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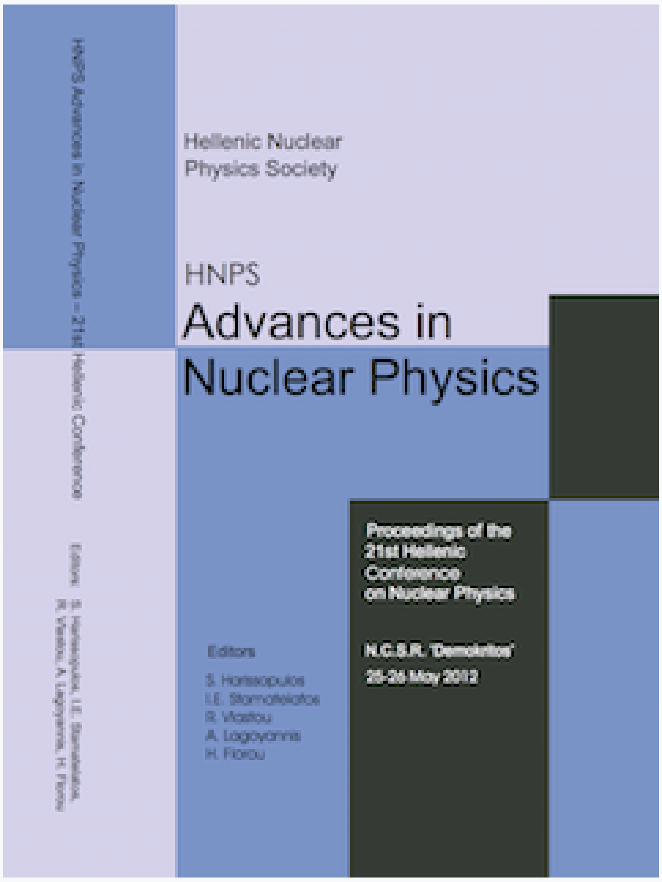


## Electron-capture and beta-decay modes with realistic nuclear structure calculations


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# Electron-capture and beta-decay modes with realistic nuclear structure calculations

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**Abstract**

In this work, we concentrate on the $e^-$-capture and $\beta$-decay processes in isotopes that are important for searching the explosive nucleosynthesis in massive stars. To this aim, we improved our codes which use compact analytical expressions for the required reduced matrix elements of all basic multipole operators (isospin representation). The ground state of the nuclear isotopes chosen, is constructed in the context of the BCS method while their excited states are calculated by solving the QRPA equations, using as residual two-body interactions that of the Bonn C-D one-meson exchange potential. We focus on the role of the charged Gamow-Teller and Fermi transitions in Fe group nuclei that are the main constituents of the core in presupernovae formation.

## 1 Introduction

It is well known that the nuclear $\beta$-decay modes ($\beta^-$ and $\beta^+$ decays),

$$(A,Z) \to (A,Z+1) + e^- + \nu_e\,, \quad (A,Z) \to (A,Z-1) + e^+ + \bar{\nu}_e\,, \tag{1}$$

the electron- or positron-capture ($e^{\pm}$-capture) on nuclei,

$$(A,Z) + e^- \to (A,Z-1)^* + \nu_e\,, \quad (A,Z) + e^+ \to (A,Z+1)^* + \bar{\nu}_e\,, \tag{2}$$

as well as the charged and neutral current neutrino-nucleus reactions,

$$\nu_l(\overline{\nu_l}) + (A,Z) \to (A,Z\pm1)^* + l^-(l^+)\,, \quad \nu_l(\overline{\nu_l}) + (A,Z) \to (A,Z)^* + \nu_l^{\prime}(\overline{\nu_l^{\prime}}), \tag{3}$$

are very important processes to study the fundamental electroweak interactions and understand the late stages of stellar evolution [1–4].

The evolution of massive stars in the late stages are strongly influenced by the weak interactions which determine, among others physical observables (for example the core entropy), the electron-to-baryon ratio $Y_e = n_e/n_b$ of the presupernova star. Obviously, the $e^-$-capture process reduces the number of electrons available for pressure support while $\beta^-$-decay acts in the opposite direction. Processes (1) and (2), generate neutrinos (or antineutrinos), which for densities smaller than $10^{11} g\, cm^3$ escape the star carrying away energy and entropy from the core. This is a very effective cooling mechanism of exploding massive stars [5,6].

Recent studies on stellar collapse indicate that $e^-$-capture on nucleons and nuclei play important role, mostly in the early stages of the collapse of massive stars. Various nuclear methods used for calculating $e^-$-capture on heavy nuclei during the collapse phase have shown that the capture on free protons dominates due to the significantly lower Q-value [7,8]. Moreover, $e^-$-capture on nuclei produces neutrinos with rather low energies, in contrast to the inelastic neutrino-nucleus reactions occurring in supernova. We mention that the number of free protons divided by the total number of nucleons ($Y_p = n_p/n_N$) is quite low ($Y_p \sim 10^{-6}$ in a presupernova model for a star of mass $M = 15M_\odot$) [1,8]. Electron capture on nuclei takes place in very dense environment of the stellar core where the Fermi energy (or equivalently the chemical potential) of the degenerate electron gas is sufficiently large to overcome the threshold energy given by negative Q values of the reactions involved in the interior of the stars. This high Fermi energy of the degenerate electron gas leads to enormous $e^-$-capture on nuclei and reduces the electron to baryon ratio $Y_e$ [5,6]. As a consequence, the nuclear composition is shifted to more neutron-rich and heavier nuclei (including those with $N > 40$) which dominate the matter composition for densities larger than about $10^{10} g\, cm^{-3}$ [1,10].

During the final stage of evolution of massive stars, $e^-$-capture and nuclear $\beta$-decay are dominated by Fermi (F) and Gamow-Teller (GT) transitions (allowed transitions) of the daughter nuclei. In addition to the determination of the nuclear structure, an appropriate description of both transition types (specifically those of GT transitions in nuclei) affects directly the information extracted from the early phases of core collapse supernova type II. As we will see (see also Ref. [5,6,9,10]) $e^-$-capture rates are predominantly determined by the F and GT transitions but also by first- and second-forbidden transitions that contribute to the $e^-$-capture rates in the supernova environment.

In the present work, we address $e^-$-capture rates and $\beta$-decays within a refined version of the Quasi-Particle Random Phase Approximation (QRPA) which is applied to construct all the accessible final (excited) states. For the description of a correlated nuclear ground state we determine the single-particle occupation numbers, calculated in BCS theory as shown below. Our primary focus is on the GT transitions in the region of medium-heavy nuclei (around A = 56) which are of special significance and offer the main constituents of the stellar core in presupernovae formations.

## 2 Semileptonic-Charged current reactions in nuclei

In this work,we study in detail the $e^-$-capture and $\beta$-decay modes on medium-heavy nuclei. Even thought the investigation of these processes started decades ago, more accurate transition rates are needed due to their significant importance in core-collapse supernova dynamics and other astrophysical phenomena. To this purpose, we first, are indented to do extensive cross sections calculations for $e^{\pm}$-capture in a set of promising isotopes by employing the Donnelly-Walecka multipole decomposition method [12]. Since $e^-$-capture is a particle conjugate process of the corresponding charged-current neutrino-nucleus reaction Eq.(3), from a nuclear theory point of view the above mentioned processes can be studied with the same nuclear methods. For the computation of the many-body nuclear wave functions we utilize the pn-QRPA method as described below [11–13].

### *2.1 Interaction Hamiltonian for charge changing reactions*

The nuclear calculations for all the semi-leptonic processes that occur in the presence of nuclei (including $e^-$-capture) start by writing down the weak interaction Hamiltonian, $\hat{H}_w$, in current-current form. By denoting the leptonic current as $j_\mu^{lept}$ and the hadronic current as $\hat{\mathcal{J}}_\mu$, the Hamiltonian $\hat{H}_w$ is written as

$$\hat{\mathcal{H}}_w = \frac{G\,\cos\theta_c}{\sqrt{2}} j_\mu^{lept}(\mathtt{x})\hat{\mathcal{J}}^\mu(\mathtt{x}) \quad (4)$$

where G is the weak interaction coupling constant and $\theta_c$ is the Cabibbo angle ($\cos\theta_c = 0.974$). The hadronic current, $\mathcal{J}_\mu$, which is of primary interest from a nuclear theory point of view, consists of the polar-vector and the axial-vector components written in terms of vector, axial-vector and pseudoscalar form factors that depend on the four-momentum transfer and read

$$\hat{\mathcal{J}}^\mu = g_V \hat{J}_V^\mu - g_A \hat{J}_A^\mu \quad (5)$$

In Eq. (5), $g_V$ ($g_A$) represent the static weak interaction coupling constants of the polar (axial) vector component (at four momentum transfer $q_\mu^2 = 0$).

One of the main goals of this effort is to construct an advantageous code for computing $e^-$-capture cross sections in various currently interesting nuclear isotopes. This code is based on the present formalism and the existing pn-QRPA code that uses realistic Bonn C-D residual interaction [12–15].

## 3 Evaluation of the nuclear wave functions within QRPA.

As it is well known, in a first approximation, the constituents of the studied nucleus, i.e. the protons and the neutrons, can be considered as independent particles attracted by the nuclear center through a central strong nuclear force. This attraction can be described by a mean field, as for example the harmonic oscillator and the Woods-Saxon potential [21,22]. In the present paper, for the description of the nuclear field, we use a Woods-Saxon potential (with Coulomb corrections and a spin orbit part) of the form

$$U(r) = \frac{V_0}{1+exp(r-R_0)/a_0} + 2(\frac{\hbar}{m_\pi c})^2 \frac{1}{r}\frac{d}{dr}\frac{V_{S0}}{1+exp(r-R_{S0})/a_{S0}} \ell \cdot \mathbf{s} + \begin{cases} \frac{Ze^2}{2R_C}\left(3-\frac{r^2}{R_C^2}\right) & \text{for } r < R_C \\ \frac{Ze^2}{r} & \text{for } r \geq R_C \end{cases} \tag{6}$$

For the latter potential we adopted two parameterizations: (i) that of Bohr and Motelson [16], (ii) that of the IOWA group [17,18](the results shown in Tables 1 and 2 have been obtained with the use of Bohr and Motelson parametrization).

For a construction of a reliable nuclear Hamiltonian, the two-nucleon correlations (also known as the residual two-body interaction) are necessary to be included. To this aim, we assumed the Bonn C-D one-boson exchange potential. The initially evaluated bare two-body (nucleon-nucleon) matrix-elements of the latter potential refer to an isotope of mass number A. For the specific isotope $(A, Z)$ studied, a renormalization of the two-body interaction is required and this is achieved with the use of four multiplicative parameters. The first two known as pairing parameters $g_{pair}^{p,n}$, for protons (p) and neutron (n) renormalize the monopole (pairing) interaction. The third, $g_{pp}$, tunes the particle-particle channel and the fourth, $g_{ph}$, renormalizes the particle-hole interaction.

The ground state of the nucleus in question, is obtained in the context of the BCS theory by solving the relevant BCS equations which give us the quasi-particle energies and the amplitudes V and U that determine the probability for each single particle level to be occupied or unoccupied, respectively [12,13]. The excited states of the studied isotope are constructed by solving the QRPA equations. Their solution is an eigenvalue problem, which provides the X and Y amplitudes for forward and backward scattering, respectively, as well as the QRPA excitation energies [13]. In the Donnelly-Walecka method the diagonalization of the QRPA equations is carried out separately for each multipole set of states $|J^\pi\rangle$ which means that the $g_{pp}$ and $g_{ph}$ parameters are adjusted for each multipole individually.

## 4 Calculations of the Gamow-Teller and Fermi contributions to cross sections

In this paper, we first calculate the pronounced $\lambda = 1^+$ and $\lambda = 1^-$ transitions, which in the long wavelength approximation ($q \to 0$) are mediated by the Fermi and GT operator $\tau_\pm$ and $\sigma \cdot \tau_\pm$, respectively. Then, we consider the momentum-transfer dependence of the corresponding complete operators within the QRPA [12–14]. It is worth mentioning that the common RPA is quite appropriate for the description of the Fermi, the GT and the forbidden contributions to the cross sections of all semi-leptonic reactions. However, this method does not include all nuclear correlations needed to correctly reproduce the quenching of the GT distribution. They are better described within modern approaches like the QRPA, the large-scale shell-model, etc. Thus, the correction due to the momentum-transfer dependence needs a considerable computational effort to be estimated by performing complete QRPA calculations as done for other processes [12–14] using the complete $\lambda = 1^\pm$ multipole operators [6,9,23].

For astrophysical purposes, the rate of the charge changing weak processes mentioned in the Introduction ($e^\pm$-capture, $\beta$-decay etc.) is obtained by

$$w^\alpha = \sum_i P_i \sum_j w_{ij}^\alpha = \frac{ln2}{K} \sum_i \frac{(2J_i+1)e^{-E_i/(kT)}}{G(A,Z,T)} \sum_j B_{ij} \Phi_{ij}^\alpha \tag{7}$$

where the sums over i and j run over states belonging to the parent and daughter nucleus, respectively, and the superscript $\alpha$ stands for $e^\pm$-capture, $\beta^\pm$-decay etc. [9]. $P_i$ represents the probability of occupation of the parent excited states and satisfy the normal Boltzmann distribution. The phase space integral is represented by $\Phi_{ij}^\alpha$ [9] while the quantities $B_{ij}$ contain the sum of the reduced transition probabilities of the F and GT transitions, B(F) and B(GT), respectively, given by

$$B_{ij} = B(F)_{ij} + (g_A/g_V)^2_{eff}\; B(GT)_{ij} \tag{8}$$

with

$$B(F)_{ij} = \frac{1}{2J_i+1} \left| \langle j \middle|\middle| \sum_k t_\pm^k \middle|\middle| i \rangle \right|^2, \quad B(GT)_{ij} = \frac{1}{2J_i+1} \left| \langle j \middle|\middle| \sum_k t_\pm^k \sigma^k \middle|\middle| i \rangle \right|^2 \tag{9}$$

In Eq. (7) the constant K can be determined from the superalloweded Fermi transitions (usually the value $K = 6146 \pm 6s$ is used) and $G(Z,A,T) = \sum_i exp(-E_i/(kT))$ denotes the partition function of the parent nucleus [9].

At this point, we mention that in our calculations we adopt the Brink's hypothesis [5,6,9] which states that: the GT strength distribution originated from an excited states of the parent nucleus $[B(GT)_{ij}$, with $|i\rangle \neq g.s.]$, is identical to that from the ground state, shifted only by the excitation energy of the state $|i\rangle$.

It should be noted that, for comparison with the experimental data, we have to multiply the calculated GT strengths by an additional quenching factor of typical value 0.6 [5,6,9] so that, instead of $(g_A/g_V)^2_{bare} = (-1.254)^2$, in Eq. (8) we employ

$$(g_A/g_V)_{eff} = 0.74(g_A/g_V)_{bare} \tag{10}$$

For comparison with laboratory studies, the summation over (i) in Eq.(7) is not needed (the nuclear target is assumed to be in the ground state). It is worth mentioning that, for the reliability of the results when using the contributions of the Fermi and Gamow-Teller operators ($\tau_{\pm}$, $\sigma \cdot \tau_{\pm}$) one is usually checking the reproducibility of the Ikeda sum rules [20] for the parent nucleus (with Z protons and N neutrons) which are written as:

$$\sum B(F_-) - \sum B(F_+) = (N - Z) \tag{11}$$
$$\sum B(GT_-) - \sum B(GT_+) = 3(N - Z) \tag{12}$$

where $B(F_{\pm})$ and $B(GT_{\pm})$ represent the Fermi and Gamow-Teller reduced transition probabilities for $\beta^{\pm}$-decays, respectively.

## 5 Results and Discussion

In the first step of our calculations, the adjustment of the parameters $g^{p,n}_{pair}$ was performed. In Table 1 the values of the BCS parameters are listed through which we construct the wave function for the initial (ground) state of $^{56}Fe$. The parameters $g_{pp}, g_{ph}$ determined for the reproducibility of the final states of the daughter nuclei ($^{56}Mn$ for the $\beta^+$-decay and $^{56}Co$ for the $\beta^-$-decay) are shown in Table 2.

| | $b$ $(h.o.)$ | $g^n_{pair}$ | $g^p_{pair}$ | $\Delta^{exp}_n$ $(MeV)$ | $\Delta^{theor}_n$ $(MeV)$ | $\Delta^{exp}_p$ $(MeV)$ | $\Delta^{theor}_p$ $(MeV)$ |
|---|---|---|---|---|---|---|---|
| $^{56}Fe$ | 1,996 | 0,8885 | 0,8725 | 1,36268 | 1,36240 | 1,56828 | 1,56820 |

Table 1
Parameters re-normalizing the interaction of proton (neutron) pairs, $g^p_{pair}$ ($g^n_{pair}$), adjusted in such way that the corresponding experimental gaps $\Delta^{exp}_p$ and $\Delta^{exp}_n$, to be reproduced.

| | $^{56}Mn$ | | | $^{56}Co$ | |
|---|---|---|---|---|---|
| State | $g_{ph}$ | $g_{pp}$ | State | $g_{ph}$ | $g_{pp}$ |
| $1^+$ | 1.838 | 0.691 | $0^+$ | 1.595 | 1.162 |
| $2^+$ | 0.614 | 0.961 | $1^+$ | 1.767 | 0.679 |
| $3^+$ | 0.030 | 1.085 | $2^+$ | 0.733 | 0.956 |
| $4^+$ | 0.200 | 1.450 | $3^+$ | 0.062 | 1.085 |
| $5^+$ | 0.102 | 1.595 | $4^+$ | 0.001 | 1.450 |
| $2^-$ | 0.001 | 0.761 | $5^+$ | 0.075 | 1.585 |
| $3^-$ | 0.001 | 1.089 | $7^+$ | 1.717 | 1.616 |

Table 2
Strength parameters for the particle-particle ($g_{pp}$) and particle-hole ($g_{ph}$) interaction for low-spin multipolarities $J^{\pi} \leq 8^{\pm}$.

In the next step cross-sections results for studied processes are obtained. As is known, they are proportional to the square of the relevant nuclear matrix elements which, in this work, are calculated in the context of the QRPA [12,13]. Up to now, we have received preliminary results for the reduced transition probabilities $B(GT_-)$ and $B(GT_+)$. For the studied isotopes the comparison with experiments is rather good in the case of $B(GT_-)$, but for the $B(GT_+)$ values more detailed study is required in order to improve our method.

The reliability of such calculations, could also come out of the comparison of our $e^-$-capture cross sections with those evaluated in other similar processes, e.g. the corresponding charged-current neutrino nucleus reactions, which is the particle conjugate process of the $e^-$-capture.

As discussed in the Introduction, during the presupernova and collapse phase, $e^-$-captures on nuclei (and in the late stage also on free protons) play an important role, as the nuclear $\beta$-decay during silicon burning does. Electron-captures become increasingly possible as the density in the star's center is increased. It is accompanied by an increase of the chemical potential (Fermi energy) of the degenerate electron gas and it reduces the electron-to-baryon ratio $Y_e$ of the matter composition.

## 6 Summary and Conclusions

In this work, we use an advantageous numerical approach, constructed by our group recently, to calculate all basic multipole transition matrix elements needed for obtaining e-capture cross sections and $\beta$-decay rates. The required nuclear wave functions are obtained within the context of the QRPA using realistic two-body forces (Bonn C-D potential). Results for the cross sections are expected to be received soon which may be used in supernova simulations.

## 7 Acknowledgments

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