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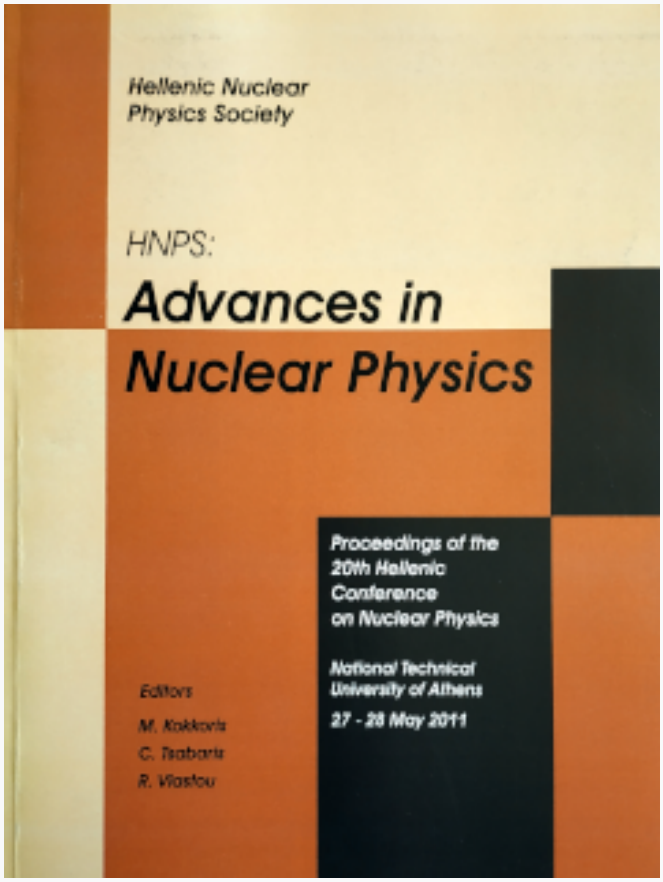

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The Deformation Dependent Mass Davidson Model

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Abstract

The Deformation Dependent Mass Davidson Model is an extension of the well known Bohr-Mottelson Hamiltonian for the atomic nuclei. It primarily refers to the mass dependence on the deformation and secondary to the Davidson behavior for the potential of the β-vibration. This article will be devoted solely in the solution of the radial equation. Fitting results for the ^{162}Dy and ^{238}U ground state, β_1 and γ_1 bands are also presented.

1 SUSYQM for the β vibrations

Quesne and Tkachuk began to study SUSYQM methods for non-pointlike quantum oscillators [1,2], that is harmonic oscillators with non-pointlike excitations. In principle, in such an oscillator the Heisenberg uncertainty relations are modified and this guides the modification of the canonical commutation relations. In [3] the equivalence of such an oscillator with a Schroedinger equation of a position dependent mass problem was established. In [4] a Schroedinger equation of the Bohr-Mottelson type was presented for the case of a mass dependent on the β degree of freedom. Therefore, based on the Quesne and Tkachuk equivalence, we construct the phase space of the β degree of freedom with commutation relations,

$$[\beta, p_\beta] = i\hbar f(\beta). \tag{1}$$

The function $f(\beta)$ is called the deformation function. Because of its presence, in the second quantization procedure, the ladder operators will not be as usual but

modified in generally as,

$$A^{\pm} \rightarrow A^{\pm}(a,\mu,\nu) = \mp\sqrt{f(a;\beta)}\frac{d}{d\beta}\sqrt{f(a;\beta)} + W(\mu,\nu;\beta). \tag{2}$$

Here a deformed momentum operator is introduced through the deformation function $f(a;\beta)$ and the superpotential $W(\mu,\nu;\beta)$ which signals the SUSYQM method. From the parameters (a,μ,ν), only a will remain free. The Hamiltonian corresponding to these ladder operators will of course give good quantum numbers for the stationary states of β-vibrations, characterized by a function $R(\beta)$.

The principal SUSYQM demands states that the action of the Hamiltonian operator to the ground state shall give zero. The parameter ε_0 is introduced, which is assumed to be the energy of the ground state, and therefore SUSYQM method is valid for the Hamiltonian,

$$A^{+}(a,\mu,\nu)A^{-}(a,\mu,\nu) = H - \varepsilon_0, \tag{3}$$

which gives zero eigenvalue for the vacuum. This Hamiltonian shall correspond to the radial equation which is [5],

$$HR = -\left(\sqrt{f}\frac{d}{d\beta}\sqrt{f}\right)^2 R + 2uR = 2\epsilon R. \tag{4}$$

This correspondence emerges the equation,

$$W^2(\mu,\nu;\beta) - f(\beta)W'(\mu,\nu;\beta) + \varepsilon_0 = 2u(\beta). \tag{5}$$

Now, the main result of [3] is that the Schroedinger equation (4) is also obtained for the case of a position dependent mass problem, as discussed in [4,6] with a change in the potential,

$$u \rightarrow u_{eff} = u + \frac{1}{4}ff'' + \frac{1}{6}(f')^2. \tag{6}$$

Therefore the energy of the ground state ε_0 can be determined from for the u_{eff}. This can be done if the specific potential $u_{eff}(\beta)$ has known superpotential and deformation function.

1.1 Shape invariance and the Davidson potential

Schroedinger equation is known to be exactly solvable for the Davidson potential [7] . Shape invariance states that a potential gives exact solutions if and only if retains the same functional dependence under the change of its parameters. In the figure below, shape invariance is shown for the Davidson's parameter β_0 which is fitted to a specific nucleus, namely the minimum reflects the ground state of the β vibrations.

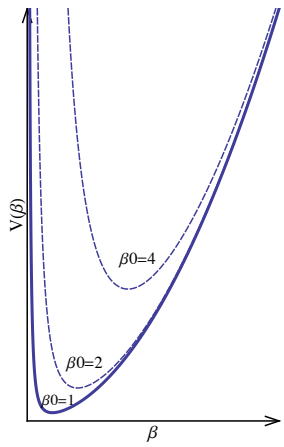

Fig. 1. The Davidson Potential $V(\beta) = \beta^2 + \frac{\beta_0^4}{\beta^2}$ and its shape invariant behavior under a parameter shift.

We extend the shape invariance condition for the effective potential. That means that u_{eff} should retain the Davidson behavior for every change in the parameters, with

$$W^2(\mu,\nu;\beta) - f(\beta)W'(\mu,\nu;\beta) + \varepsilon_0 = 2u_{eff}(\beta) = k_1\beta^2 + k_0 + \frac{k_{-1}}{\beta^2}. \quad (7)$$

In [8], classes of shape invariant potentials have been studied with the identification of their corresponding superpotentials and deformation functions. The superpotential and deformation function for the Davidson case are,

$$W(\beta) = \frac{\mu}{\beta} + \nu\beta \qquad , \quad f(\beta) = 1 + a\beta^2. \quad (8)$$

2 Energy spectrum

¿From [4] the coefficients of the effective potential are,

$$k_1 = 2 + a^2(12+\Lambda), \qquad k_0 = a(13+2\Lambda), \qquad k_{-1} = 2 + \Lambda + 2\beta_0^4, \quad (9)$$

with $\Lambda = \tau(\tau+3)$ for γ unstable and $\Lambda = \frac{L(L+1)-K^2}{3} + (6c)(n_\gamma+1)$ for the axially symmetric behavior. The parameter c controls γ stiffness and n_γ is the quantum number for γ vibrations.

With these equations the energy of the ground state is determined from the expressions,

$$\mu(\mu+1) = k_{-1}, \qquad \nu(\nu-a) = k_1, \qquad 2\mu\nu + \mu a - \nu + \varepsilon_0 = k_0, \quad (10)$$

$$\mu = -\frac{1}{2}\left(1+\sqrt{9+\Lambda+8\beta_0^4}\right), \qquad \nu = \frac{a}{2}\left(1+\sqrt{1+\frac{8+4a^2(12+\Lambda)}{a^2}}\right). \tag{11}$$

¿From these equations the energy of the ground state ϵ_0 is found to be

$$\begin{aligned}\epsilon_0 &= \frac{19}{4}a+\frac{5}{2}a+\frac{1}{2}\sqrt{a^2+4k_1} \\ &+\frac{a}{2}\sqrt{1+4k_{-1}}+\frac{1}{4}\sqrt{(a^2+4k_1)(1+4k_{-1})}+a\Lambda.\end{aligned} \tag{12}$$

Actually this is the energy of the ground state band for the β vibration, because of the Λ dependence in each case. The energy for all the bands, for the β vibrations is found to be

$$\epsilon_n = \frac{1}{2}[k_0+\frac{1}{2}a(3+2\Delta_1+2\Delta_2+\Delta_1\Delta_2)+2a(2+\Delta_1+\Delta_2)n+4an^2], \tag{13}$$

with $n = 0, 1, 2, \ldots$ and $\Delta_1 \equiv \sqrt{1+4k_{-1}}, \quad \Delta_2 \equiv \sqrt{1+4\frac{k_1}{a^2}}$. The ground state band is obtained from $n = 0$, while the quasi-β_1 band is obtained from $n = 1$, and the quasi-β_2 band is obtained from $n = 2$.

3 Fitting

In [5] the above energy spectrum was fitted for the cases of γ-unstable and axially symmetric nuclei. The fitting measure was the Gaussian error,

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(E_i(exp)-E_i(th))^2}{(n-1)E(2_1^+)^2}}. \tag{14}$$

In the axially symmetric prolate case the free parameters are (β_0, a, c_γ) were a shows in a certain nucleus the mass dependence on the deformation and c_γ the γ stiffness. Results for the spectra are encouraging for the majority of γ-unstable and axially symmetric nuclei, apart from the X(5) candidates, where it is expected that the appropriate potential is not the Davidson. Here the cases of ^{238}U and ^{162}Dy are presented. The agreement is very good for large angular momenta for the ground state and the γ_1 bands. In the case of the β_1 band the agreement is restricted to the low-lying states. Also in [5] $B(E2)$ are compared to the experiment and reveal an overall good agreement apart from the transitions from the β_1 to the ground state band, which are in general overestimated.

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Table 1
Normalized [to the energy of the first excited state, $E(2_1^+)$] energy levels of the ground state band (gsb) and the β_1 and γ_1 bands of ^{162}Dy and ^{238}U, obtained from the Bohr Hamiltonian with β-dependent mass for axially symmetric prolate deformed nuclei using the parameters given in Ref. [5], compared to experimental data [9].

	^{162}Dy	^{162}Dy	^{238}U	^{238}U		^{162}Dy	^{162}Dy	^{238}U	^{238}U
L	exp	th	exp	th	L	exp	th	exp	th
	gsb	gsb	gsb	gsb		γ_1	γ_1	γ_1	γ_1
0	0.00	0.00	0.00	0.00	2	11.0	11.2	23.6	24.7
2	1.00	1.00	1.00	1.00	3	11.9	12.1	24.6	25.5
4	3.29	3.30	3.30	3.31	4	13.2	13.3	25.9	26.7
6	6.80	6.80	6.84	6.86	5	14.7	14.7	27.4	28.1
18	47.58	47.28	48.78	48.98	11	29.0	28.9	41.7	41.9
26			89.46	90.55	15			55.7	55.5
28			100.57	102.08	16			59.7	59.4
30			112.10	113.99	17			63.9	63.4
					18			68.2	67.7
	β_1	β_1	β_1	β_1	19			72.7	72.0
0	17.3	15.7	20.6	20.6	20			77.3	76.6
2	18.0	16.7	21.5	21.6	21			82.1	81.3
4	19.5	19.0	23.5	24.0	22			87.0	86.1
6	21.9	22.6			23			91.9	91.0

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