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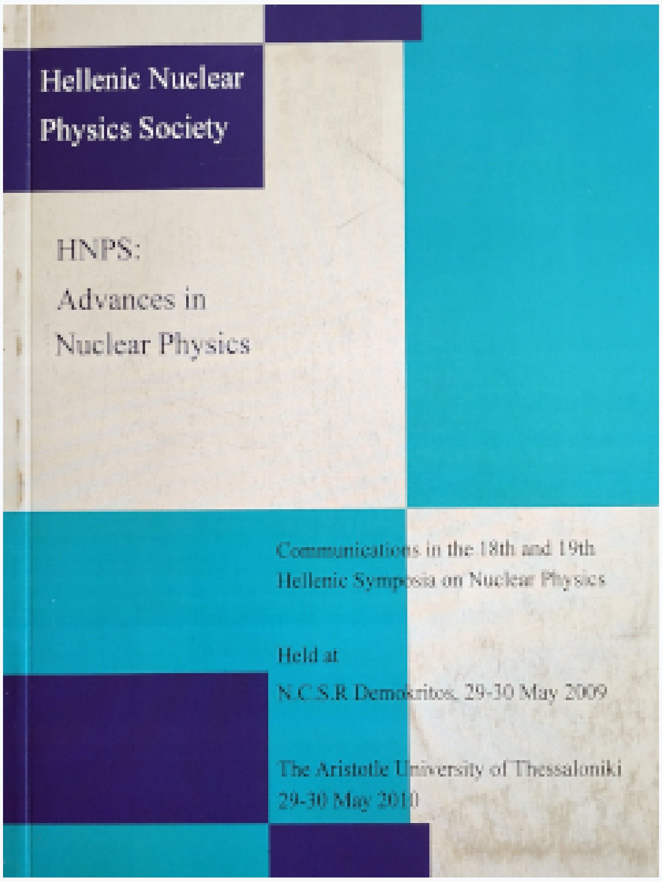

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Application of Information and Complexity Measures to Neutron Stars Structure

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Abstract

We apply several information and statistical complexity measures to neutron stars structure. Neutron stars is a classical example where the gravitational field and quantum behaviour are combined and produce a macroscopic dense object. We concentrate our study on the connection between complexity and neutron star properties, like maximum mass and the corresponding radius, applying a specific set of realistic equation of states. Moreover, the effect of the strength of the gravitational field on the neutron star structure and consequently on the complexity measure is also investigated. It is seen that neutron stars, consistent with astronomical observations so far, are ordered systems (low complexity), which cannot grow in complexity as their mass increases. This is a result of the interplay of gravity, the short-range nuclear force and the very short-range weak interaction.

Keywords: Shannon Entropy, Disequilibrium, Statistical Complexity, Self-Organisation, Equation of State, Neutron Stars.

1. Information and Complexity Measures

The Shannon information entropy S for a continuous probability distribution $\rho(\mathbf{r})$, denoting a measure of the amount of uncertainty associated with a probability distribution, is defined as

$$S = -\int \rho(\mathbf{r}) \, \ln \rho(\mathbf{r}) \, \mathrm{d}\mathbf{r}, \tag{1}$$

while the disequilibrium D, being a quadratic distance from equiprobability, is defined as

$$D = \int \rho^2(\mathbf{r}) \, \mathrm{d}\mathbf{r}, \tag{2}$$

with dimension of inverse volume [1].

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In order to study the statistical complexity defined by López-Ruiz, Mancini and Calbet (LMC), we use a slightly modified definition

$$C = H \cdot D, \tag{3}$$

where $H = \mathrm{e}^S$ is the information content of the system, while the exponential functional preserves the positivity of C [2].

The aforementioned definitions of information entropy and disequilibrium in the case of neutron stars are modified as follows:

$$S = -b_0 \int \bar{\epsilon}(r)\, \ln \bar{\epsilon}(r)\, \mathrm{d}\mathbf{r}, \quad \text{and} \quad D = b_0 \int \bar{\epsilon}(r)^2\, \mathrm{d}\mathbf{r}, \tag{4}$$

where $b_0 = 8.9 \times 10^{-7}$ Km^{-3} is a proper constant satisfying the condition that both information entropy S and disequilibrium should be dimensionless quantities, while $\bar{\epsilon}(r)$ is the dimensionless energy density of the system. It is equivalent to the density mass $\rho(r)$, obtained by solving the structure equations characterising the system [3].

2. Neutron Star Structure Equations and Nuclear Equation of State

In order to calculate the gross properties of a neutron star, we assume that the star has a spherically symmetric distribution of mass in hydrostatic equilibrium and is extremely cold ($T = 0$). Effects of rotations and magnetic fields are neglected and the equilibrium configurations are obtained by solving the Tolman-Oppenheimer-Volkoff (TOV) equations

$$\begin{aligned} \frac{\mathrm{d}P(r)}{\mathrm{d}r} &= -\frac{GM(r)\rho(r)}{r^2}\left(1+\frac{P(r)}{c^2\rho(r)}\right)\left(1+\frac{4\pi r^3 P(r)}{c^2 M(r)}\right)\left(1-\frac{2GM(r)}{c^2 r}\right)^{-1}, \\ \frac{\mathrm{d}M(r)}{\mathrm{d}r} &= 4\pi r^2 \rho(r) = \frac{4\pi r^2 \epsilon(r)}{c^2}, \end{aligned} \tag{5}$$

where $P(r)$ and $M(r)$ are the pressure and the mass functions of the star respectively, The radius R and the total mass of the star, $M \equiv M(R)$, depend on the value of P_c. Also, we have to know the energy density $\epsilon(r)$ (or the density mass $\rho(r)$) in terms of the pressure $P(r)$. This relationship is the equation of state (EOS) for neutron star matter and here, has been calculated applying a phenomenological nuclear model.

In general, the energy per baryon of neutron-rich matter may be written to a very good approximation as

$$\frac{E(n,x)}{A} = \frac{E(n,\frac{1}{2})}{A} + (1-2x)^2 E_{\mathrm{sym}}(n) \tag{6}$$

where n is the baryon density ($n = n_n + n_p$) and x is the proton fraction ($x = n_p/n$). The symmetry energy $E_{\mathrm{sym}}(n)$ can be expressed in terms of the difference of the energy per baryon between neutron ($x = 0$) and symmetrical ($x = 1/2$) matter.

The density dependent potential $V(u)$ of the symmetric nuclear matter is parameterised as follows

$$V(u) = \frac{1}{2}\,Au + \frac{Bu^{\sigma}}{1+B'u^{\sigma-1}} + 3\sum_{i=1,2} C_i \left(\frac{\Lambda_i}{p_F^0}\right)^3 \left(\frac{p_F}{\Lambda_i} - \arctan\frac{p_F}{\Lambda_i}\right), \qquad (7)$$

where p_F is the Fermi momentum, related to p_F^0 by $p_F = p_F^0 u^{1/3}$. The parameters Λ_1 and Λ_2 parameterise the finite-range forces between nucleons, while the parameters A, B, B', σ, C_1 and C_2 are determined using the constraints provided by the empirical properties of symmetric nuclear matter and the saturation density n_0 [4].

To a very good approximation, the nuclear symmetry energy $E_{\rm sym}$ can be parameterised as

$$E_{\rm sym}(u) \simeq 13\,u^{2/3} + 17\,F(u), \qquad (8)$$

where the first term of the right-hand side part of Eq. (8) is the contribution of the kinetic energy and the second term comes from the interaction energy (function $F(u)$ parametrises the interaction part of the symmetry energy).

Now the total energy and total pressure of charge neutral and chemically equilibrium nuclear matter are

$$\epsilon_{tot} = \epsilon_b + \sum_{l=e^-,\mu^-} \epsilon_l, \quad \text{and} \quad P_{tot} = P_b + \sum_{l=e^-,\mu^-} P_l\ . \qquad (9)$$

From equations (9) we can construct the equation of state in the form $\epsilon = \epsilon(P)$. In order to calculate the global properties of the neutron star, i.e. the radius and mass, we solved numerically the TOV equations (5) with the given equations of state constructed employing the present model.

The model of our study, both in the sense of the information and statistical complexity measures, and the neutron star structure equations and nuclear equation of state, along with extensive bibliography, is presented in [3].

3. Results

In Fig. 1(a), we plot the nuclear symmetry energy $E_{\rm sym}$, in Fig. 1(b) the corresponding equations of state and in Fig. 1(c) the mass-radius diagrams for each of the three cases. Actually every pair (R, M) in a mass-radius diagram is the outcome of the structure equations for an arbitrary chosen initial value of the pressure P_c in the center of the star. Thus, varying the value of P_c in a reasonable range, we can have a picture of the behaviour of those substantial structure characteristics. We have to note here that the region where $\mathrm{d}M/\mathrm{d}R < 0$ corresponds to a stable neutron star, while $\mathrm{d}M/\mathrm{d}R > 0$ to an unstable one. Another important feature of a neutron star is the value of $M_{\rm max}$ corresponding to the maximum mass for which the star can exist for the specific equation of state. As displayed in Fig. 1(c), $M_{\rm max}$ is strongly dependent on the equation of state, while a stiffer equation leads to larger $M_{\rm max}$. Note that we vary the parameter

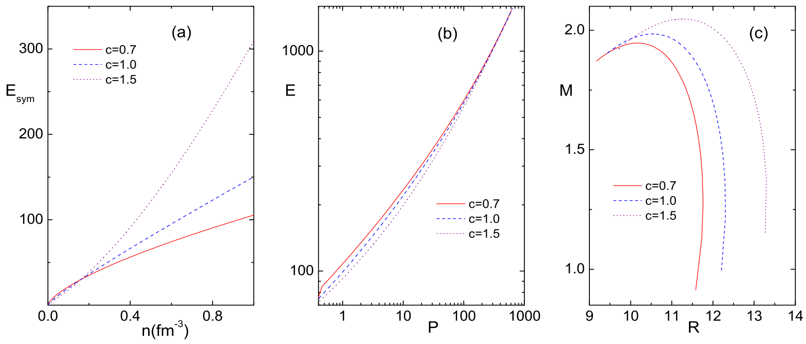

Figure 1: (a) Symmetry Energy vs Baryon Density n, (b) Energy vs Pressure, and (c) Mass vs Radius.

c, which characterises the density dependence of the nuclear symmetry energy, from $c = 0.7$ (soft equation of state) and $c = 1.0$, to $c = 1.5$ (stiff equation of state).

In Fig. 2(a), we present the information entropy S, as a function of the mass M. We find that S is a decreasing function of M in the region denoting a stable neutron star, while in the unstable region S increases with M, as expected intuitively (in Fig. 2(b) we present $H(M)$). In Fig. 2(c) we display the disequilibrium $D(M)$. Increasing M corresponds to greater concentration of the density distribution, its energy density becomes more localised, resulting to a monotonically increasing D. Complexity C, is plotted in Fig. 2(d). C is a monotonically decreasing function of the star mass M. In the unstable region, where $M > M_{\max}$, C increases with M, as indicated in the detailed (inset) figure, but this refers to a case with no physical meaning. The most interesting result in this figure is that a neutron star can not grow in complexity as its mass increases towards the limit of $M_{\max}$.

This result becomes more striking in the following set of figures, Fig. 3, where we plot in three-dimensions information and complexity measures, as functions of both M and R, taking advantage of the fact that each choice of initial values in the equation of state provides a different pair (R, M), reflecting the competition between the gravitational and degenerate gas pressures. The fact that the most probable radii of a neutron star are close to 10 Km, together with the aforementioned comment on the most likely masses, lead us to the conclusion that a neutron star is in general, a system of minimum complexity. Furthermore, it can not grow in complexity as mass or radius increase inside the regions imposed and commented above. The neutron star is an ordered system.

Finally in Fig. 4, we present the direct dependence of complexity C on the parameters c and G. It is seen from Fig. 4(a) that complexity for a given $M_{\max}$, is a decreasing function of the equation of state parameter c, while it increases exponentially with the parameter of the gravitational field Fig. 4(b).

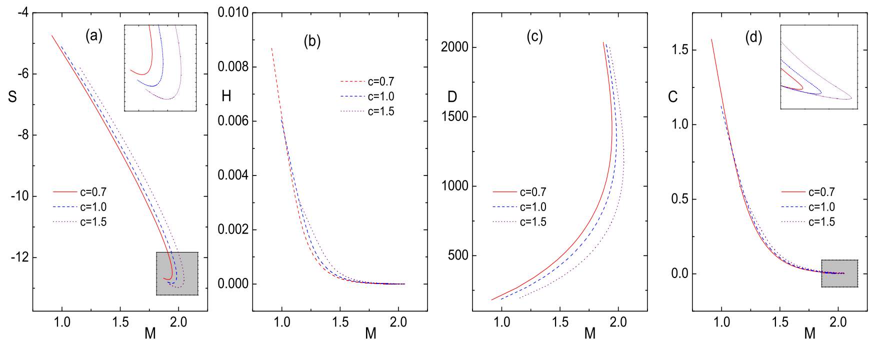

Figure 2: (a) Entropy $S(M)$, (b) Information Content $H(M) = \mathrm{e}^S$, (c) Disequilibrium $D(M)$, and (d) Complexity $C(M)$.

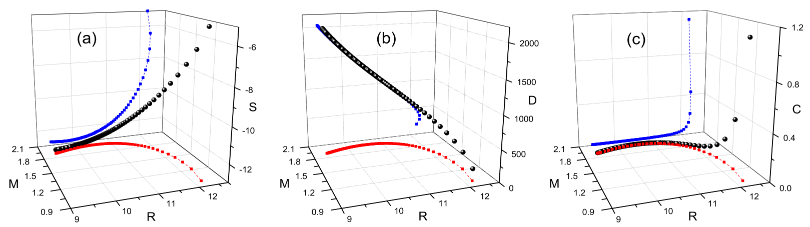

Figure 3: Three-dimensional display of (a) Entropy $S(R, M)$, (b) Disequilibrium $D(R, M)$, and (c) Complexity $C(R, M)$, projected for each case on two planes: (a) $R - M$ and $S - R$, (b) $R - M$ and $D - R$, (c) $R - M$ and $C - R$.

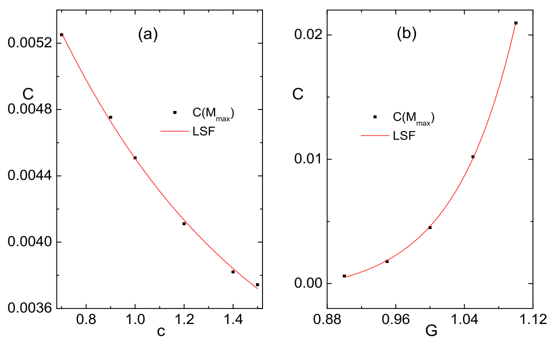

Figure 4: (a) Complexity vs the equation of state parameter c, and (b) Complexity vs the gravitational parameter G, for a given $M = M_{\max} = 1.5\ \mathrm{M}_{\odot}$.

4. Summary

We present a study of neutron stars from the point of view of information and complexity theories [3], continuing a recent application to white dwarfs structure [5]. It is shown that the measures of information entropy S and disequilibrium D can serve as indices of structure of a neutron star. More specifically, S is a decreasing function of the mass of the star, while it is an increasing one of its radius. The result is consistent with the fact that as a neutron star's mass increases, its radius decreases resulting to more localised energy and mass densities. The disequilibrium D shows an inverse behaviour. It is an increasing function of the mass and a decreasing one of its radius. More localised energy and mass densities correspond to a distribution far from equiprobability and as a result the disequilibrium of the system is higher.

The complexity C of a neutron star is a decreasing function of its mass. It almost vanishes for a vast set of pairs of values (R, M), while it increases rapidly for masses less than $1.5M_\odot$ and radii greater than 12 km, a not so favourable case for a neutron star (present astronomical observations). The favourable case, for masses larger than $1.5M_\odot$ and radii less than 12 km, corresponds to almost vanishing complexity, supporting the conclusion that a neutron star is an ordered system, which cannot grow in complexity as its mass increases.

Furthermore, we investigate the impact of the equation of state parameter c and the gravitational parameter G on S and C. The behaviour of information and complexity measures is equivalent in both cases. Complexity decreases exponentially with the mass, while it increases linearly with the radius. In direct calculations, complexity decreases exponentially with the equation of state parameter c, while it increases exponentially with the gravitational parameter G.

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