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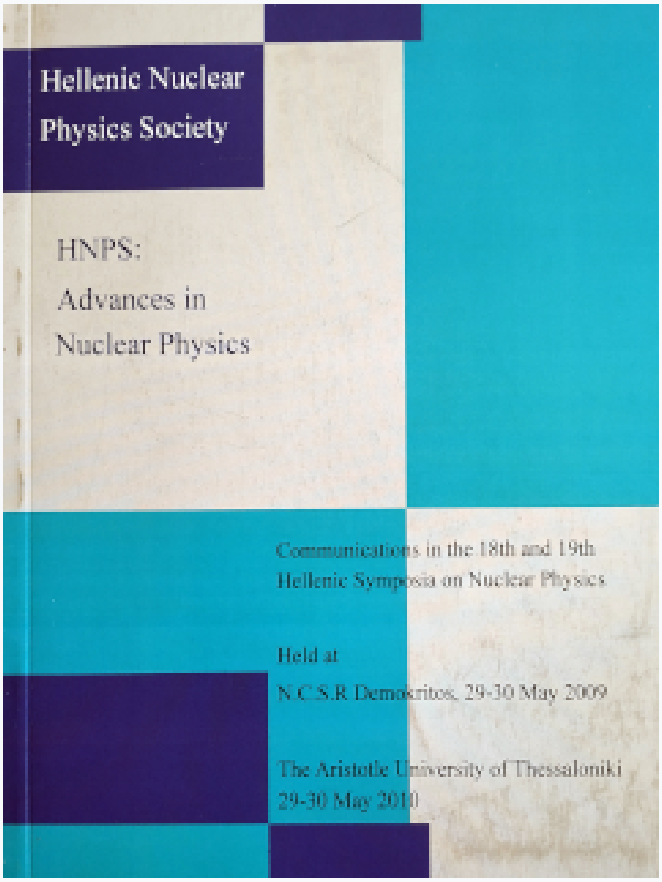


# Detailed calculations for muon capture rates within the quasi-particle RPA

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# Detailed calculations for muon capture rates within the quasi-particle RPA

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**Abstract**

Detailed calculations for bound muon capture in complex nuclei are performed by employing the quasi particle random phase approximation (QRPA). The required bound muon wavefunctions for the large and the small components of the Dirac muon wavefunctions are obtained by using the genetic algorithm approach. We obtained contributions for 2p muon orbit; that is to say wavefunctions for atomic excited state of the muonic atoms in nucleus $^{28}Si$. As a byproduct the above method give the corresponding energies to these wavefunctions which are compared with those of other methods. Our goal is to use the method developed recently by Laganke, Zinner and Vogel and our advantageous numerical approach to obtain state by state calculations of the muon capture rates within the QRPA.

## 1 Introduction.

A bound muon in a muonic atom could be captured either from 1s or from 2p state. In order to calculate this capture rate the wavefunctions for both states are required. In the present work we calculated these wavefunctions for 2p state using Genetic Algorithms(GAs). In the past, numerical methods which have been used for solving the Schrödinger and Dirac equations are the Artificial Neural Networks (ANN)[1–3]. Even though there are no big differences between the two methods, Genetic Algorithms is the most realistic one because it chooses the first point randomly. The main advantage of this new technique is that it produces precise analytic solutions for these wave equations expressed as linear combinations of sigmoid functions.

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## 2 The genetic algorithms.

GAs are modelled loosely on the principles of the evolution via natural selection, employing a population of individuals that undergo selection in the presence of variation-inducing operators such as mutation and recombination (crossover) [5].

(1) Generate an initial random population N(0) of chromosomes
(2) Evaluate and save the fitness function f(n) that is used to evaluate chromosomes, and their reproductive success in the current population
(3) Define selection probabilities p(n) for each individual n. p(n) is proportional to f(n)
(4) Generate new population of chromosomes via genetic operators and replace the worst chromosomes
(5) Repeat step 2 until a solution that is satisfying is obtained.

In this article a modified version [6] of the standard genetic algorithm is used. The modified version utilizes three modifications namely: a) a new stopping rule, b) a new mutation scheme and c) a periodical application of a local search procedure.

## 3 Solving the Dirac equation for the excited state (2p) with the Genetic Algoritmhs method

The solution of the Dirac equation in the case of the potential $V(\mathbf{r})$ related to the extended nuclear Coulomb field, for example the one originating from the point-nucleon charge distribution $\rho(r)$ which is given by

$$V(\mathbf{r}) = -\mathbf{e^2} \int\limits_{-\infty}^{\infty} \frac{\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} \mathbf{d^3 r'} \tag{1}$$

requires numerical integration by using the appropriate algorithm.The nuclear charge density $\rho(r)$ can be estimated using the two parameter Fermi model [7]:

$$\rho(r) = \frac{\rho(r_0)}{1+e^{\frac{r-c}{z}}} \tag{2}$$

The Dirac Equation in a central force system is described by the following equation

$$E\psi = [-i\gamma_5\sigma_r(\frac{\partial}{\partial r} + \frac{1}{r} - \frac{\beta}{dr}K) + V(r) + m_i\beta]\psi \tag{3}$$

We use the following parametrized solutions for the small and large component of the Dirac equation

$$f(r) = re^{-\beta r}N(r, \mathbf{u_f}, \mathbf{v_f}, \mathbf{w_f}) \ ; \ g(r) = re^{-\beta r}N(r, \mathbf{u_g}, \mathbf{v_g}, \mathbf{w_g}) \tag{4}$$

In order to evaluate wavefunctions of excited muon states we attempted to implement the solution of the Dirac equation for the 2p state. The energy for this state(2p) is:

$$E = \frac{\int\limits_0^\infty (m_\mu[g^2(r) + f^2(r)] + V(r)[g^2(r) - f^2(r)] - \frac{2}{r}f(r)g(r))dr}{\int\limits_0^\infty [g^2(r) - f^2(r)]} \tag{5}$$

The error function that has to be minimized in order to evaluate the binding energy:

$$\begin{aligned} &\frac{1}{\int_0^\infty [g^2(r) + f^2(r)]dr} \sum_{i=1}^{n} \Bigg\{ [\frac{df(r_i)}{dr} - [m_\mu - E + V(r_i)]g(r_i)]^2 \\ &\quad + \sum_{i=1}^{n} [\frac{dg(r_i)}{dr} - \frac{2g(r_i)}{r_i} - [m_\mu - E + V(r_i)]f(r_i)]^2 \Bigg\} = 0 \end{aligned} \tag{6}$$

## 4 Results

The goal of this work is to use the exact muon wave functions in the evaluation of the muon capture rate and especially focus on the contributions coming from the 2p, 3p etc low-lying orbits of a bound muon. To this aim, we have constructed an advantageous and very efficient numerical approach providing us with these wave functions by solving the Dirac equation. By exploiting the aforementioned computational tools and the (rather complicated) formalism of the muon capture rate (see Eq. below [8], [14]) we are going to study systematically and throughout the chart of nuclides the muon capture process.

$$
\begin{aligned}
\mathcal{H} = \frac{2G_F \cos\theta_c N^*}{\sqrt{2}} \times [\sum_{J=0}^{\infty} \sqrt{4\pi}[J](-i)^J [i\delta_{m,-1/2}[M'_{J,0} - \mathcal{L}'_{J,0}] \\
+a(J-1,J,m+1/2)\mathcal{T}_1(J-1,J,m+\frac{1}{2}) \\
+a(J+1,J,m+\frac{1}{2})\mathcal{T}_1(J+1,J,m+1/2) \\
-i\beta_+(J,J,m)\mathcal{T}_2(J,J,m+\frac{1}{2}) \\
-i\beta_+(J+1,J,m)\mathcal{T}_2(J+1,J,m+\frac{1}{2}) \\
-i\beta_+(J+1,J+2,m)\mathcal{T}_2(J+1,J+2,m+\frac{1}{2}) \\
-i\beta_-(J-1,J-2,m)\mathcal{T}_3(J-1,J-2,m+\frac{1}{2}) \\
-i\beta_-(J-1,J,m)\mathcal{T}_3(J+1,J,m+\frac{1}{2}) \\
-i\beta_-(J,J,m)\mathcal{T}_3(J,J,m+\frac{1}{2})] \\
+\sum_{J=0}^{\infty} \sqrt{4\pi}[J](-i)^J [i\delta_{m,-1/2}[\mathcal{T}'^{el}_{J,1} - \mathcal{T}'^{mag}_{J,1}] \\
-\delta(J-1,J-1,m)\mathcal{T}_4(J-1,J-1,m+\frac{1}{2}) \\
-\delta(J-1,J-1,m)\mathcal{T}_4(J-1,J-1,m+1/2) \\
-\delta(J,J,m)\mathcal{T}_4(J,J,m+1/2) \\
-\delta(J,J+1,m)\mathcal{T}_4(J,J+1,m+1/2) \\
-\delta(J+1,J+1,m)\mathcal{T}_4(J+1,J+1,m+1/2) \\
+i\eta_+(J,J,m)\mathcal{T}_2(J,J,m+1/2) \\
+i\eta_+(J+1,J,m)\mathcal{T}_2(J+1,J,m+1/2) \\
+i\eta_+(J+1,J+2,m)\mathcal{T}_2(J+1,J+2,m+1/2) \\
-i\eta_-(J-1,J,m)\mathcal{T}_3(J-1,J,m+1/2) \\
-i\eta_-(J,J,m)\mathcal{T}_3(J,J,m+1/2) \\
-i\eta_-(J-1,J-2,m)\mathcal{T}_3(J-1,J-2,m+1/2)]]
\end{aligned}
$$

Where the tensors operators in the nuclear Hilbert space is:

$$
\begin{aligned}
\mathcal{M}'_{J,M} &= \int d^3\vec{x} g(r) Y_{00} j_J(kx) Y_{JM} J_0 \\
\mathcal{L}'_{J,M} &= \frac{i}{k}\int d^3\vec{x} g(r) Y_{00} \nabla (j_J(kx) Y_{JM}) J_0 \\
\mathcal{J}'^{mag}_{J,M} &= \int d^3\vec{x} g(r) Y_{00} j_J(kx) \vec{\mathcal{Y}}^M_{J,J,1} \vec{J} \\
\mathcal{J}'^{el}_{J,M} &= \frac{1}{k}\int d^3\vec{x} g(r) Y_{00} \nabla \wedge (j_J(kx) \vec{\mathcal{Y}}^M_{J,J,1}) \vec{J}
\end{aligned}
$$

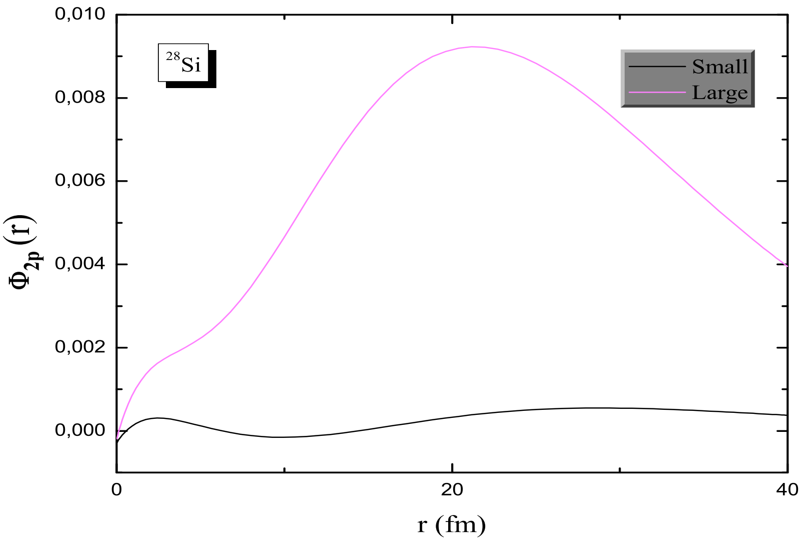


Fig. 1. The two components of Dirac spinor for the 2p state of a muon bound in $^{28}Si$

$$\mathcal{T}_1(\gamma,\rho,\mu) = \int d^3\vec{x} f(r) j_\rho(kx) Y_{\gamma,\mu} J_0$$
$$\mathcal{T}_2(\gamma,\rho,J,\mu) = \int d^3\vec{x} f(r) j_{J+1}(kx) \vec{\mathcal{Y}}^{\mu}_{\gamma,\rho,1} \vec{J}$$
$$\mathcal{T}_3(\gamma,\rho,J,\mu) = \int d^3\vec{x} f(r) j_{J-1}(kx) \vec{\mathcal{Y}}^{\mu}_{\gamma,\rho,} \vec{J}$$
$$\mathcal{T}_4(\gamma,\rho,J,\mu) = \int d^3\vec{x} f(r) j_J(kx) \vec{\mathcal{Y}}^{\mu}_{\gamma,\rho,1} \vec{J}$$

where $\Upsilon$ are the spherical harmonics and $\vec{\Upsilon}$ are the vector harmonics.

In this work we calculated the muon wavefunction (see Fig. 1) and the corresponding binding energy of the $\mu^-$ in a $^{28}Si$ muonic atom by solving Dirac's equation. We concluded that the estimated binding energy and the bound muon wavefunctions (small and large component for the $2p$ state) for the $^{28}Si$ nucleus are in very good agreement with that expected from other theoretical models. In the future we will calculate such partial muon capture rates for other shells(p,d,f). These calculations have already been estimated with a different method of ours [13].

## 5 ACKNOWLEDGMENTS

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