form $\mathbb{J} : L \to gl(m/n) \subset \begin{matrix} P_{bf} \\ P_{fb} \end{matrix}$ from the LS L to an isomorphic copy of the general linear superalgebra $gl(m/n)$ embedded into either P_{bf} or into P_{fb}. These maps have been shown to be graded Hopf algebra homomorphisms or more generally braided group isomorphisms. From the pure mathematics viewpoint, such maps may be

[1] or: interacting parabosonic and parafermionic degrees of freedom, in a language more suitable for physicists

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considered as generalizations of the Ado-Iwasawa theorems [2] for Lie and super-Lie algebras or even of the Cayley theorem [3] of group theory. From the viewpoint of mathematical physics, these maps generalize -in various aspects- the usual bosonic-fermionic Jordan-Scwinger realizations of Quantum mechanics. In [1, 3], we have further proceeded in building and studying a class of irreducible representations [4] for the simplest case of the $P_{bf}^{(1,1)}$ algebra in a single parabosonic and a single parafermionic degree of freedom (a 4-generator algebra).

2. Prospects-Research objectives

The carrier spaces of the Fock–like representations of $P_{BF}^{(1,1)}$ constitute a family parametrized by the values of a positive integer p. They have the general form $\bigoplus_{n=0}^{p} \bigoplus_{m=0}^{\infty} \mathcal{V}_{m,n}$ where p is an arbitrary (but fixed) positive integer. The subspaces $\mathcal{V}_{m,n}$ are 2–dim except for the cases $m = 0$, $n = 0, p$ i.e. except the subspaces $\mathcal{V}_{0,n}$, $\mathcal{V}_{m,0}$, $\mathcal{V}_{m,p}$ which are 1–dim for all values of m and n. These subspaces can be visualized as

$$\begin{array}{cccccccc}
\mathcal{V}_{0,0} & \mathcal{V}_{0,1} & \dots & \mathcal{V}_{0,n} & \dots & \dots & \mathcal{V}_{0,p-1} & \mathcal{V}_{0,p} \\
\mathcal{V}_{1,0} & \mathcal{V}_{1,1} & \dots & \mathcal{V}_{1,n} & \dots & \dots & \mathcal{V}_{1,p-1} & \mathcal{V}_{1,p} \\
\vdots & \vdots & \dots & \vdots & \dots & \dots & \vdots & \vdots \\
\mathcal{V}_{m,0} & \mathcal{V}_{m,1} & \dots & \mathcal{V}_{m,n} & \mathcal{V}_{m,n+1} & \dots & \dots & \mathcal{V}_{m,p} \\
\vdots & \vdots & \dots & \mathcal{V}_{m+1,n} & \dots & \dots & \vdots & \vdots \\
\vdots & \vdots & \dots & \vdots & \dots & \dots & \vdots & \vdots
\end{array}$$

Our research will focus on both aspects of Pure Mathematics (developing or generalizing techniques for building new representations and studying their properties i.e. computing characters, eigenvalues of Casimirs, formulae for the action of the generators, irreducibility, ... etc.) and aspects of applying these representations in constructing a realistic model for the interaction of monochromatic radiation with a multiple level system:

[2] which roughly state that any f.d. Lie (or super-Lie) algebra is actually isomorphic to a matrix subalgebra of $gl(n)$ (or a graded matrix subalgebra of $gl(m/n)$)

[3] that any fin. group is isomorphic to some subgroup of the permutation group S_n

[4] we have used in [1, 3] the terminology "Fock-like reprs." because of these reprs. apparently generalize the well known boson-fermion Fock spaces of Quantum Field theory

2.1. Representation-theoretical aspects:

Our first objective is to study representation theoretic implications and applications of the above mentioned constructions: We intend to present an abstraction of the Fock representations methodology, in such a way that it can be applicable to an arbitrary algebra given in terms of generators and relations. This work has already begun [6]. Our next task, will be apply the method in order to extend the results of [1] to the case of the $P_{fb}^{(1,1)}$ algebra as well and then to proceed in studying the general cases of arbitrary degrees of freedom for either P_{bf} or P_{fb}. Combining this study with the results of [2, 4] we will "translate" the constructed paraparticle representations in terms of an arbitrary Lie superalgebra. Finally, we will proceed in studying and computing properties of the constructed Lie Superalgebra representations such as computation of characters, explicit formulae for the action of the generators, eigenvalues for the Casimirs, reduction in reducible modules, classification of the cyclic irreducible modules etc. This work has also already begun, by considering the simplest case of $P_{bf}^{(1,1)}$: In [5] we are building Lie superalgebra representations starting from an arbitrary Lie Superalgebra (LS) possessing a 2-dim. graded matrix representation.

2.2. Towards the construction of a realistic model for the interaction of monochromatic radiation with a multiple level system:

Our second objective has to do with a potential physical application of the of the paraparticle and LS Fock-like representations discussed above, in the extension of the study of a well-known model of quantum optics: the Jaynes-Cummings model [7] is a fully quantized -and yet analytically solvable- model describing (in its initial form) the interaction of a monochromatic electromagnetic field with a two-level atom. Using the Fock-like modules described above, we will attempt to proceed in a generalization of the above model in the study of the interaction of a monochromatic parabosonic field with a $(p+1)$-level system. The Hamiltonian for such a system might be of the form

$$\begin{gathered}\mathcal{H} = \mathcal{H}_b + \mathcal{H}_f + \mathcal{H}_{interact} = \omega_b N_b + \omega_f N_f + \lambda(Q^+ + Q^-) = \\ = \tfrac{\omega_b}{2}\{b^+, b^-\} + \tfrac{\omega_f}{2}[f^+, f^-] + \tfrac{(\omega_f - \omega_b)p}{2} + \tfrac{\lambda}{2}(\{b^-, f^+\} + \{b^+, f^-\})\end{gathered} \tag{1}$$

or more generally:

$$\mathcal{H} = \mathcal{H}_b + \mathcal{H}_f + \mathcal{H}'_{interact} = \omega_b N_b + \omega_f N_f + \lambda(Q^+ + Q^-) =$$

$$= \frac{\omega_b}{2}\{b^+, b^-\} + \frac{\omega_f}{2}[f^+, f^-] + \frac{(\omega_f - \omega_b)p}{2} + \lambda_1 b^- f^+ + \lambda_2 f^+ b^- + \lambda_3 b^+ f^- + \lambda_4 f^- b^+ \tag{2}$$

where ω_b stands for the energy of any paraboson field quanta (this generalizes the photon, represented by the Weyl algebra part of the usual JC-model), ω_f for the energy gap between the subspaces $\mathcal{V}_{m,n}$ and $\mathcal{V}_{m,n+1}$ (this generalizes the two-level atom, represented by the $su(2)$ generators of the usual JC-model) [5] and λ or λ_i, $(i = 1, .., 4)$ suitably chosen coupling constants. The $\mathcal{H}_b + \mathcal{H}_f$ part of the above Hamiltonian represents the "field" and the "atom" respectively, while the $\mathcal{H}_{interact} = \lambda(Q^+ + Q^-)$, $\mathcal{H}'_{interact} = \lambda_1 b^- f^+ + \lambda_2 f^+ b^- + \lambda_3 b^+ f^- + \lambda_4 f^- b^+$ terms represent the "field-atom" interactions causing transitions from any $\mathcal{V}_{m,n}$ subspace to the subspace $\mathcal{V}_{m-1,n+1} \oplus \mathcal{V}_{m+1,n-1}$ (absorptions and emissions of radiation). The Fock-like representations, the formulas for the action of the generators and the corresponding carrier spaces, will provide a full arsenal for performing actual computations in the above conjectured Hamiltonian and for deriving expected and mean values for desired physical quantities. A preliminary version of these ideas, for the simplest case of $P_{bf}^{(1,1)}$ has already appeared (see Section 5 of [1]). The spectrum generating algebra of $\mathcal{H}$ may be considered to be either $P_{bf}^{(1,1)}$ or $P_{fb}^{(1,1)}$ or more generally any other mixed paraparticle algebra whose representations can be directly deduced (see [6] for details) from those of $P_{bf}^{(1,1)}$ or $P_{fb}^{(1,1)}$. In this way, we will actually construct a family of exactly solvable, quantum mechanical models, whose properties will be studied quantitatively (computation of energy levels, eigenfunctions, rates of transitions between states, etc) and directly compared with theoretical and experimental results. Last, but not least, it is expected that the study of such models will provide us with deep inside into the process of Quantization itself: We will be able to proceed in direct comparison between mainstream quantization methods of Quantum Mechanics and the idea of Algebraic (or Statistical) Quantization (using Hamiltonians which contain no explicit dynamical interaction terms but including the interaction implicitly into the relations of the spectrum generating algebra itself) as this is outlined in works such as [8].

[5] actually ω_b and ω_f might be some functions of m or n or both.

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