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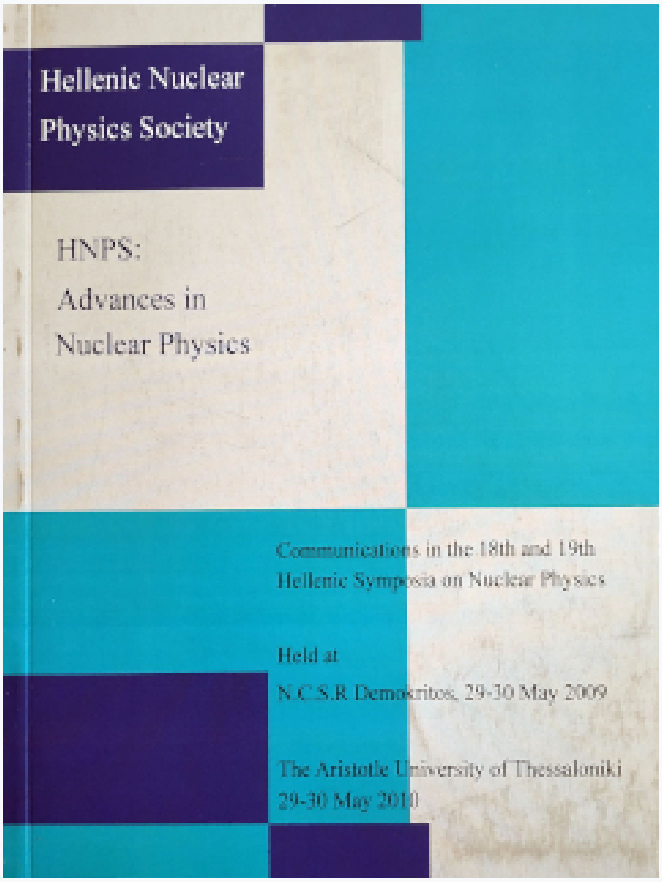

Constraints on the inner edge of neutron star crusts from relativistic nuclear energy density functionals

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Constraints on the inner edge of neutron star crusts from relativistic nuclear energy density functionals

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Abstract

The transition density n_t and pressure P_t at the inner edge between the liquid core and the solid crust of a neutron star are analyzed using the thermodynamical method and the framework of relativistic nuclear energy density functionals. Starting from a functional that has been carefully adjusted to experimental binding energies of finite nuclei, and varying the density dependence of the corresponding symmetry energy within the limits determined by isovector properties of finite nuclei, we estimate the constraints on the core-crust transition density and pressure of neutron stars: $0.086\ \mathrm{fm}^{-3} \leq n_t < 0.090\ \mathrm{fm}^{-3}$ and $0.3\ \mathrm{MeV\ fm}^{-3} < P_t \leq 0.76\ \mathrm{MeV\ fm}^{-3}$ [1].

Keywords: Nuclear density functional, Equation of state, Neutron star crust.

Neutron stars are extraordinary astronomical laboratories for the physics of dense neutron-rich nuclear matter [2]. They consists of several distinct layers: the atmosphere, the surface, the crust and the core. The latter, divided into the outer core and inner core, has a radius of approximately 10 km and contains most of the star's mass. The crust, of ≈ 1 km thickness and containing only a few percent of the total mass, can also be divided into the outer crust and inner crust. One of the most important prediction of a given equation of state (EOS) is the location of the inner edge of a neutron star crust. The inner crust comprises the region from the density at which neutrons drip-out of nuclei, to the inner edge separating the solid crust from the homogeneous liquid core. At the inner edge, in fact, a phase transition occurs from the high-density homogeneous matter to the inhomogeneous matter at lower densities.

All theoretical studies have shown that the core-crust transition density and pressure are very sensitive to the density dependence of the nuclear matter symmetry energy. In particular, it has been shown that the $E_{sym}(n)$ constrained in the same sub-saturation density range as the neutron star crust by the isospin diffusion data in heavy-ion collisions at intermediate energies [3, 4], limits the

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transition density and pressure to 0.040 fm^{-3} $\leq n_t \leq$ 0.065 fm^{-3} and 0.01 MeV fm^{-3} $\leq P_t \leq$ 0.26 MeV fm^{-3}, respectively. In the present work we apply a class of relativistic density functionals in a systematic investigation of the transition density n_t and pressure P_t at the inner edge separating the liquid core from the solid crust of neutron stars by employing the thermodynamical method.

The core-crust interface corresponds to the phase transition between nuclei and uniform nuclear matter. The uniform matter is nearly pure neutron matter, with a proton fraction of just a few percent determined by the condition of beta equilibrium. Weak interactions conserve both baryon number and charge [5], and from the first low of thermodynamics, at temperature $T = 0$:

$$\mathrm{d}u = -P\mathrm{d}v - \hat{\mu}\mathrm{d}q, \tag{1}$$

where u is the internal energy per baryon, P is the total pressure, v is the volume per baryon ($v = 1/n$ where n is the baryon density) and q is the charge fraction ($q = x - Y_e$ where x and Y_e are the proton and electron fraction in baryonic matter respectively). The stability of the uniform phase requires that $u(v, q)$ is a convex function [6]. This condition leads to the following two constraints for the pressure and the chemical potential

$$-\left(\frac{\partial P}{\partial v}\right)_q - \left(\frac{\partial P}{\partial q}\right)_v \left(\frac{\partial q}{\partial v}\right)_{\hat{\mu}} > 0, \tag{2}$$

$$-\left(\frac{\partial \mu}{\partial q}\right)_v > 0. \tag{3}$$

It is assumed that the total internal energy per baryon $u(v, q)$ can be decomposed into baryon (E_N) and electron (E_e) contributions

$$u(v, q) = E_N(v, q) + E_e(v, q). \tag{4}$$

In this work the well know parabolic approximation is used for the baryon energy $E_N(v, q)$

$$E_N(v, q) \simeq V(v) + E_{sym}(v)(1 - 2x)^2\ . \tag{5}$$

The condition of charge neutrality $q = 0$ requires that $x = Y_e$. This is the case we will consider in the present study. After some algebra, it can be shown that the conditions (2) and (3) are equivalent to

$$\begin{aligned} C_I(n) &= n^2\frac{\mathrm{d}^2V}{\mathrm{d}n^2} + 2n\frac{\mathrm{d}V}{dn} \qquad (6)\\ &+ (1-2x)^2\left[n^2\frac{\mathrm{d}^2E_{sym}}{\mathrm{d}n^2} + 2n\frac{\mathrm{d}E_{sym}}{\mathrm{d}n} - 2\frac{1}{E_{sym}}\left(n\frac{\mathrm{d}E_{sym}}{\mathrm{d}n}\right)^2\right] > 0, \end{aligned}$$

$$C_{II}(n) = -\left(\frac{\partial q}{\partial \hat{\mu}}\right)_v = \frac{1}{8E_{sym}} + \frac{3Y_e}{\hat{\mu}} > 0. \tag{7}$$

The second inequality (7) is usually valid. The transition density n_t is determined from the first inequality (7). For a given EOS, the quantity $C_I(n)$ is

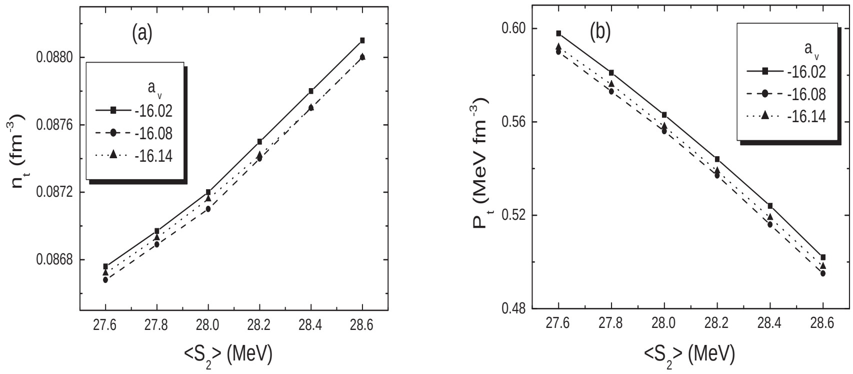

Figure 1: The transition density n_t (a), and the transition pressure P_t (b), as functions of $\langle S_2 \rangle$ for three values of the nuclear matter volume energy coefficient a_v.

plotted as a function of the baryonic density n, and the equation $C_I(n_t) = 0$ defines the transition density n_t.

The framework of nuclear energy density functionals (NEDF) provides, at present, the most complete microscopic approach to the rich variety of structure phenomena in medium-heavy and heavy complex nuclei, including regions of the nuclide chart far from the valley of β-stability. By employing global functionals parameterized by a set of ≈ 10 coupling constants, the current generation of EDF-based models has achieved a high level of accuracy in the description of ground states and properties of excited states, exotic unstable nuclei, and even nuclear systems at the nucleon drip-lines [7]. Starting from the relativistic energy density functional DD-PC1, in this work we examine the sensitivity of the core-crust transition density n_t and pressure P_t of neutron stars, on the density dependence of the corresponding symmetry energy of nucleonic matter.

Fig. 1 displays the values of the transition density n_t (in fm^{-3}) and transition pressure P_t (in Mev fm^{-3}), calculated in the thermodynamical model, as functions of $\langle S_2 \rangle$ for three values of the nuclear matter volume energy coefficient a_v. For a given value of the parameter a_v, the values of n_t rise with increasing $\langle S_2 \rangle$, whereas the opposite is found for the values of P_t. For the considered interval of $\langle S_2 \rangle$, however, the changes are small. An increase of 3.5% in $\langle S_2 \rangle$ leads to an increase of 1.5% in the value of n_t. The transition pressure exhibits a somewhat more pronounced dependence (the corresponding decrease is around $16 - 20\%$). Both n_t and P_t display a negligible dependence on a_v, even though $a_v = -16.02$ MeV and $a_v = -16.14$ MeV lie outside the interval of values for which the absolute deviations between calculated and experimental masses are smaller than 1 MeV.

In Fig. 2 we plot the transition pressure P_t as a function of the transition density n_t for the three sets of nuclear matter EOS and symmetry energy

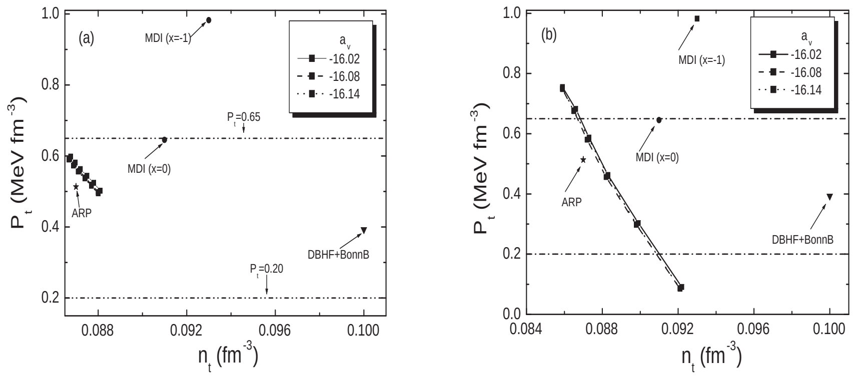

Figure 2: (a) The transition pressure P_t as a function of the transition density n_t. For a fixed value of the symmetry energy at saturation $a_4 = 33$ MeV, and three values of the nuclear matter volume energy coefficient a_v, the parameter $\langle S_2 \rangle$ is varied in the interval between 27.6 MeV and 28.6 MeV. (b) The same but for fixed $\langle S_2 \rangle = 27.8$ MeV, and the symmetry energy at saturation in the interval 30 MeV $\leq a_4 \leq$ 35 MeV.

described above, in comparison with results of recent calculations performed using an isospin and momentum-dependent modified Gogny effective interaction (MDI) [8]. The different values of the parameter x in the MDI model correspond to various choices of the density dependence of the nuclear symmetry energy. In addition to the MDI EOS, in Fig. 2 we also show the result obtained by Akmal et al. [9] with the $A18 + \delta v + UIX^*$ interaction (ARP), and the value obtained in the recent Dirac-Brueckner-Hartree-Fock (DBHF) calculation [10] with the Bonn B One-Boson-Exchange (OBE) potential (DBHF+Bonn B) [11]. We notice that by keeping $\langle S_2 \rangle$ constant and varying a_4 in the interval between 30 MeV and 35 MeV, the density dependence of the symmetry energy can be modified in a controlled way, i.e. the corresponding energy density functionals still reproduce ground-state properties of finite nuclei in fair agreement with data.

In Fig. 3 we display the corresponding values of the transition density n_t (in fm^{-3}) and transition pressure P_t (in Mev fm^{-3}) as functions of a_4 for three values of the nuclear matter volume energy coefficient a_v. The transition pressure P_t as a function of the transition density n_t for the three sets of nuclear matter EOS and symmetry energy is also plotted.

Finally, in Fig. 4 we compare the present prediction for the range of values of the transition density n_t with the results of Horowitz and Piekarewicz who, in Ref. [12], also used the framework of relativistic mean-field effective interactions to study the relationship between the neutron-skin thickness of a heavy nucleus and the properties of neutron star crusts. For the solid crust of a neutron star, the effective RMF interactions were used in a simple RPA calculation of the transition density below which uniform neutron-rich matter becomes unstable

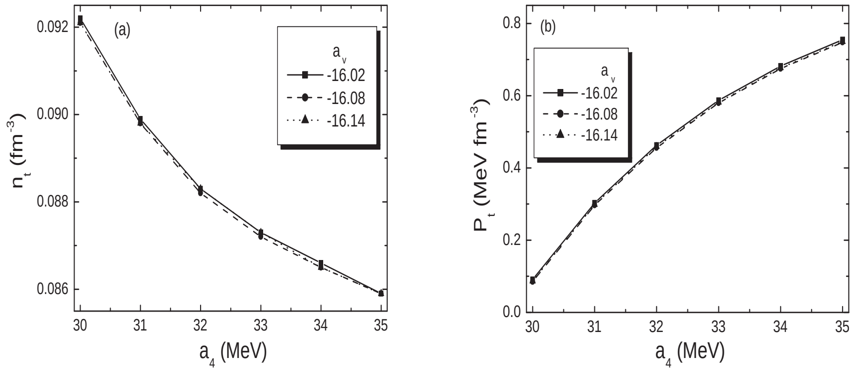

Figure 3: The transition density n_t (a), and the transition pressure P_t (b), as functions of the symmetry energy at saturation density a_4, for three values of the nuclear matter volume energy coefficient a_v.

against small amplitude density fluctuations. The resulting transition densities are plotted in Fig. 4 as a function of the predicted difference between neutron and proton *rms* radii in ^{208}Pb. This inverse correlation was parameterized [12]

$$n_t \approx 0.16 - 0.39(R_n - R_p), \tag{8}$$

with the skin thickness expressed in fm. In the present analysis, using a different type of relativistic effective interactions and varying the density dependence of the symmetry energy by explicitly modifying $\langle S_2 \rangle$ or a_4, we find a much weaker dependence n_t on the neutron-skin thickness of ^{208}Pb.

The framework of relativistic nuclear energy functionals has been employed to analyze and constrain the transition density n_t and pressure P_t at the inner edge between the liquid core and the solid crust of a neutron star, using the thermodynamical method. Starting from a class of energy density functionals carefully adjusted to experimental masses of finite nuclei, we have examined the sensitivity of the core-crust transition density n_t and pressure P_t on the density dependence of corresponding symmetry energy of nucleonic matter. Instead of an unrestricted variation of the parameters of the Taylor expansion of the symmetry energy around the saturation density of nuclear matter, that is the slope parameter and the isovector correction to the compression modulus, we modify the density dependence by varying the value of the nuclear symmetry energy at a point somewhat below the saturation density $\langle S_2 \rangle$ (the symmetry energy at $n = 0.12$ fm^{-3}), and at the saturation density a_4 (the symmetry energy at $n = 0.152$ fm^{-3}, the saturation density for this class of relativistic density functionals). In the former case, for a given value of the volume energy coefficient a_v, $\langle S_2 \rangle$ has been varied in a rather narrow interval of values 27.6 MeV $\leq \langle S_2 \rangle \leq$ 28.6 MeV determined by a fit to the experimental binding

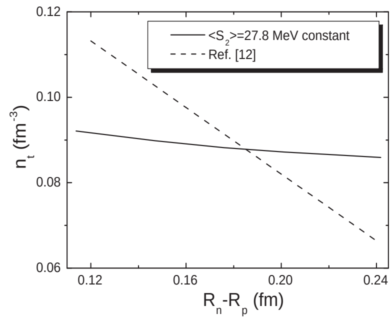

Figure 4: The transition density n_t as function of the neutron-skin thickness $R_n - R_p$ of ^{208}Pb. The values of n_t calculated using the thermodynamical model in the present work (solid), are compared with those of Ref. [12] (see text for description).

energies. Both n_t and P_t display a negligible dependence on a_v. The variation of the parameter a_4 has been in the range of values: 30 MeV $\leq a_4 \leq$ 35 MeV, allowed by the empirical thickness of the neutron-skin and excitation energies of isovector dipole resonances, for a fixed value of $\langle S_2 \rangle$. Again, there is virtually no dependence on a_v, but now both n_t and P_t span much wider intervals.

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