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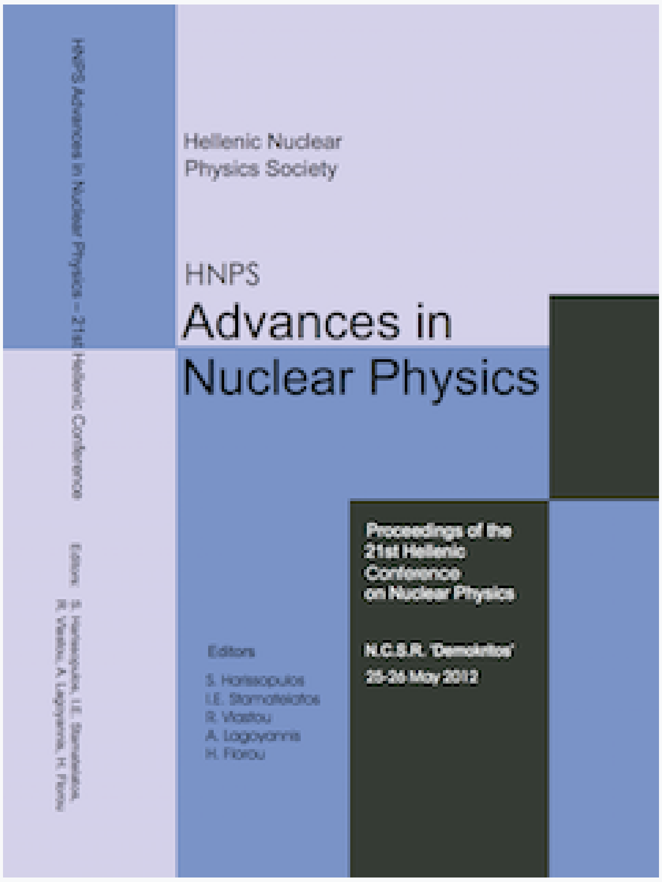

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Nuclear responses to astrophysical neutrinos through the neutral Gamow-Teller strength

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Abstract

We study nuclear responses to low-energy neutrinos through the neutral Gamow-Teller strength B(GT0). As a concrete example we adopt the ^{56}Fe isotope which plays important role in core collapse supernova. By using the quasi-particle RPA method we found that this strength is mainly concentrated on the resonance at around $\sim$ 10 MeV. Within the QRPA, we may also examine the changes in B(GT0) induced by non-zero values of the momentum transfer in astrophysical neutrino scattering on nuclei.

1 Introduction

The low-energy neutrinos created in the stellar interior may interact with nuclei mainly via neutral current reactions described by

$$\nu_\ell(\tilde{\nu}_\ell) + (A, Z) \longrightarrow \nu'_\ell(\tilde{\nu'}_\ell) + (A, Z)^*\,, \tag{1}$$

where $\ell = e, \mu, \tau$. While ν_μ, $\tilde{\nu}_\mu$, ν_τ, $\tilde{\nu}_\tau$ could not participate in charged current reactions (they do not have sufficient energies to produce the heavy leptons $\mu^\pm$ and $\tau^\pm$) [1–3], the ν_e neutrinos (and $\tilde{\nu}_e$ anti-neutrinos) may, in addition, interact through charged–current reactions as

$$\nu_e(\tilde{\nu}_e) + (A, Z) \longrightarrow e^-(e^+) + (A, Z \pm 1)^*\,. \tag{2}$$

Obviously, the (+) sign on the *r.h.s.* corresponds to neutrino reactions while the (-) sign to anti-neutrino ones. The absence of charged–current reactions for ν_x and $\tilde{\nu}_x$ ($x = \mu, \tau$) neutrinos in collapsing stars, justifies their emission with higher average energies than those of ν_e and $\tilde{\nu}_e$ [4–7].

In the investigations of the structure and evolution of distant stars, the neutrino-driven explosion mechanisms in massive stars, etc., the emitted (supernova) neutrinos play a key role. Also, for searching the fundamental electroweak interactions in the interior of stars and the nuclear weak responses, the astrophysical neutrinos are important messenger particles. Towards this aim, the behavior of the cross sections of the reactions (1) and (2), as functions of the initial and final lepton energies for a range of supernova relevant mean neutrino energies, has been extensively explored previously (see Refs. [7–9] and references therein). However, a plethora of open questions related to these applications still remain unanswered.

In the present work, we focus on the study of the neutral-current neutrino-nucleus reactions of Eq. (1) for a number of selected nuclei that are relevant: (i) for supernova simulations [7–9], and (ii) for terrestrial experiments aiming at neutrino astrophysics as well as neutrino-nucleus scattering cross sections measurements. From a nuclear theory point of view, such studies allow us to estimate (or improve existed calculations on) nuclear responses to low energy neutrinos ($\varepsilon_\nu \leq 100$ MeV) in the light of the operation of neutrino detectors with very-low threshold and very high sensitivity.

Specifically we concentrate on the calculation of the strength distribution of the neutral Gamow-Teller operator which gives the cross sections of the pronounced 1^+ multipole transitions (in neutral-current ν-nucleus scattering) at zero momentum transfer ($q \to 0$), an approximation known as "long wavelength limit". As it is well known, the allowed transitions represent an approximation that neglects the variation of the lepton wave functions inside the nucleus and, hence, the various moments are independent of the positions of the nucleons.

2 Brief description of the formalism

In the long wavelength approximation, the only surviving multipoles are the $\hat{M}_{00}$ and $\hat{T}^{el5}_{1M} = \sqrt{2}\hat{L}^5_{1M}$ (see notation of Ref. [7]). In order to determine inelastic neutrino scattering on nuclei through Gamow-Teller strength distributions, experimental electromagnetic M1 data are required to yield the desired $GT0$ information [4]. To the extent that the isoscalar and orbital pieces present in the M1 operator can be neglected, reliable theoretical methods can also be employed (specifically at the low energy of supernova neutrinos) [4,5] to evaluate ν-nucleus cross sections.

In the rest of the paper we will focus on the neutral Gamow-Teller operator and its contribution to the cross sections of neutrino scattering off nuclei based on electromagnetic M1 data.

2.1 The electromagnetic Gamow-Teller operator

In the case of the electromagnetic interactions, one of the fundamental low-energy excitations of the target nucleus is the M1 response that can be well explored by means of inelastic electron scattering [6]. Such transitions are mediated by the operator (magnetic dipole operator)

$$\widehat{O}(M1) = \sqrt{\frac{3}{4\pi}} \sum_{k=1}^{A} \left[g_\ell(k)\ell_k + g_s(k)\mathbf{s}_k \right] \mu_N \,, \tag{3}$$

where ℓ_k ($\mathbf{s}_k$) is the orbital (spin) angular momentum operator of the k-th nucleon and the sum runs over all nucleons of the nucleus in question. The orbital g_ℓ and spin g_s gyromagnetic factors are equal to g_ℓ^p=1 and g_s^p=5.586 for protons, and g_ℓ^n=0 and g_s^n=-3.826 for neutrons, while μ_N stands for the nuclear magneton. Thus, $g_s^{p,n}$ represent the proton (p) or neutron (n) magnetic moments.

Using nucleon isospin quantum numbers ($t = 1/2$, with $m_t = 1/2$ for protons and $m_t = -1/2$ for neutrons) and the 3rd component of the isospin operator $\mathbf{t}_0 = \tau_0/2$, Eq. (3) can be rewritten in isoscalar and isovector parts as

$$\begin{aligned} \widehat{O}(M1) &= \sqrt{\frac{3}{4\pi}} \left[\sum_{k=1}^{Z} (g_\ell^p \ell_k + g_s^p \mathbf{s}_k) + \sum_{k=1}^{N} (g_\ell^n \ell_k + g_s^n \mathbf{s}_k) \right] \mu_N \\ &= \sqrt{\frac{3}{4\pi}} \left[\sum_{k=1}^{A} \left(g_\ell^{IS} \ell_k + \frac{g_s^{IS}}{2} \sigma_k \right) - \left(g_\ell^{IV} \ell_k + \frac{g_s^{IV}}{2} \sigma_k \right) \tau_0(k) \right] \mu_N \end{aligned} \tag{4}$$

where $g_\ell^{IS(IV)}$ and $g_s^{IS(IV)}$ are gyromagnetic factors for the isoscalar (isovector) orbital and spin terms, respectively, defined as

$$g_\ell^{IS} = \frac{g_\ell^p + g_\ell^n}{2}, \quad g_\ell^{IV} = \frac{g_\ell^p - g_\ell^n}{2}, \quad g_s^{IS} = \frac{g_s^p + g_s^n}{2}, \quad g_s^{IV} = \frac{g_s^p - g_s^n}{2} \,. \tag{5}$$

From Eqs. (4)-(5) we see that, the M1 transition strengths consist of the isovector and isoscalar parts, and each of them stems from the orbital and spin contributions. Using the free-nucleon values for g_ℓ and g_s, we find that the gyromagnetic factor for the isoscalar (IS) spin term, which is estimated to be $g_s^{IS} = 0.880$, is much smaller (in the magnitude) than that for the isovector (IV) spin term, $g_s^{IV} = -4.706$. Therefore, the isoscalar spin contribution for the M1 transition strength is about thirty times smaller than the isovector contribution which means that electro-magnetic probes are sensitive to the isovector spin part. To obtain the isoscalar spin part hadronic probes are useful [6]. In the case of proton scattering off nuclei at forward angles, for example, the spin part of the M1 transition strength is dominant.

The isovector and isoscalar spin-flip M1 strengths are defined by

$$B(\sigma\tau_0) \;=\; \frac{1}{2J_i+1}\frac{3}{16\pi}\,|\langle f|\sum_{k=1}^{A}\sigma_k\tau_0(k)|i\rangle|^2 \tag{6}$$

$$B(\sigma) \;=\; \frac{1}{2J_i+1}\frac{3}{16\pi}\,|\langle f|\sum_{k=1}^{A}\sigma_k|i\rangle|^2 \tag{7}$$

As can be seen from Eq. (4), there is a simple relation between the isovector spin-flip M1 strength and the GT strength. Within a factor $\sqrt{3/16\pi}(g_s^p - g_s^n)\,\mu_N = 2.2993\,\mu_N$, the spin part of the isovector M1 operator is equal to the neutral Gamow-Teller (usual shorthand notation $GT0$) operator which reads

$$\widehat{O}(GT0) = \sum_{k=1}^{A}\sigma_k t_0(k) = \sum_{k=1}^{A}\frac{1}{2}\sigma_k\tau_0(k)\,, \tag{8}$$

Inelastic neutrino-nucleus scattering at low energies, where finite momentum transfer corrections can be neglected, is dominated by allowed (Fermi and Gamow-Teller) transitions.

2.2 The neutral Gamow-Teller operator in ν-nucleus scattering

The matrix elements of the operator $\widehat{O}(M1)$ between the initial $|i\rangle$ and a final $|f\rangle$ nuclear state that contain the nuclear dependence of the Gamow-Teller operator (if the isovector part in Eq. (4) dominates due to a strong cancellation of the g factors in the isoscalar part) is defined by

$$B_{if}(GT0) = g_A^2\frac{1}{2J_i+1}|\langle f|\sum_{k=1}^{A}\frac{1}{2}\sigma_{\mathbf{k}}\tau_0(k)|i\rangle|^2\,. \tag{9}$$

The latter quantity is known as reduced transition probability between the nuclear states $|i\rangle$ and $|f\rangle$. One can also show easily that the following double equation holds

$$-ig_A\sqrt{\frac{1}{12\pi}}\langle f|\sum_{k=1}^{A}\frac{1}{2}\sigma_k\tau_0(k)|i\rangle = \langle f|L_1^5|i\rangle = -\sqrt{\frac{1}{2}}\langle f|T_1^{el5}|i\rangle\,, \tag{10}$$

(for definitions of L_1^5 and T_1^{el5} for a nucleus see e.g. Ref. [7]).

We will now see how can one use the experimental electromagnetic M1 data in order to extract the desired $B(GT0)$ information, required subsequently to determine inelastic neutrino scattering on nuclei [4] and specifically that of the low energy of supernova neutrinos. As we concluded before, since the leading term in the nuclear vector-current does not contribute to the cross sections of (A,Z)(ν,ν')(A,Z)* [7], to the extent that the isoscalar and orbital pieces present in the M1 operator can be neglected, one expects that the isovector component of the operator (4) dominates over the isoscalar piece. In addition, the major strength of the orbital spin M1 responses are energetically well separated in the nuclear system of interest [1]. Thus, the axial-current term, $J_{axial} = g_A \widehat{O}(GT0)$, where g_A is the ratio of the axial to vector weak coupling constants, is the leading term contributing to (ν,ν').

The neutral Gamow-Teller operator $\widehat{O}(GT0)$ connects the state $|J_iT_i\rangle$ with the states $|J_fT_f\rangle$ if the following relations hold: (i) $J_f - J_i = 0, \pm 1$ (but not $J_i = J_f = 0$) and (ii) $T_f - T_i = 0(\delta \mathrm{T} = 0)$. Thus, the δT = 0 transitions involve a change in the angular momentum, $J_f - J_i = 0, \pm 1$, but not a change in isospin. The $T_f - T_i = \pm 1(\delta \mathrm{T} = 1)$ transitions caused by the $\widehat{O}(GT\pm)$ operator (not discussed in this work) may have a lower (T_i - 1) or higher (T_i + 1) isospin in addition to a change in the angular momentum.

2.3 ν-nucleus cross sections through the neutral Gamow-Teller strength

The cross section for a transition from an initial nuclear state $|i\rangle$ to a final state $|f\rangle$ is given by

$$\sigma_{i,f}(\varepsilon_\nu) = \frac{2G_F^2 g_A^2}{\pi(2J_i+1)}(\varepsilon_\nu - \omega)^2 |\langle f| \sum_{k=1}^{A} \frac{1}{2}\sigma_k \tau_0(k)|i\rangle|^2, \tag{11}$$

where G_F and g_A are the Fermi and axial vector coupling constants, respectively, ε_ν is the energy of the scattered neutrino and ω is the difference between final (E_f) and initial E_i nuclear energies, $\omega = E_f - E_i$. Note that for ground state transitions $\omega = E_f - E_{gs} = E_x$.

After substituting in Eq. (11) the known value of G_F and g_A, for a $J_i^\pi = 0^+$ ground state nucleus we obtain

$$\sigma_{i\to f}(\varepsilon_\nu) = 4.2299 \times 10^{-45} (\varepsilon_\nu - \omega)^2 |g_A \langle \sigma\tau_0\rangle|^2 cm^2, , \tag{12}$$

where ε_ν and ω are in units of MeV. The quantity $\langle \sigma\tau_{\mathbf{0}}\rangle$ is a shorthand notation for the reduced nuclear matrix element $\langle f \parallel \sum_{k=1}^{A} \sigma_k \tau_0(k)/2 \parallel i\rangle$.

It should be pointed out that, when Eq. (11) is a valid approximation, the scattering of neutrinos and antineutrinos are indistinguishable. From now on, unless otherwise specified, we shall use the term neutrino to indicate both kinds of particles, neutrinos and antineutrinos. Equation (12) is the general expression for allowed nuclear transitions in low energy (ν, ν') and $(\tilde{\nu}, \tilde{\nu'})$ reactions, but in fact for those transitions where both $B(M1)$ and $|g_A\langle\sigma\tau_{\mathbf{0}}\rangle|^{\mathbf{2}}$ are known experimentally. We see that, by putting the 3-momentum of the outgoing electron equal to zero, $\mathbf{q} = 0$, the (ν, ν') cross section can be reduced to simpler expression giving the total cross-section. In the next section we present and discuss results for the reaction (1) obtained for the quantity of Eq. (9) in the case of the reaction $Fe(\nu, \nu')Fe^*$.

3 Results and discussion

In this work, the adopted incoming neutrino energy range is extended to energies up to 100-120 MeV, so as to consider both allowed (Fermi and Gamow-Teller) as well as forbidden multipole contributions to the ν-nucleus cross sections. Such contributions are calculated within the quasi-particle random phase approximation (QRPA) by using realistic two-body forces (the Bonn C-D potential that is a slightly modified version of the Bonn-C one-meson exchange potential) for the residual interaction of the nuclear Hamiltonian. The QRPA approach, with a rich model space comprising of 10-25 single-particle orbits (in the present paper we employed 14 orbits), allows an accurate representation of the low–lying strength distribution of B(GT0).

3.1 Distribution of the GT0 ground state transition strength

At first, in order to check our codes we verified the double equality of Eq. (10). Then, since the relevant nuclear structure information resides in the matrix elements B_{if}(GT0) of Eq. (9) that define the strength for the Gamow-Teller operator of Eq. (8) between an initial (ground state) and all final states, we performed detailed nuclear structure calculations for the GT0 ground state transition strength.

In Figs. 1-2 we present the GT0 strength distributions for the ^{56}Fe isotope. We also include quenched results corresponding to $(f_A/f_V)^2 = 0.6$ [10]. From these figures we observe that the peak of the distribution appears at ∼10 MeV. In general the peak for the δT $= 0$ $(T_i \rightarrow T_i)$ GT0 strength is at around 8-12 MeV. As found previously, the strongest δT $= 1$ $[T_i \rightarrow (T_i + 1)]$ transitions lie a bit higher, in the energy range of 10-15 MeV. As can be implied from Fig. 2, once neutrinos have sufficiently large energies to excite the GT0 centroid,

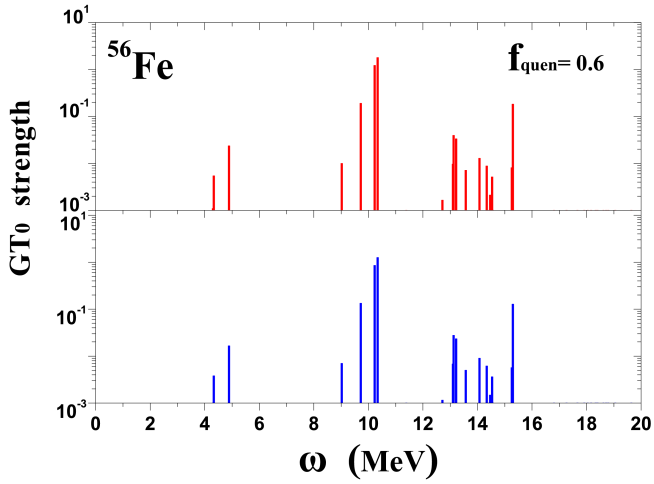

Fig. 1. State-by-State calculation of the Gamow-Teller strength distribution (up to about 20 MeV) for the neutral current reaction $^{56}Fe(\nu,\nu')56Fe^{*}$. In the upper panel the quenched values are shown.

the cross section is dominated by this transition. At high neutrino energies, other multipoles contribute to the cross section as well, where the excitation is again mainly due to the collective excitations [8–10].

In future studies, within the QRPA we can examine the changes in B(GT0) induced by a non-zero value of momentum transfer q. We note that, in previous studies in a series of iron isotopes with increasing $N-Z$ and especially for very neutron rich nuclei, it has been shown that fairly significant changes can occur in B(GT0) for non-zero momentum transfer.

Similar calculations were reported [4,5] and were already used in several other neutrino-nucleus reaction studies for both charged- and neutral-current calculations and in Ref. [4] total cross-sections and normalized neutrino spectra for neutral-current neutrino reactions on ^{56}Fe were presented. It is also worth mentioning that, previous studies exploring the effect of the strangeness in the nucleon (assuming the larger EMC value for the maximum effect) upon B(GT0) in a number of $T=0$ nuclei found a change about 36 % and a small shift for the centroid of the B(GT0).

4 Summary and Conclusions

In the present work we found that in the ^{56}Fe isotope the GT0 strength is mainly concentrated in the resonance at ~ 10 MeV. As it is known, in general, for even-even nuclei the GT0 strength is mainly concentrated in the resonance at around $\sim 8-12$ MeV. For several isotopes some low-lying strength develops once nucleons start to occupy higher orbitals.

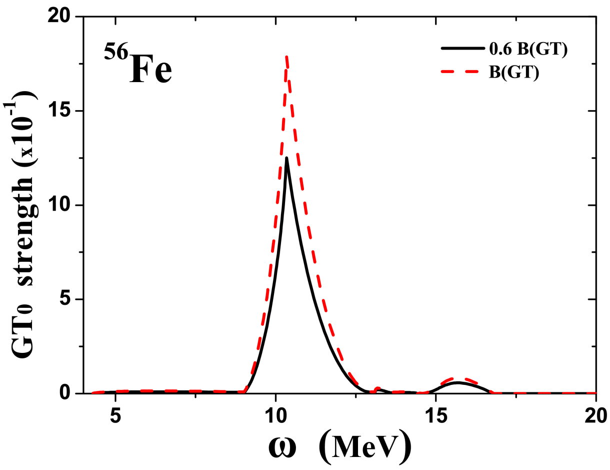

Fig. 2. Gamow-Teller strength for the neutral current neutrino nucleus reaction $Fe(\nu,\nu')Fe^{*}$. The centroid occurs at about 10.3 MeV. The solid line represent the quenched results corresponding to $(f_A/f_V)^2 = 0.6$.

We have also studied the effect of the pairing structure of the nuclear ground state (a BCS ground state) on the position of the centroid of the B(GT0) distribution in the ^{56}Fe isotope and found that it does not indicate a pronounced sensitivity. Moreover, the GT0 strength distribution in ^{56}Fe is not fragmented, result in good agreement with previous calculations based on large scale shell model.

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