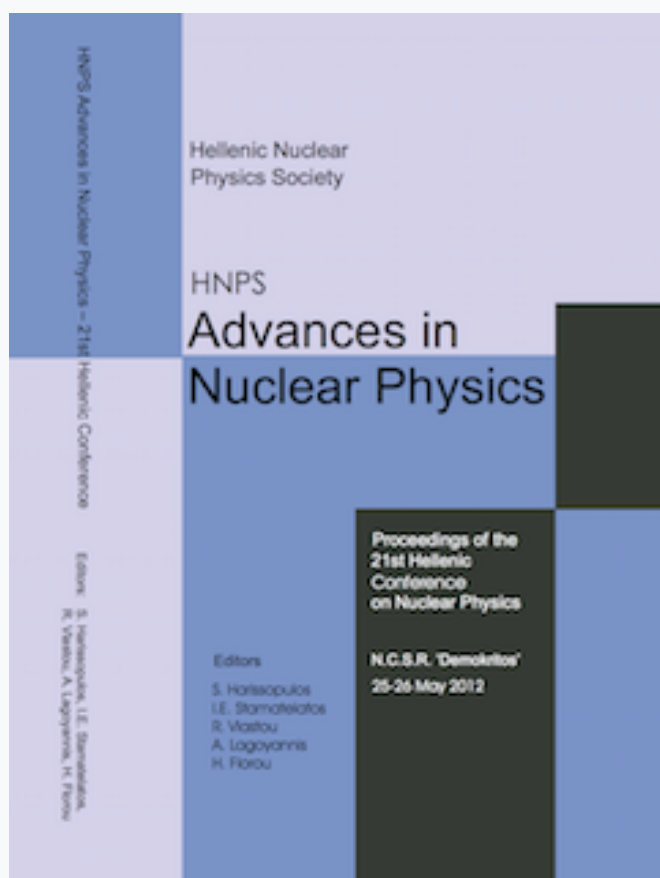**Image Reconstruction in the Positron Emission Tomography**

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Image Reconstruction in the Positron Emission Tomography

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Abstract

Positron Emission Tomography (PET) has become a valuable tool with a broad spectrum of clinical applications in nuclear imaging. PET scanners can collect *in vivo* information from positron radiotracer distributions, which is further reconstructed to a tomographic image with the help of well established analytical or iterative algorithms. In this current work, an innovative PET image reconstruction method from raw data based on a simple mathematical model is presented. The developed technique utilizes the accumulated density distribution in a predefined voxelized volume of interest. This distribution is calculated by intersecting and weighting the two-gamma annihilation line with the specified voxels. In order to test the efficiency of the new algorithm, GEANT4/GATE simulation studies were performed. In these studies, a cylindrical PET scanner was modeled and the photon interaction points are validated on an accurate physical basis. An appropriate cylindrical phantom with different positron radiotracers was used and the reconstructed results were compared to the original phantom.

Key words: PET, Image Reconstruction Algorithms, Monte-Carlo Simulation, GEANT4/GATE

1 Introduction

In the resent years Positron Emission Tomography (PET) has become a valuable tool and is finding a growing clinical acceptance for the detection of

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functional abnormalities as well as for the evaluation of various clinical conditions. PET scanners collect data from the *in vivo* β^+-radiotracer distribution injected to a patient. The algorithms used for image reconstruction in this modality can be divided in analytical and iterative ones. Under the analytical category falls the *Back Projection* (BP) and *Forward Back Projection (FBP)* techniques [1,2]. These algorithms use very complex mathematics in order to acquire a tomographic image, which in many cases contain misrepresentations and artifacts. The second category includes iterative algorithms such as the *Ordered Subset Expectation Maximization* (OSEM) and the *Maximum Likelihood Expectation Maximization* (MLEM) [3,4], with the latter being the most commonly used algorithm in commercial applications. It produces a better image but consumes lots of computational time.

A new PET image reconstruction algorithm from raw data, called *REC3D*, based on simple mathematics is presented in this study. REC3D is an algorithm that transforms the difficult mathematical problem of the PET image reconstruction in a simple geometrical one. The developed technique utilizes the accumulated density distribution in a predefined voxelized volume with appropriate dimensions which covers the field of interest.

2   The REC3D Reconstruction Algorithm

The position of two anti-diametrically and simultaneously detected annihilation photons can be acquired from raw data in all types of PET scanners. Let's assume that the space inside the FOV of the detector is a big volume divided in Nx, Ny, Nz cubic voxels along the X, Y and Z axis respectively. Now, the new reconstruction problem basically becomes an intersection of a plane and a line.
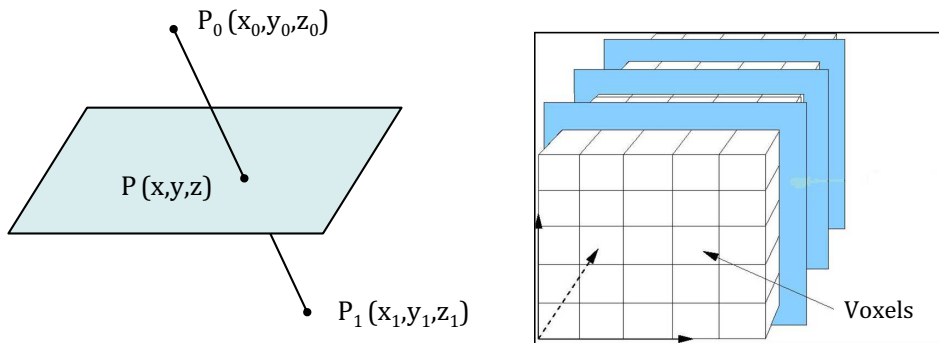

Fig. 1. *Left:* Intersection of the annihilation line defined by the two opposite 511-keV photons detected at $P_0(x_0, y_0, z_0)$ and $P_1(x_1, y_1, z_1)$ respectively with a plane. *Right:* Definition of the voxelized volume of interest.

The annihilation line is primarily defined by the two opposite 511-keV photons detected at the points $P_0(x_0, y_0, z_0)$ and $P_1(x_1, y_1, z_1)$ respectively (Figure 1). This line intersects a given plane at the point $P(x, y, z)$ which fulfills the analytic equations:

$$\frac{x - x_0}{x_1 - x_0} = \frac{y - y_0}{y_1 - y_0} = \frac{z - z_0}{z_1 - z_0} \tag{1}$$

For a given plane, parallel to one of the axes X, Y or Z, one of the fractions is arithmetically defined and therefore the full coordinates of the intersection point P are simply calculable. For example, for the plane $z = \mathcal{Z}$, parallel to the XY-plane and intersecting the Z-axis at the value $\mathcal{Z}$, its intersection point with the line $P_1 P_2$ is given by:

$$P = \left[\frac{\mathcal{Z} - z_0}{z_1 - z_0}(x_1 - x_0) + x_0 \ , \ \frac{\mathcal{Z} - z_0}{z_1 - z_0}(y_1 - y_0) + y_0 \ , \ \mathcal{Z} \right] \tag{2}$$

Furthermore, for each voxel interesting the annihilation line at the two points $\mathcal{C}_1(u_1, v_1, w_1)$ and $\mathcal{C}_2(u_2, v_2, w_2)$, the Euclidean distance $\mathcal{D}is$ is calculated

$$\mathcal{D}is = |\mathcal{C}_1 - \mathcal{C}_2| = \sqrt{(u_1 - u_2)^2 + (v_1 - v_2)^2 + (w_1 - w_2)^2} \tag{3}$$

and it is used as an accumulative weight factor for the reconstruction of the 3D image.

The REC3D algorithm utilizes this technique to scan the whole volume, voxel by voxel, creating the final 3D image. Additionally, an acceleration method can be applied to this algorithm. Instead of scanning the whole volume, only the voxels along the Line of Interest (LOR) are scanned. This can be accomplished by using the first derivative to determine the direction of the annihilation line in the volume space and consequently to meet only the active voxels in a continuous way. Following this technique for each pair of the detected photons the computation time is significantly reduced.

3   Results and Discussion

In order to test the efficiency of the new method an experiment is simulated, using the GEANT4/GATE software package [5]. An appropriate five cylinder phantom is constructed and scanned with a cylindrical PET detector. The PET detector simulated in this study (Figure 2) is a Small-Animal PET scanner called Sherbrooke 16-ring detector [6].

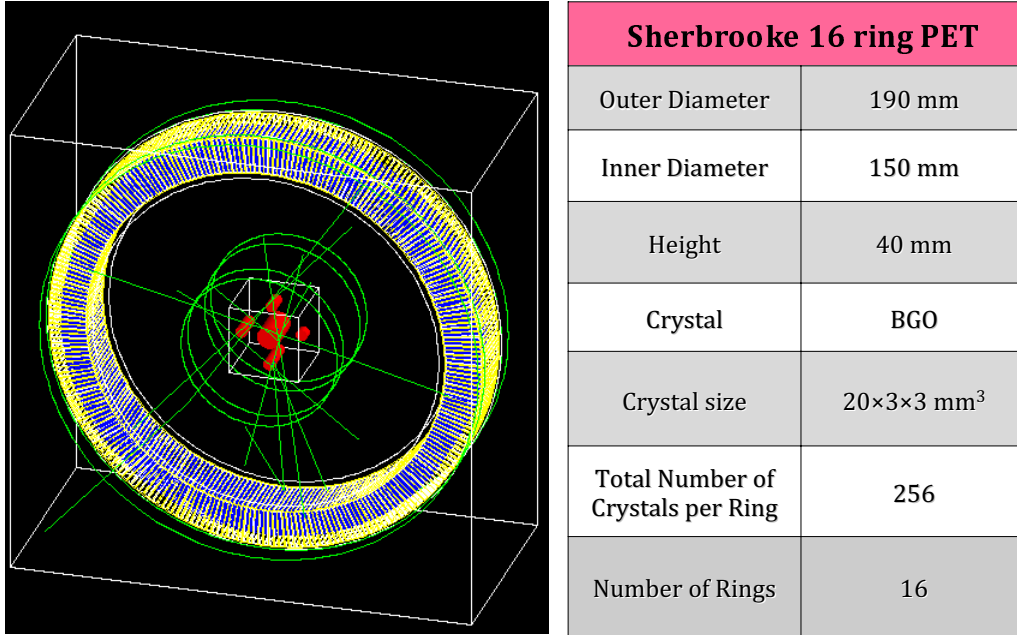| Sherbrooke 16 ring PET	
Outer Diameter	190 mm
Inner Diameter	150 mm
Height	40 mm
Crystal	BGO
Crystal size	20×3×3 mm^3
Total Number of Crystals per Ring	256
Number of Rings	16

Fig. 2. The GEANT4/GATE simulated Sherbrooke 16-ring Small-Animal PET Scanner with its technical characteristics.

The 16 ring version of the Sherbrooke Animal PET scanner does not really exist. This is a scanner that have only been modeled within the GATE environment in order to assess the performance of some image reconstruction algorithms for bigger scanner configuration than the originally existing 2-ring Sherbrooke PET scanner. It has however similar radial geometry as the 2-ring real version; it differs only in the available number of rings, a fact that improves its axial resolution [6].

The Sherbrooke Animal PET scanner is a 2-ring PET Camera composed of 256 detectors per ring. It is based on the EG&G C30994 detector module consisting of two BGO scintillators, each coupled individually to a silicon APD. The detectors are enclosed in an hermetic package of dimensions $3.8\ mm \times 13.2\ mm \times 30\ mm$ which determines the channel packing density ($4\ channels/cm^2$). The detector modules are physically and logically grouped into cassettes which also incorporate the front-end electronics. The dimensions and shape of the cassettes are such that they can be used in the construction of various diameter rings without modification. The port diameter is 135 mm, which is suitable for small laboratory animals, such as rats, rabbits or the brain of small monkeys. Since each layer of modules consists of two adjacent rings of detectors, three image planes (two direct, one cross) are defined within a 10.5 mm thick transaxial slice.

The phantom used in this study is an asymmetrical phantom that consists of five cylindrical sources of various heights and diameters. In order to be freed from the artifacts created by positron emitting sources, due to the positron

mean free path, two gamma back-to-back sources are here simulated. The cylinders are placed in a cross like formation inside the detector as shown in Figure 2. The phantom characteristics are shown in detail in the following Table 1.

Source	Radius	Height	(x,y) Placement	Element	Activity
0	10mm	8cm	(+0.0,+0.0) [cm]	^{18}F	8.0 kBq
1	5mm	2cm	(+2.5,+0.0) [cm]	^{18}F	0.5 kBq
2	5mm	4cm	(+0.0,+2.5) [cm]	^{18}F	1.0 kBq
3	5mm	6cm	(-2.5,+0.0) [cm]	^{18}F	1.5 kBq
4	5mm	8cm	(+0.0,-2.5) [cm]	^{18}F	2.0 kBq

Table 1
Geometry and activity of the 5-cylinder phantom used in the GEANT4/GATE simulation.

Reconstruction results based on the REC3D algorithm are shown in Figure 3. The method is accurate and efficient enough and produces good results in a

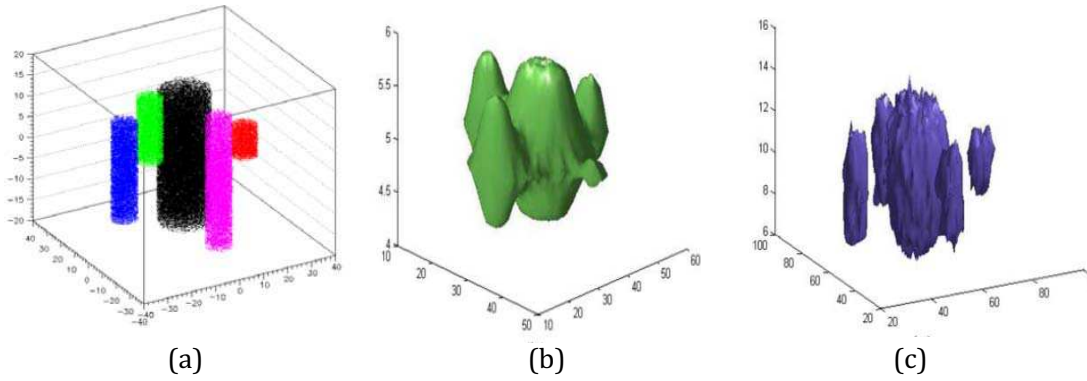(a)   (b)   (c)

Fig. 3. Reconstruction results with the REC3D algorithm developed in this study. (a) The original phantom (b) Reconstruction results without energy cuts (c) With energy cut-off (Compton photons removed).

relatively small time interval. The analysis has shown that the quality of the reconstruction procedure can be further improved by applying energy cuts to the detected photons. The rejection of possible Compton events or other scattered photons in false coincidence by inserting an energy cutoff around the annihilation photopeak improves significantly the spatial resolution of the obtained image, as shown in the above Figure 3c.

Finally, a direct comparison of the REC3D reconstructed image with results obtained by applying other established reconstruction techniques in PET is shown in Figure 4. Here, a central slice of the phantom is reconstructed. The Ordered Subset Expectation Maximization (OSEM) tomographic image re-
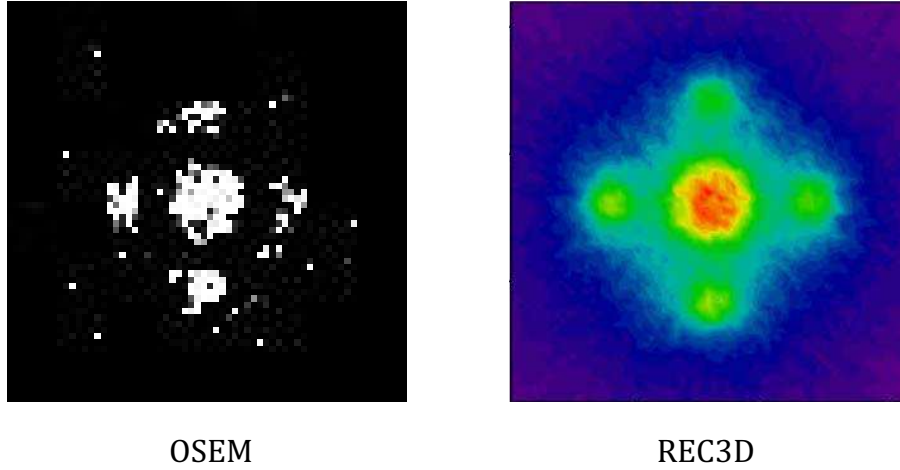
OSEM   REC3D

Fig. 4. Comparison of reconstruction images for a central phantom slice. Left is shown the reconstrcuted tomographic image with the Ordered Subset Expectation Maximization (OSEM) technique. Right is the reconstruction result of the same phantom slice performed with the REC3D algorithm.

flects the inability of this statistical method to correctly reproduce the density of the original image within a reasonable resolution.

4   Concluding Remarks

REC3D is an analytical PET reconstruction algorithm based on simple geometrical techniques. It produces efficiently accurate images without artifacts in small computational time. Due to the simplicity of the required input information - only the raw position of the opposite detected annihilation photons is acquired - the algorithm can be used in any scanner architecture, regardless the detector's geometrical characteristics. Contrary to other iterative methods, the REC3D technique is not limited by any data quality restrictions (data in the middle of the Poisson distribution) in order to obtain good results.

Furthermore, the number of voxels in the predefined range of interest and their physical dimensions are totaly free. As a consequence, the REC3D reconstruction algorithm can be adjusted so to meet the specific needs regarding the spatial resolution. The acceleration method of the algorithm can be used by scanning only the voxels that are in the LOR of the two input points, a fact that reduces the computation time significantly.

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