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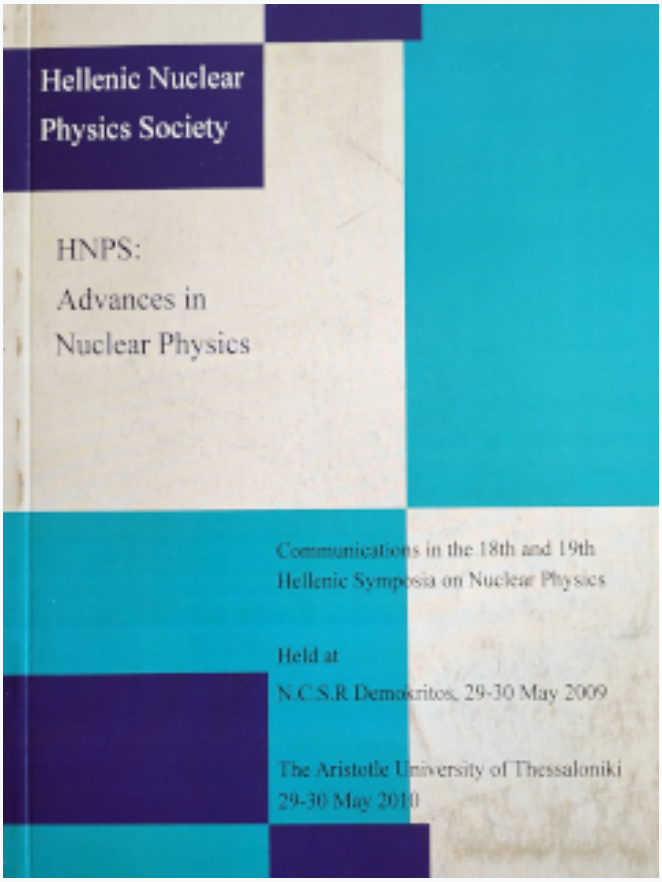

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Nuclear muon capture rates by using relativistic muon wavefunctions.

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1 Introduction.

The numerical solutions of ordinary and partial differential equations (ODEs and PDEs) among others are of special importance for quantum mechanics problems. The wavefunctions of elementary particles in nuclei and the binding energies are calculated by the Schrödinger and Dirac equations. The last two decades or so, a large number of numerical algorithms have been employed to solve the eigen value problem. Such numerical methods the Artificial Neural Networks (ANN), genetic algorithms, direct diagonalization, and the finite elements method (FEM) among others. ANN and FEM were presented as the most reliable algorithms (e.g. MERLIN) to solve non-homogeneous ODEs and PDEs [1,3,2].

We apply the genetic algorithm in order to solve the Dirac equation of muonic atoms. For the case of ^{208}Pb the muon binding energy $E_b \sim -10.54MeV$ and the muon wavefunction using the ANN method [1] are in good agreement with our calculations obtained by a genetic algorithm. The calculation of the wavefunctions can be used to estimate the total muonic capture rates. For these purpose, we make use of the Fermi's golden rule and the semi-empirical Primakoff's method [4] and highlight the limitation of the method.

2 Dirac equation

In this paragraph, we solve the Dirac equation for muonic atoms. In our investigation we consinder both the large and small component [5,6]. The Dirac

Equation in a central force system is described by the following equation

$$E\psi = [-i\gamma_5\sigma_r(\frac{\partial}{\partial r} + \frac{1}{r} - \frac{\beta}{dr}K) + V(r) + m_i\beta]\psi \tag{1}$$

where γ_i are the well known Dirac matrices and

$$\sigma_r = \begin{pmatrix} \sigma \cdot r & 0 \\ 0 & \sigma \cdot r \end{pmatrix} \; ; \; K = \begin{pmatrix} \sigma \cdot l + 1 & 0 \\ 0 & \sigma \cdot l + 1 \end{pmatrix} \tag{2}$$

where μ_i is the reduced mass, s are the Pauli matrices, l is the orbital angular momentum. For the case $l = 0$ (s-$state$), the two components of Dirac's equation could be given by

$$\frac{d}{dr}f(r) + \frac{1}{r}f(r) = \frac{1}{\hbar}(\mu c^2 - E + V(r))g(r) \tag{3}$$

$$\frac{d}{dr}g(r) - \frac{1}{r}g(r) = \frac{1}{\hbar}(\mu c^2 + E - V(r))f(r) \tag{4}$$

where $f(r)$ and $g(r)$ are the small and large components of the reduced radial wavefunction of the muon bound respectively. The total energy is calculated by

$$E = \frac{\mu c^2 \int_0^\infty [g^2(r) + f^2(r)]dr + \int_0^\infty V(r)[g^2(r) - f^2(r)]dr}{\int_0^\infty [g^2(r) - f^2(r)]dr} \tag{5}$$

and the extended nuclear Coulomb field, for example the one originating from the point-nucleon charge distribution $\rho(r)$ is given by

$$V(\mathbf{r}) = -\mathbf{e^2} \int_{-\infty}^{\infty} \frac{\rho(\mathbf{r'})}{|\mathbf{r} - \mathbf{r'}|}\mathbf{d^3r'} \tag{6}$$

The nuclear charge density $\rho(r)$ can be estimated using the following models [7]:
1) Two parameter Fermi model

$$\rho(r) = \frac{\rho(r_0)}{1 + e^{\frac{r-c}{z}}} \tag{7}$$

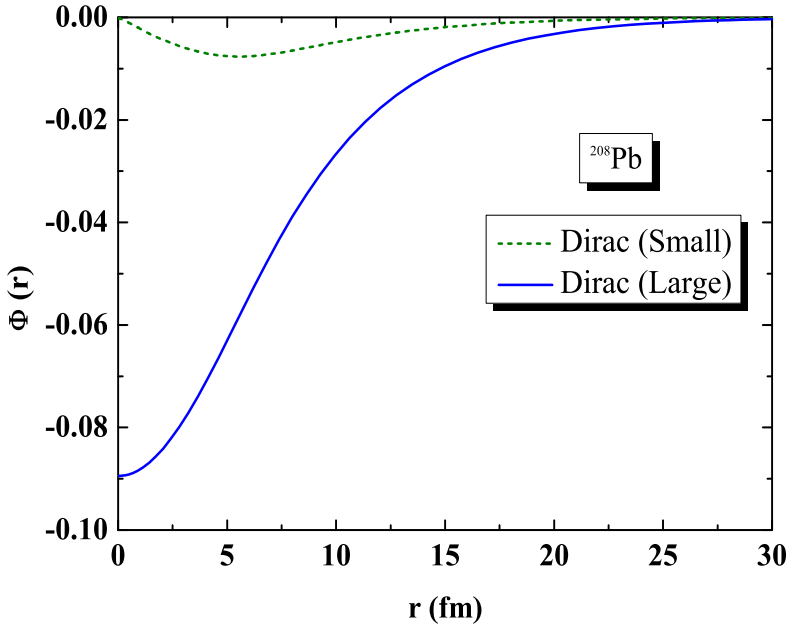

Fig. 1. The large, small component of Dirac spinor solution for muon bound in ^{208}Pb.

where c, z are the radius and thickness parameter respectively.
2) Fourier-Bessel model

$$\rho(r) = \begin{cases} \sum_n \alpha_n j_0(n\pi r/R) & \text{for } r \leq R \\ 0 & \text{for } r \geq R \end{cases} \tag{8}$$

where $j_0(qr)$ denotes the Bessel function of order zero.
3) Sum of Gaussians

$$\rho(r) = \sum_n A_n (e^{-[(r-R_n)/\gamma]^2} + e^{-[(r+R_n)/\gamma]^2}) \tag{9}$$

where the coefficients are defined in Ref. [7].

We use the following parametrized solutions for the small and large component of Dirac equation

$$f(r) = re^{-\beta r} N(r, \mathbf{u_f}, \mathbf{v_f}, \mathbf{w_f}) \tag{10}$$

$$g(r) = re^{-\beta r} N(r, \mathbf{u_g}, \mathbf{v_g}, \mathbf{w_g}) \tag{11}$$

with $\beta > 0$ and the parameters $\mathbf{u_g}$, $\mathbf{v_g}$, $\mathbf{w_g}$ being ANN parameters. $N(r, \mathbf{u_g}, \mathbf{v_g}, \mathbf{w_g})$ is feed-forward artificial neural network

The following error function has to be minimized for estimating the binding energy

$$\frac{\sum_{i=1}^{n}[\frac{df(r_i)}{dr} + \frac{f(r_i)}{r_i} - \frac{\mu c^2 - E + V(r_i)}{\hbar c} g(r_i)]^2 + [\frac{dg(r_i)}{dr} - \frac{g(r_i)}{r_i} - \frac{\mu c^2 + E - V(r_i)}{\hbar c} f(r_i)]^2}{\int_0^\infty [g^2(r) + f^2(r)] dr} \quad (12)$$

The minimization has been achived using a genetic algorithm. Genetic algorithms are biologically inspired global optimization methods that are based on the so called genetic operations of natural selection, reproduction and mutation. These techniques have been used with success in many scientific fields such as combinatorial problems [8], neural network training [9,10], electromagnetics [11], design of water distribution networks [12] etc. The main advantages of genetic algorithms are: a) they can be implement easily, b) they can be parallelized and c) they do not require the computation of derivatives of any order. The main steps of the used genetic algorithm are shortly described as follows:

- Step 1 (**initialization**):
  - Generate N uniformly distributed random points (chromosomes) and store them to the set S.
  - Set iter=0
- Step 2 (**evaluation**): Evaluate the function value of each chromosome.
- Step 3 (**termination check**): If termination criteria are hold terminate. The termination criteria of the used algorithm are based on asymptotic considerations. At every generation denoted by iter, the variance $\sigma^{(\text{iter})}$ of the best located value is recorded. If there is not any improvement for a number of generations, it is highly possible that the global minimum is already found and hence the algorithm should terminate.
- Step 4 (**genetic operations**):
  - **Selection**: Select $m \leq N$ parents from S. The selection is performed using the tournament selection technique.
  - **Crossover**: Create m new points (offsprings) from the previously selected parents.
  - **Mutation**: Mutate the offsprings produced in the crossover step with probability p_m.
- Step 5 (**replacement**): Replace the m worst chromosomes in the population with the previously generated offsprings.
- Step 6 (**local technique**): Create using the local technique procedure a trial point $\tilde{x}$. If $f(\tilde{x}) \leq f(x_h)$ where x_h is the current worst point in S, then replace x_h by $\tilde{x}$.
- Step 7:

- A local search procedure is applied to the best located chromosome x_l every $K_{\rm ls}$ generations, where $K_{\rm ls}$ is a user defined constant that denotes how frequent the local search procedure has to be applied.
- **Set** iter=iter+1
- **goto** step 2

In this article the modified genetic algorithm was used with a population of 200 chromosomes. The maximum number of generations allowed was set to 20 and the mutation rate was set to 5%. The binding energy ϵ_b could be given by $\epsilon_b = E - \mu c^2$.

3 Results for muon capture

The capture of a muon bound in the field of a nucleus (A, Z) can be represented by the following nuclear process

$$\mu^- + (A, Z) \rightarrow (A, Z-1) + \nu_\mu \tag{13}$$

The total muon capture rates could be estimated by the following relation [4]

$$\lambda_{\mu c} = G^2 \frac{\nu}{2\pi\hbar^2 c} \left(\frac{m_{\mu_\mu} c^2 \alpha}{\hbar c} \right)^3 \frac{Z_{eff}^4}{Z} \left[Z - F_\sigma(A, Z) \right] \tag{14}$$

where α is the fine structure constant, $F_\sigma(A, Z)$ is the correlation function and the effective atomic number Z_{eff} is given by

$$\left\langle \Phi_\mu^{1s} \right\rangle^2 = \frac{\int |\Phi_\mu|^2 \rho(\mathbf{r}) d^3\mathbf{r}}{\int \rho(\mathbf{r}) d^3\mathbf{r}} = \frac{\alpha^2 m_\mu^3}{\pi} \frac{Z_{eff}^4}{Z} \tag{15}$$

where Φ_μ^{1s} is the muon wavefunction for the 1s state (see Fig. 1.) It is worth mentioning that the effective atomic number is less than the atomic number Z because a large part of the muon wavefunction is inside the nucleus.

The correlation function $F_\sigma(A, Z)$ can be estimated by microscopic methods and Primakoff's semi-empirical method. The Primakoff's semi-empirical method is an approximation which works for light nuclei as is presented in table 1. For heavy nuclei, $F_\sigma(A, Z)$ gets four times larger value than the microscopic method.

Table 1
The correlation function for light and heavy nuclei for two different models.

$Nuclei$	$Microscopic\ method$	$Primakoff\ method$
(A, Z)	$F_\sigma(A, Z)$	$F_\sigma(A, Z)$
(4, 2)	-1.72	-1.68
(208, 82)	-17.80	-78.18

4 Summary and Conclusions

In summary, in the present work we have used a genetic algorithm in order to calculate the muon wavefunction and the corresponding binding energy of the μ^- in a ^{208}Pb muonic atom. More specifically, we have solved the Dirac's equation and concluded that the estimated binding energy and the bound muon wavefunctions (small and large component for the $1s$ state) are in very good agreement with those obtained by solving this equation by utilizing artificial neural network (ANN) techqniques [1]. Lastly, we have discussed and point out the frame of the muonic capture process by using the semi-empirical Primakoff's method.

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