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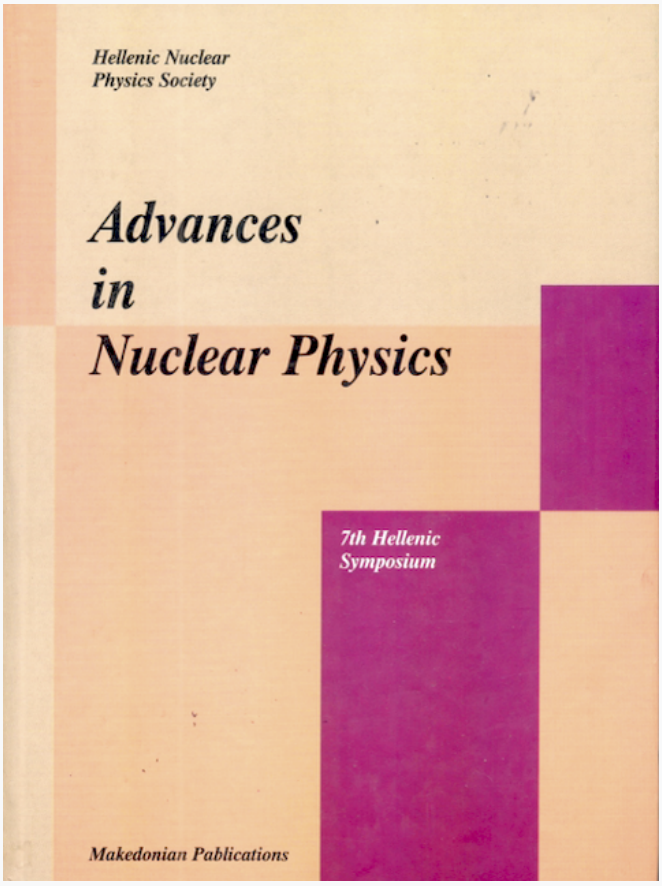

Effects of Short and Long Range Correlations on the Charge Densities and Radii of Ca Nuclei

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Effects of Short and Long Range Correlations on the Charge Densities and Radii of Ca Nuclei

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Abstract

The experimental data for the charge (proton) density differences of the even Ca nuclei is analyzed by means of a simple phenomenological model where the effects of certain type short and long range correlations have been accounted. Short range correlations are approximated through the Jastrow type correlation function while for long range correlations the fluctuations of the nuclear surface are considered. The analysis shows that the combined effects of these correlations lead to a better description of the experimental charge (proton) density differences. Moreover, the calculated charge mean square radii of the even Ca nuclei exhibit a parabolic behaviour and compare well with the experimental isotope shifts from the laser spectroscopy measurements.

1 Introduction

Calcium nuclei have been of great experimental as well as theoretical interest. It is the only magic element for which precision measurements on isotope shifts [1, 2] (see also Andl et al. [3]) have been carried out over a full neutron shell, namely the $1f_{7/2}$ shell between the two doubly magic isotopes ^{40}Ca and ^{48}Ca.The empirical data for the isotope shifts [1] show an anomalous A dependence. The addition of neutrons to the ^{40}Ca core leads to an increase of the charge radii up to ^{44}Ca. Then adding more neutrons the charge radii start to decrease. The very interesting feature is that the charge radii of the two doubly magic nuclei ^{40}Ca and ^{48}Ca have practically the same value. It is noted, however, that the electron scattering experiments have shown that the charge distributions of these magic nuclei are not identical [4]. Moreover, muon spectroscopy, electron and hadron scattering provide more information on the charge and mass distributions [5]. Experimental data for the form factors and

their isotopic change for some even stable isotopes is available. Therefore, the rich experimental input makes Ca nuclei attractive for theoretical study.

In a very recent publication [6] the role of short range correlations (SRC) in reproducing the empirical data for the charge (proton) density differences of even Ca nuclei was examined. In that approach the cluster expansion [7, 8] truncated at the two body term was employed and SRC of Jastrow type [9] were considered. The parameters of the model were adjusted to reproduce the experimental isotope shifts [1, 2] of Ca nuclei. It turned out that the calculated values for the differences of the density distributions exhibited the correct trend. It should be noted, however, that the comparison with the data was not very good in all cases. The maximum for the proton density difference of $^{48}Ca - {}^{40}Ca$ (see fig. 4 of ref. 6) was not reproduced well. This was an indication that additional correlations were necessary to improve the agreement with the experiment. On the other hand, recently, the effect of fluctuations of the nuclear surface was also included in the model [10]. It was shown that the combined effects of SRC and of the surface fluctuation correlations (SFC) improved the description of the experimental charge form factors of ^{16}O and ^{40}Ca nuclei.

Here the study of ref. 6 is extended by including the effect of SFC. Specifically, we investigate the effects of SFC on the charge (proton) density differences of Ca nuclei, while simultaneously approximating SRC through the Jastrow correlation factor.

In Secs. 2 and 3 the formalism for the SRC and SFC is briefly discussed, while in Sec. 4 the numerical results are reported and commented.

2 Correlated charge form factors, densities and m.s radii

Expression for the correlated charge form factors, $F_{ch}(q)$, of the closed s-p and s-d shell nuclei were derived [11-15] in the framework of the factor cluster expansion of Ristig, Ter Low and Clark [7, 8] using the Jastrow ansatz for the correlated wave-functions. This type of correlations is characterized by the correlation parameter λ_{nlS} which enters in the normalized correlated wave functions of the relative motion:

$$\psi_{nlS}(r) = N_{nlS}[1 - \exp(-\lambda_{nlS} r^2/b^2)]\phi_{nl}(r) \qquad (1)$$

where N_{nlS} are the normalization factors, $\phi_{nl}(r)$ are the harmonic oscillator (HO) wave functions and $b = \sqrt{2}b_1$ ($b_1 = \sqrt{\hbar/m\omega}$) is the harmonic oscillator parameter for the relative motion.

In this approach the expression for the point proton form factor, $F(q)$, takes the form:

$$F(q) = F_1(q) + F_2(q) \tag{2}$$

where $F_1(q)$ is the contribution of the one-body term to $F(q)$, which can be written easily in closed form [6]. The contribution of the two-body term, $F_2(q)$, to the form factor $F(q)$ can be expressed in a rather simple way in a closed form by means of the matrix elements:

$$A_{nlS}^{n'l'S'}(j_k) = < \psi_{nlS}|j_k(qr/2)|\psi_{n'l'S'} > \tag{3}$$

These are simple polynomials and exponential functions of q^2 [11-15]. The correlation parameter λ_{nlS} is taken state independent ($\lambda_{nlS} = \lambda$). It is noted that it has been shown in [15], that the effect of the state dependence of the short range correlations is small. Then the charge form factor, $F_{ch}(q)$, is written: $F_{ch}(q) = f_p(q) \times f_{CM}(q) \times F(q)$ with $f_p(q)$ and $f_{CM}(q)$ being the corrections due to the finite proton size [11] and the centre of mass motion [16] respectively.

The interesting feature of the method is the possibility of finding an analytic form for the correction to the uncorrelated charge (proton) density distribution by means of a Fourier transform of $F_2(q)$. Thus the correlated proton density distribution is written:

$$\rho_{SRC}(r) = \rho_1(r) + \rho_2(r) \tag{4}$$

3 Surface fluctuation correlations

The role of ground state (long range) correlations has been a matter of detailed investigation long ago [17-22]. Esbensen and Bertcsh [21] have shown that fluctuations of the nuclear surface due to zero-point motions coming from low-lying collective states affect the ground state charge density. Barranco and Broglia [22] have found that the ground state correlations associated with the surface modes of the Ca isotopes are important and qualitatively explain the observed behaviour of the mean square (m.s.) radii with the mass number.

In the present work we follow ref. 23, i.e. we consider the ground state correlations which are indroduced due to zero point motion of collective surface vibrations. According to ref. 23 the proton (or charge) density of a nucleus, deformed through the zero-point fluctuations, has the form (see also [17-19]

for a rather similar expression):

$$\rho_{cor}(r) = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{\infty} \rho_1(r-\xi) \exp\left[-\frac{(\xi - s_0)^2}{2\sigma^2}\right] d\xi \tag{5}$$

where $\rho_1(r)$ is the uncorrelated density, s_0 is a correction needed to conserve the number of particles in the correlated ground state and σ is a measure of the effect of the zero point fluctuations. The value of σ is related to β_λ, the deformation parameters for the states of multipolarity λ, with the relation:

$$\sigma^2 \simeq \frac{R_0^2}{4\pi} \sum_\lambda \beta_\lambda^2 (\tau = 0) \tag{6}$$

The β_λ parameters can be determined from the values of $B(E_\lambda)$ [21, 23]. In our approach σ is taken as a free parameter.

4 Numerical results and comments

The study of the combined effect of SFC and SRC is done by substituting in (5) the uncorrelated proton density distribution $\rho_1(r)$ with $\rho_{SRC}(r)$ (formula (4)) where the effect of SRC is accounted through Jastrow type correlations and then following the procedure of ref 6. We note here, that the use of HO orbitals for the uncorrelated proton density distribution, although it is a simplification, has certain advantages. The correction of the centre of mass motion can be done exactly. Most of the calculations are analytic and closed form expressions can be derived for various quantities such as the correlated form factor and density distribution as well as the moments of the density. In addition the computation time is reduced considerably.

There are three parameters for each nucleus i.e. the HO parameter b_1, the parameter λ which describes the effect of SRC and the parameter σ which gives a measure of SFC effect.

For the determination of the parameters of the model the available data for the charge (proton) density differences of the even Ca nuclei is used. It is clear that such an experimental input provides much more detailed information on nuclear structure than the isotope shifts measurements (used in ref. 6) which give information only about the changes of the ms radii. The parameters are determined by an overall (global) fit of the correlated charge(proton) density differences

$$\Delta\rho_{cor}(40+2n) = \rho_{cor}(40+2n) - \rho_{cor}(40) \tag{7}$$

to the experimental data. More specifically the expression

$$\Delta\rho_{cor}(40+2n)r^2 \qquad n=1,2,4 \tag{8}$$

is fitted to the experimental charge distribution differences of $^{42}Ca - {}^{40}Ca$ and $^{44}Ca - {}^{40}Ca$ (n=1,2) and the proton density difference of $^{48}Ca - {}^{40}Ca$ (n=4). The experimental values are taken from refs [24] and [4] respectively. In the fitting procedure the isotopic change of the charge radii of the doubly magic nuclei ^{40}Ca and ^{48}Ca [1] it is also taken into consideration.

Table 1
The values of the HO parameters b_1 (in fm), the SRC parameters μ (in fm) and the SFC parameters σ for the even Ca nuclei.

A	b_1	μ	σ
40	1.9664	0.5805	0.5284
42	2.0105	0.5788	0.4333
44	1.9845	0.6158	0.4748
46	1.9934	0.6143	0.3401
48	2.0023	0.6129	0.2054

In table 1 the parameters b_1, μ and σ for each even Ca isotope are shown. It is noted that instead of the SRC parameter λ, and in accord with ref. 6, we report in table 1 the parameter μ ($\mu = \sqrt{b_1^2/\lambda}$), which is the "actual correlation parameter". In the same table our predictions for the parameters of the ^{46}Ca nucleus are also given. These values are determined by an interpolation of the calculated values for ^{44}Ca and ^{48}Ca nuclei.

Using the values of Table 1 the charge (proton) form factors, density distributions as well as the differences of the density distrbutions $\Delta\rho(40+2n) = (\rho(40+2n) - \rho(40))$ can be easily calculated. In Figures 1-2 the quantity $\Delta\rho_{ch}(40+2n)r^2$ for the charge distribution differences of $^{42}Ca - {}^{40}Ca$ and $^{44}Ca - {}^{40}Ca$ (dashed lines) respectively are compared with the empirical data (solid lines). The same holds also for Figure 3 for the difference $^{48}Ca - {}^{40}Ca$. In this case the available experimental values correspond to the proton density distributions. The two solid lines correspond to the upper and lower values of the proton density difference. It is seen that all the theoretical curves have the correct trend and compare well with the experimental values. In Figure 4 the prediction of the model for the proton density difference of $^{46}Ca - {}^{40}Ca$ is shown.

In Figure 5 the calculated isotope shifts of the even Ca nuclei are compared with the experimental data from the high precision laser spectroscopy measurement. It is seen that the model reproduces very well the empirical data.

It is noted that in the calculation of the m.s. charge radii of Ca isotopes the proton and neutron m.s. charge radii were taken into account [25].

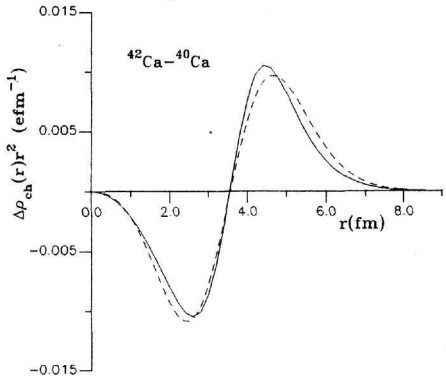

Fig.1 The difference of the charge distributions of $^{42}Ca - {}^{40}Ca$, multiplied by r^2, (dashed line) calculated in the present approach together with the experimental data (solid line) taken from ref. Frosch et al. [24].

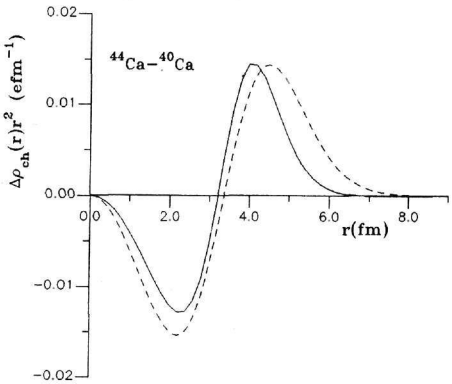

Fig.2 The same as in Fig.1, for the charge distribution difference of $^{44}Ca - {}^{42}Ca$.

In summary, our simple phenomenological model, where the combined effects of SRC and SFC are accounted, is able to provide a satisfactory description of the empirical data of the density distribution differences of even Ca nuclei. It is also able to reproduce well the trend of the variation of the charge radii of Ca nuclei with the mass number. Moreover the model improves the description of the experimental charge form factors [10].

Concluding, we would like also to point out that our analysis shows that by considering only SRC the calculated charge (proton) density differences, though they have the correct trend, are not very good. The charge form factors reproduce well all the diffraction minima. The mean square charge radii exhibit a parabolic behavior. However, the maximum is in the wrong place (^{42}Ca).

On the other hand, accounting only SFC the charge (proton) density differ-

ences are not reproduced well. Especially for the difference $^{44}Ca - {}^{40}Ca$ the comparison with the experiment is very bad. The calculated isotope shifts have the correct trend, but the value for ^{44}Ca is unnaturally large. Finally the third diffraction minimum in the charge form factors is not reproduced at all.

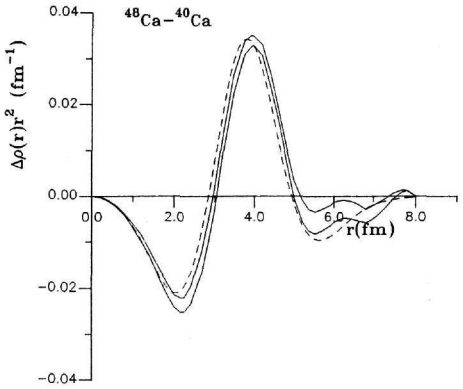

Fig.3 The difference of the point proton distributions of $^{48}Ca - {}^{40}Ca$ multiplied by r^2 (dashed line) calculated in the present approach together with the empirical data taken from Emrich et al. [4]. The two solid lines correspond to the upper and lower values of the experimental proton density difference.

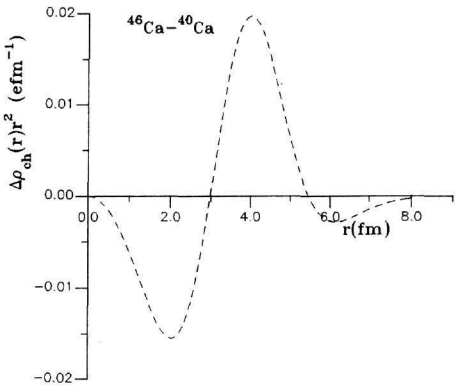

Fig.4 The difference of the charge distributions of $^{46}Ca - {}^{40}Ca$, multiplied by r^2, (dashed line) calculated in the present approach.

It is therefore the combination of SRC and SFC that gives the correct description of the density differences as well as of the charge form factors [10], while simultaneously the parabolic behavior of the charge radii is reproduced well. One could say that the putative roles of the mean-field, short range correlations and surface fluctuation effects get mixed up to some degree. The interplay between SRC and SFC improves the correction to the independent particle model leading thus to a better description of the experimental data of Ca nuclei.

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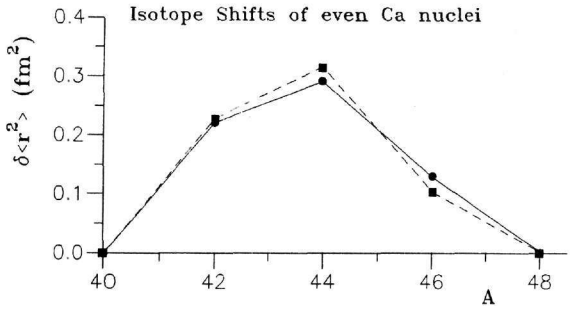

Fig.5 The isotope shifts of even Ca nuclei, calculated in the present work (dashed line) together with the experimental values (solid line) obtained from the laser spectroscopy measurements (see Otten [1]).

References

[1] E.W. Otten, in *Treatise on Heavy-Ion Science*, edited by D.A. Bromley (Plenum, New York, 1989) Vol 7, p. 515

[2] J. Billowes and P. Campbell, J. Phys. G: Nucl. Part. Phys. **21** 707 (1995).

[3] A. Andl, K. Bekk, S. Göring, A. Hanser, G. Nowicki, H. Rebel, G. Schatz, and R.C. Thompson, Phys. Rev. C **26** (1982) 2194.

[4] H.J. Emrich, G. Fricke, G. Mallot, H. Miska, and H.G. Sieberling, Nucl. Phys. **A396**, 401c (1983).

[5] H. Rebel, H.J. Gils, and G. Schatz (eds), Proc. Int. Discussion-Meeting on *What do we know about the radial shape of nuclei in the Ca region?* KFK-Rep. 2830, Karlsruhe (1979).

[6] G.A. Lalazissis, S.E. Massen, Phys. Rev. C **53**, 1599 (1996).

[7] M.L. Ristig, W.J. Ter Low and J.W. Clark, Phys. Rev. C **3**, 1504 (1971).

[8] J.W. Clark, Prog. Part. Nucl. Phys. **2** (1979) 89.

[9] R. Jastrow, Phys. Rev. **98**, 1479 (1959).

[10] S.E. Massen, V.P. Garistov, M.E. Grypeos, Nucl. Phys. **A597**, 19 (1996).

[11] S.E. Massen, H.P. Nassena and C.P. Panos, J. Phys. G **14**, 753 (1988).

[12] S.E. Massen, and C.P. Panos, J. Phys. G **15**, 311 (1989).

[13] S.E. Massen, J. Phys. G **16**, 1713 (1990).

[14] S.E. Massen, in "First Hellenic Symposium on Theoretical Nuclear Physics", Thessaloniki, Greece, (1990) p.62.

[15] G.A. Lalazissis, S.E. Massen, and C.P. Panos, Z. Phys. A **348**, 257 (1994).

[16] L. J. Tassie and F. C. Barker, Phys. Rev **111**, 940 (1958).

[17] V.P.Garistov and I.J.Petkov, Bulg.J.Phys. **3**, 6 (1976).

[18] A.N. Antonov,V.P. Garistov and I.J. Petkov Phys. Let. **68B**, 305 (1977).

[19] V.P.Garistov, Int. Journ. of Modern Physics E **4**, 371 (1995).

[20] P.G. Reinhard and D. Drechsel, Z. Physik **A290**, 85 (1979).

[21] H. Esbensen, and G.F.Bertsch Phys. Rev. **C 28**, 355 (1983).

[22] F. Barranco and R.A. Broglia, Phys. Let. **151B**, 90 (1985).

[23] S. Shlomo Phys. Lett. **209B**, 23 (1988).

[24] R.F. Frosch, R. Hofstadter, J.S. McCarthy, G.K. Nöldeke, K.J. Van Oostrum, M.R. Yearian, B.C. Clark, R. Herman, and D.G. Ravenhall, Phys. Rev. **174**, 1380 (1968).

[25] H. Chandra, and G. Sauer, Phys. Rev. C **13**, 245 (1976).