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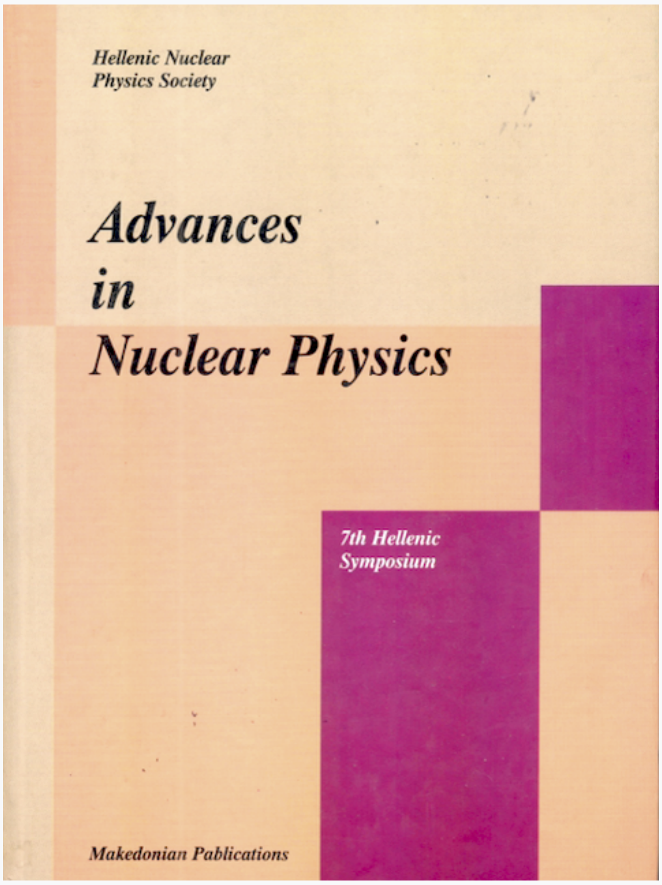

New Directions in Nuclear Theory

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1. Introduction

It is not possible in a brief survey of new developments in nuclear theory to do more than to select a few topics that seem to be important, and inevitably they are linked to my own interests.

First of all, however, I want to make a general comment on the way we think about what we are doing. How do we think about the structure of a nucleus and about the mechanisms of a nuclear reaction? Do we write down a Schrodinger many-body wave equation and try to solve it subject to specified boundary conditions? Certainly that is not what I do. On the contrary, I see the nucleus as a cloud of little balls of different colours. They are in constant motion, colliding with each other, sometimes forming temporary clusters that soon dissolve again. Occasionally a collision gives one ball enough energy to escape from the cloud, and sometimes a ball comes in from outside and adds to the confusion. It may just be captured, or it may knock-out another ball or a cluster of balls.

Having visualised the reaction that interests me, I know that I must plunge into quantum-mechanical formalism, work out the predictions of my model and compare them with experiment. That is the only story that gets submitted to the Physical Review. But why are we so bashful, even ashamed, of our simple classical models? The reason, I believe, is that we have been brainwashed by the Copenhagen interpretation of quantum mechanics to believe that reality is fundamentally fuzzy and that the wavefunction contains all that ever can be known about each individual system. This is a profoundly debilitating idea and is actually harming the progress of nuclear physics. This may be illustrated by a story from the early days of nuclear physics.

In 1912, Rutherford discovered the structure of the atom, showing that it is composed of a central nucleus surrounded by electrons. It was very natural for him to probe deeper and to try to find the structure of the nucleus. Throughout the 1920s and 1930s he devised experiments to reveal this structure. He discussed the problem with Bohr, who told him that he was wasting his time, because according to his ideas about quantum mechanics the nucleus is just a structureless soup that occasionally emits particles; it is therefore meaningless to ask about nuclear structure. Discouraged by this, Rutherford abandoned his search (Wilson 1983). With hindsight the apparatus available to him was not sufficiently sensitive for his purpose, but the story is a striking illustration of the debilitating effects of the Copenhagen interpretation of quantum mechanics; it prevents us from asking perfectly reasonable questions and thus discourages innovative research.

I believe that it is false to say that reality is fundamentally fuzzy. Nucleons have precise positions and momenta (Ballentine 1970), and they interact in precisely-defined ways. At present, however, we cannot look into the nucleus and see directly what is going on. All we can do is to make measurements on a large number of nuclei, so they are inevitably statistical.

All our measurements of nuclear structure are made by recording individual events, such as the emission of a γ-ray or the inelastic scattering of a proton. Our theories of nuclear structure and reactions, however, give only the probability distributions, such as the half-life or the differential cross-section corresponding individual events. Quantum mechanics, by its very nature, enables us to calculate only the average behaviour of a large number or ensemble of similar systems. All our sophisticated nuclear theories are quantum mechanical, and so we can analyse only the average properties of nuclei. As Einstein believed, quantum mechanics is not the final theory, and it may well be that a more detailed deterministic theory will ultimately be found. So, while at present we have to calculate quantum-mechanically, we can still think about the behaviour of individual nucleons in the nucleus and their relations with each other. This is done, for example, by Anagnostatos in the work described in section 2. This may well stimulate new lines of thought and new experiments that could lead the way to a detailed microscopic theory of nuclear structure (Brody 1993).

It is helpful to think of nuclei in a simple way as a number of nucleons interacting with each other and moving around in a small approximately spherical region. Their time-averaged positions can be expressed as proton and neutron density distributions. Since these nucleons have a finite size and repel each other at small distances, their density distributions are approximately uniform inside the nucleus and, since they attract each other strongly at large distances, the density falls rapidly on the outside, giving a well defined surface. An exception to this is provided by some nuclei that are nearly unstable to neutron emission which have a neutron halo extending to large distances. The neutron-proton interaction is stronger than that between like nucleons, and this tends to lock the neutron and proton distributions together, making the spatial extent of the neutron and proton distributions very similar. Each nucleon moves on a well-defined orbit in the mean field generated by the other nucleons and has a definite energy and the quantum numbers associated with the orbit. The nucleons in each orbit have their own density distribution. Occasionally two nucleons collide and interact strongly, changing their trajectories. Sometimes two protons and two neutrons combine to form an α-particle, and this also moves in a quantum orbit until it is broken up by a further collision.

2. Nuclear Structure

The structure of the nucleus is dynamic, and not static like a crystal. It may, however, be the case that the average positions of the nucleons, the spatial density distributions, have regions of higher probability that show a regular structure.

This aspect of nuclear structure has been studied semi-classically on the nucleon level by Anagnostatos (1985) who has developed the isomorphic shell model that combines shell- and cluster-model concepts. In this model the nucleons in each shell are in dynamic equilibrium and their average positions correspond to the Leech (1957) polyhedra (figure 1) for the spatial distribution of particles on a spherical surface. The particles occupy the vertices of the polyhedra and, in each shell, neutrons and protons occupy reciprocal polyhedra. The neutrons are assigned to the stable equilibrium polyhedra and the protons to the unstable equilibrium polyhedra and together these polyhedra are stable.

The neutrons and protons are treated as hard spheres of radii $R_n = 0.974$ fm, and $r_p = 0.860$ fm; these are the only parameters of the model and are very similar to the radii of nucleon bags in the quark model. The dimensions of the shells are then determined by the close packing of the shells.

The assignment of nucleons to the vertices of polyhedra follows the principles that identical particles on a shell are interchangeable and that for each particle position there is a symmetric counterpart. The total energy of the system is minimized, and this sometimes requires that particular symmetrically distributed vertices are left unoccupied.

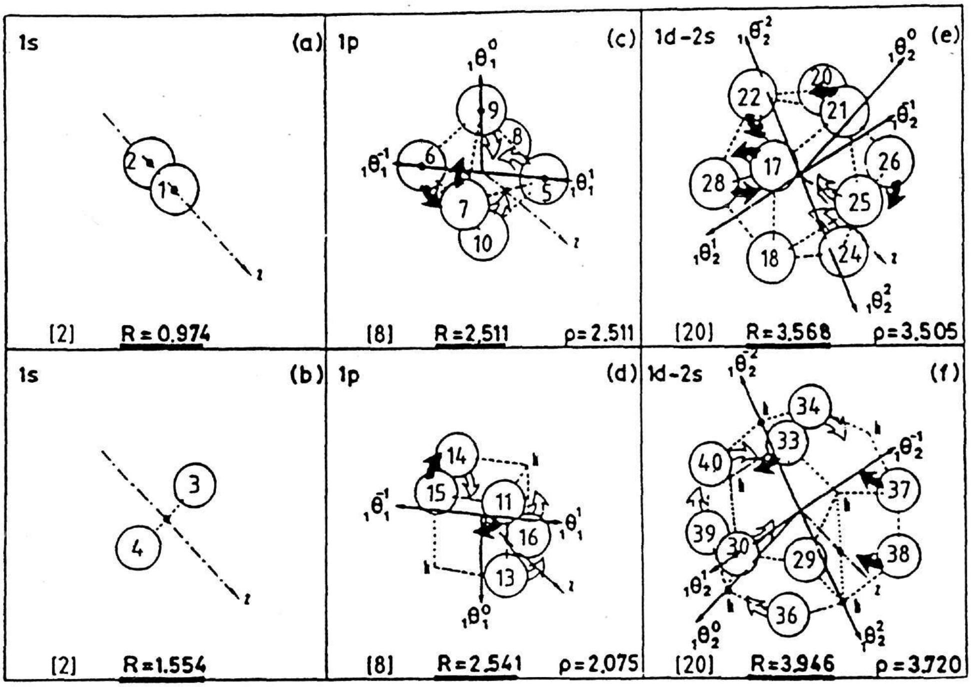

Figure 1. The isomorphic shell model (Anagnostatos, 1993).

The cumulative numbers of neutrons or protons give the magic numbers, and it is notable that these are obtained without introducing the concept of spin-orbit coupling. This suggests that the magic numbers are the result of very general symmetry considerations that can be represented in the shell model by introducing spin-orbit coupling (Anagnostatos 1985). The same magic numbers are predicted for clusters of two kinds of alkali atom (Anagnostatos 1988). This model raises many questions that deserve serious study, both experimentally and theoretically.

Many features of nuclear structure can be very simply understood by assuming that all the nucleons move independently in a mean field. We thus replace the action of all the other nucleons on a particular nucleon by a one-body potential. This can be calculated with difficulty and some approximations from the nucleon-nucleon interaction, but it is much simpler to assume that it is given by the phenomenological potential

$$V(r) = V_c(r) + Uf(r) + U_s g(r)\mathbf{L}.\sigma, \tag{1}$$

where $V_c(r)$ is the electrostatic potential, and U and U_s are the strengths of the central and spin-orbit potentials. The radial dependence $f(r)$ follows rather closely the nuclear density distribution, conveniently parametrized by the Saxon-Woods form factor $f(r) = \{1+\exp[(r-R)/a]\}^{-1}$, where $R = 1.25A^{1/3}$ fm is the radius parameter and $a = 0.65$ fm is the surface diffuseness parameter, which governs the rate of fall-off in the surface region. The radial variation of the spin-orbit term has the form $df(r)/dr)/r$, which is peaked on the nuclear surface.

This potential has a series of discrete eigenstates characterized by their energies and quantum numbers. If we allow nucleons to occupy these states, starting from the lowest and restricting the number of nucleons in each state by the Pauli principle, we obtain a useful model of nuclear structure. With potential depths $U = 50$ MeV and $U_s = 14$ MeV, the energies of the states correspond well to those found experimentally using one-nucleon transfer reactions.

The nuclear density distributions can be calculated from this model by squaring and adding the wavefunctions of all the nucleons. The charge distributions found in this way are in excellent agreement with those found from analyses of electron elastic scattering at high energies, and the matter distributions with those found similarly from analyses of proton elastic scattering (Ray and Hodgson 1979, Malaguti *et al* 1982).

This potential can also account for unbound states in the continuum with the addition of an imaginary potential to give the widths of the states. This is the optical potential that describes the scattering of nucleons by nuclei (Hodgson 1963, 1971, 1994). The strengths of the real, imaginary and spin-orbit potentials depend on energy in a smooth way, so that the nucleon mean field is able to unify bound and scattering states (Hodgson 1990). A further unification of the real and imaginary parts of the potentials can be obtained using the dispersion relations which connect them together (Hodgson 1992).

3. Nucleon correlations in nuclei

The nucleons in the nucleus are incessantly moving; so we can ask what is their momentum distribution. This is known in several ways. The cross-sections of $(p, 2p)$ and $(e, e'p)$ reactions, for example, in which an incident proton or electron knocks out a proton in the nucleus, depends on the magnitude and direction of the struck nucleon. Pion production in similar circumstances also depends on the internal momentum distribution; indeed it was first observed at incident energies lower than expected because sometimes the incident proton encounters a nucleon in the nucleus moving towards it with a high momentum.

Since the nucleon wavefunctions in momentum and coordinate space are Fourier transforms of each other, it might be thought easy to calculate the nucleon momentum distribution from the spatial distribution. This is not so; as shown in figure 2 the calculated momentum distribution is markedly below the measured distribution at high momenta. High momenta probe short distances; the source of the discrepancy at high momenta is the correlations among the nucleons of short range, which are not described by the independent particle model that is used to obtain the wavefunctions. The correlations depend on the nucleon-nucleon interaction, which is repulsive at very short distances and attactive at somewhat larger distances. When the nucleons collide, they are for a short time in a deep attractive potential with a hard-core repulsion at short distances and momentarily have high momenta. It is thus not possible to describe the spatial and momentum distributions of nucleons in the nucleus simultaneously by an independent particle model. It might be asked why these nucleons with very high momenta do not escape from the nucleus. The reason is that they are in a deep potential so that, although they have a high kinetic energy, their total energy is always less than the escape value.

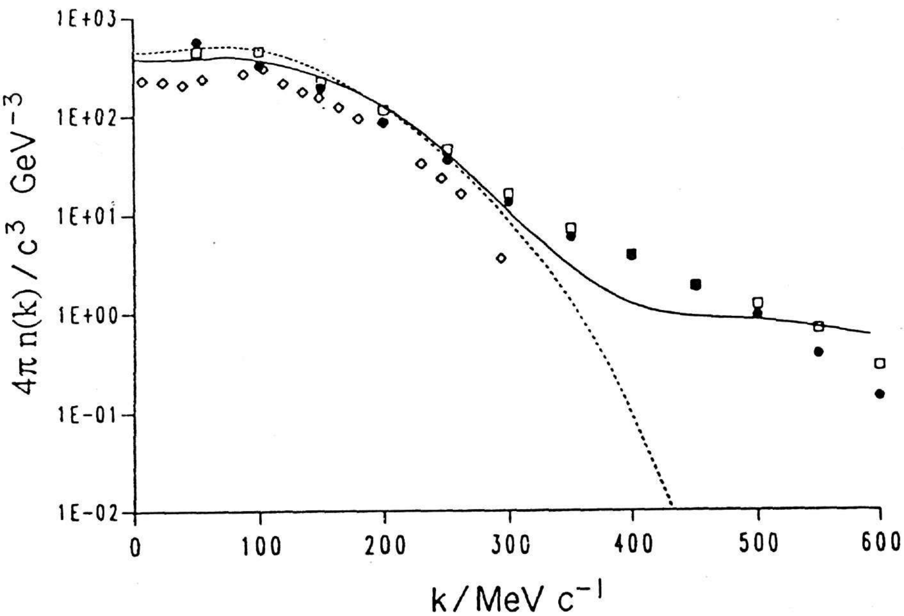

Figure 2. Nucleon momentum distribution in ^{12}C compared with calculations using the Hartree-Fock mean-field theory (...) and a theory that includes short-range correlations (—) (Ciofi degli Atti *et al* 1989).

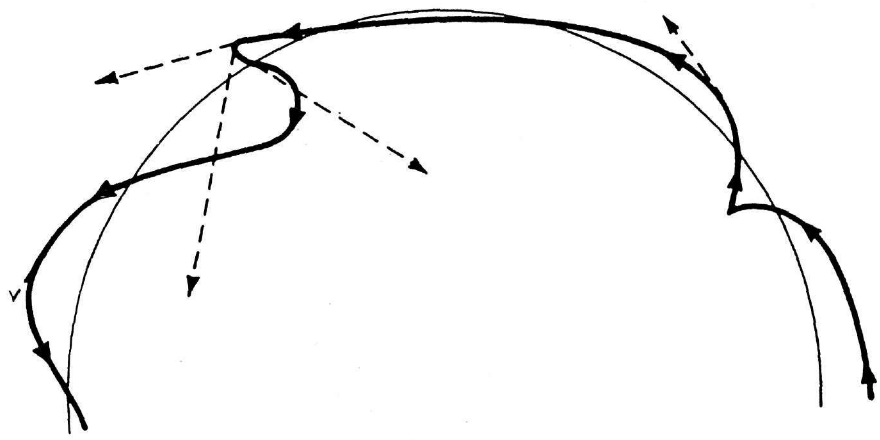

Figure 3. The classical trajectory of a nucleon in a nucleus. The heavy line shows the actual trajectory; and the light line the trajectory given by the single-particle mean-field model. The dotted arrows show the instantaneous nucleon velocities (Gottfried 1963).

The effect of the short-range correlations on the motions of the nucleons can be shown by considering the classical trajectories of the nucleons. Most of the time the nucleons move in the mean field and the trajectories are smoothly varying functions of position. Occasionally, however, they experience a close encounter with another nucleon and are sharply accelerated and deflected. The smooth parts of the trajectory correspond to the motion in the mean field and the abrupt changes to the short-range correlations with other nucleons that give rise to the high-momentum components in the nucleon momentum distribution. The difference between the trajectories with and without the short-range correlations is shown in figure 3.

The effect of the short-range correlations on the trajectories shows the difficulty of determining the high-momentum components of the nucleon momentum distribution. If this is done by measuring the momenta of the two outgoing protons in a $(p, 2p)$ reaction in which an incoming proton knocks another proton out of the nucleus, the momentum of the struck nucleon is determined by assuming that the reaction is essentially a two-body collision inside the nucleus. The effect of the other nucleons is assumed to be small and can be allowed for by assuming that the collision takes place in a nuclear medium with a complex refractive index. This model of the $(p, 2p)$ reaction is good for the smooth parts of the trajectories where the influence of the other nucleons is indeed small, and so the low-momentum part of the nucleon momentum distribution below the Fermi energy is well determined. However, the model breaks down in the region of the kinks in the trajectory because there is another nucleon present, and it is precisely these parts of the trajectory that correspond to the high-momentum components. Thus, although we know that there are high-momentum components in the nucleon momentum distribution, it is very difficult to determine them precisely (Gottfried, 1963).

In order to calculate the effects of short-range nucleon-nucleon correlations in the nucleus, it is necessary to use theories that go beyond the mean-field theory. Such theories define a correlation operator that transforms the wavefunction $\phi(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_A)$ of the uncorrelated many-body shell-model wavefunction into that of the correlated system $\psi(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_a)$. There are several ways of doing this, and among them the simplest to visualize is that of Jastrow (see Antonov *et al* (1993, chapters 3 and 4)). This method takes into account the short-range repulsion or hard core in the nucleon-nucleon interaction that prevents the nucleons coming closer than the hard-core radius r_c. This is done by introducing factors $f(r_{ij})$ that extinguish the wavefunction for all r_{ij} less than the core radius r_c, so that

$$\psi(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_A) = N \prod_{1 \leq i < j \leq A} f(r_{ij}) \phi(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_A) \qquad (2)$$

where ϕ is a Slater determinant of single-particle wavefunctions and N is the normalization constant. The correlation functions is defined to be zero for $|\mathbf{r}_i - \mathbf{r}_j| \leq r_c$ and to be $1 - \exp(-\beta|\mathbf{r}_i - \mathbf{r}_j|^2)$ otherwise. The value of the correlation range β and of the other parameters are determined by minimizing the energy of the correlated system.

The main effect of imposing the nucleon-nucleon correlations is to enhance the high-momentum components of the nucleon momentum distribution, as shown in figure 2. By calculating the mean-field potentials that give the correlation wavefunctions directly, it is found that the short-range correlations have a rather small effect on the potentials and wavefunctions corresponding to the filled states. The short-range correlations mainly affect the motion of the nucleons above the Fermi energy. The difference between the corresponding potentials is shown in figure 4. This is important for the analysis of stripping reactions that populate such states.

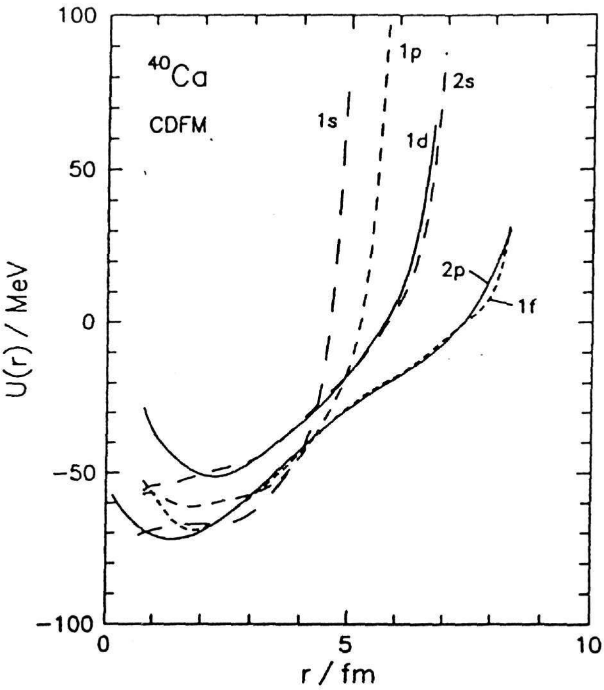

Figure 4. Effective potentials for ^{40}Ca showing the difference between those of filled states (1s, 1p, 1d and 2s) and those of unfilled states (2p and 1f) due to the short-range correlations (Antonov *et al* 1994).

4. The cluster structure of nuclei

The emission of α-particles by nuclei was interpreted by Rutherford as evidence that the α-particle is to be regarded as an 'important secondary unit in the building up of the heavier nuclei and probably of nuclei in general' (Wilson 1983, p.576). The nucleons in the nucleus can thus occasionally form transient substructures that persist until they are broken up by an encounter with another nucleon. Of all such substructures or clusters the α-particle is the most favoured because of its high symmetry and binding energy.

The α decay of nuclei was the first nuclear process studied quantum-mechanically, when Gamow proposed a simple model that was able to account for the vast range of measured half-lives. He proposed that α-particles exist inside the nucleus, and that they bounce backwards and forwards between the walls. Each time they hit the potential barrier there is a finite chance that they will overcome it and be emitted. This probability can be evaluated quantum-mechanically and is extremely sensitive to the width of the barrier. This model has proved very successful in accounting for the relative half-lives of α decay and in its latest versions for the absolute values also, to within an order of magnitude (Buck and Merchant 1989, Merchant *et al* 1989, Merchant and Buck 1989, Buck *et al* 1990, 1991a,b, 1992). Accurate calculations of the α decay half-life proved very difficult, essentially because the individual nucleons comprising the α-particle must be described by the shell model, and then a very large basis is needed to describe them as they move away from the decaying nucleus. This difficulty has been overcome by Varga *et al* (1992a,b) by adding a cluster model component to the shell-model wavefunction, so that the total wavefunction becomes

$$\psi = \psi \text{ (shell) } + \psi \text{ (cluster)}. \tag{3}$$

The calculation of the α decay width was carried out avoiding the approximations inherent in previous work and all the parameters were fixed from independent experimental data. Thus the parameters of the single-particle and harmonic oscillator potentials were chosen to fit the experimental energies of the single-particle states and the binding energies of the initial and final nuclei.

This formalism was applied to calculate the absolute decay width of ^{212}Po, which decays to ^{208}Pb. This is a particularly favourable case, as the final nucleus has a double-closed shell. The decay width was calculated form the **R**-matrix formula

$$\Lambda = 2P_L \frac{\hbar^2}{2M_\alpha a_L} g_L^2(a_L), \tag{4}$$

where M_α is the reduced mass, P_L the Coulomb penetration factor, a_L the channel radius and $g_L(a_L)$ the clustering amplitude. The result $\Lambda = 1.45 \times 10^{-15}$ MeV agrees well with the experimental value of 1.5×10^{-15} MeV. The α-particle occupancy of the state is about 20 times the best shell-model estimates, and the probability of formation of an α-particle inside the nucleus is 0.302.

A related aspect of α clustering in nuclei is the α-particle momentum distribution, and this has recently been calculated by Antonov *et al* (1992). Knowledge of this momentum distribution is important for calculations of α-particle knock-out reactions. Alpha clustering is to be expected on energy grounds, and it has been shown that a nucleon gas is likely to condense into α-particles if the density falls to a value about one third of that in the centre of the nucleus (Brink and Castro 1973).

The concept of a mean field, so useful for nucleons in nuclei, can also be applied to unify our knowledge of α-particle bound states and elastic scattering. The real part of the α-particle potential can be obtained by a double folding of the nuclear and α-particle densities with the nucleon-nucleon interaction, giving

$$V_\alpha(r) = \int\int \rho(r_1)\rho(r_2)\nu(|\mathbf{r} + \mathbf{r}_2 - \mathbf{r}_1|)d\mathbf{r}_1 d\mathbf{r}_2. \tag{5}$$

A good approximation to the radial variation may be obtained by using a delta function for the nucleon-nucleon interaction and analytical expressions for the nuclear densities. For many purposes it is more convenient to use an analytic expression for the potential, such as the Saxon-Woods potential or the cosh potential (Buck and Merchant 1989, Buck *et al* 1992):

$$V_x(r) = \frac{V[1 + \cosh(R/a)]}{\cosh(r/a) + \cosh(R/a)}. \tag{6}$$

The energies, widths and BE(E2) transition rates for the α-particle states in ^{16}O were successfully analyzed using an $\alpha-^{12}$C folded potential obtained by Buck *et al* (1975); subsequently they extended the analysis to ^{20}Ne, which has a pronounced α cluster structure Katō and Bandō 1978, Chung *et al* 1978). More recently, there has been much interest in applying the model to heavier nuclei, especially to the nuclei analogous to ^{16}O and ^{20}Ne in the fp shell, namely ^{20}Ca and ^{44}Ti.

The nucleus ^{44}Ti(=^{40}Ca + α) is a particularly favourable case, and it has been extensively studied (Horiuchi 1985, Michel, Reidemeister and Ohkubo 1986a,b, 1988, Ohkubo 1988, Wada and Horiuchi 1988, Merchant *et al* 1989). The earlier analyses were made of the cross-section of the ^{40}Ca(^{6}Li,d) reaction at 28 and 32 MeV (Fulbright *et al* 197) by applying the distorted-wave theory to determine the quantum numbers of many of the bound and unbound states of ^{44}Ti. As shown in figure 5, these states are quite well given by the α cluster model, using the α-particle optical potential that fits very well the extensive data on the elastic scattering of α-particles by ^{40}Ca (Delbar *et al* 1978). It is notable that the model predicts a $N = 13$, $K = 0^-$ negative-parity band of α-particle states in ^{44}Ti, which at that time had not been observed. The 1^-, 3^- and 5^- members of this band were subsequently found by Yamaya *et al* (1990) using an incident energy of 50 MeV (figure 6). This provides striking confirmation of the α cluster model.

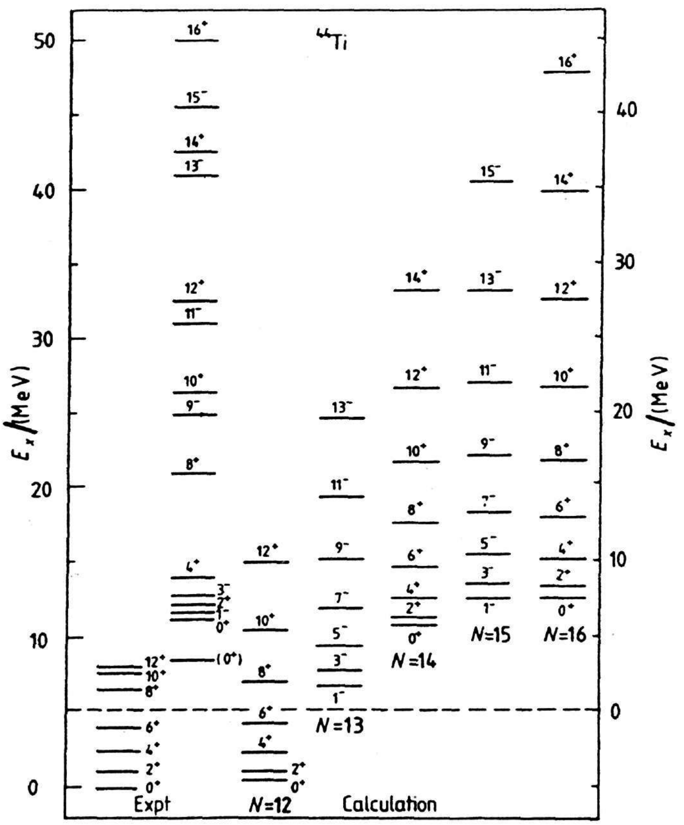

Figure 5. A comparison of the calculated energies of the bands with principal quantum numbers N=12 to N=16 the α cluster states in ^{44}Ti generated by a finite-range folded potential with the experimental data, where the right-hand energy scale refers to the α+^{40}Ca system (Merchant *et al* 1989).

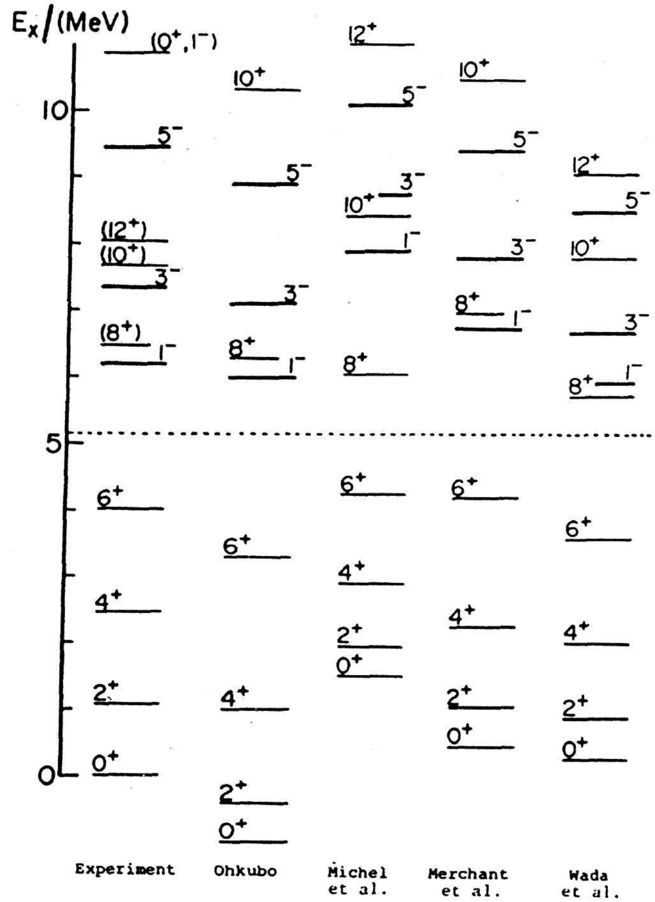

Figure 6. Observed parity doublet band in comparison with theoretical predictions of α cluster states in ^{44}Ti: (- - -), α-particle separation energy. The negative parity states observed by Yamaya *et al* are clearly shown. The 10.86 MeV state is a candidate for the head of the higher nodal band of N=14 or 15 (Yamaya *et al* 1990).

The absolute α-particle probabilities obtained in this work were however very low, ranging from 0.04 for the ground state to even lower values for the excited states. Such low values raised a serious question about the validity of the alpha-cluster model. This was further investigated by Yamaya *et al* (1993), who found that the spectroscopic factors are very sensitive to the values of the α-particle wave functions in the region of the nuclear surface. The previous analysis used a wave function obtained from a Saxon-Woods potential which was not obtained from α-particle elastic scattering, whereas later analyses such as that of Michel et al (1986a, 1988) used the more realistic squared Saxon-Woods potential which gives smaller values of the wave function in the surface region, and hence larger spectroscopic factors between 0.1 and 0.2 for the $K = 0^+$ and $K = 0^-$ bands.

Many light nuclei have a pronounced alpha-particle structure, and many aspects of their behaviour can be understood by considering them to consist of one or more alpha-particles interacting with each other and with the surrounding nucleons.

A particularly interesting class of alpha-particle states is provided by the alpha-chain states. These are usually excited states of a linear chain of alpha-particles. Such states are more stable than might be thought, because, in the case of ^{24}Mg for example, the energy of an oscillator quantum is about 24 MeV in a direction perpendicular to the chain but only about 4 MeV in the direction of the chain. It thus requires about 20 MeV to bend the chain.

Further information on the alpha-particle structure of nuclei may be obtained from analyses of the (p, α) reaction. The most likely reaction mechanisms are triton pickup and alpha-particle knockout, and many studies of the differential cross-sections and analysing powers of (p, α) reactions to discrete final states have shown that the triton pickup process is the more likely. More recently, the (p, α) reaction to the continuum has been studied by Olaniyi *et al* (1995) using the pre-equilibrium theory of Feshbach, Kerman and Koonin. Following Bonetti *et al* (1989) it was assumed that the incident proton collides with a pre-formed alpha-particle in the target nucleus and knocks it out, the proton being captured into an orbit in the residual nucleus. The transition matrix is expressed in terms of the wavefunctions of the initial and final states and the proton-alpha particle effective interaction, and is multiplied by the probability of finding the alpha-particle in the target nucleus. Double differential cross-sections for the (p, α) reaction at 30 and 44.3 MeV on several nuclei were analysed using the subtraction method, and some results are shown in figure 7. The angle-integrated cross-sections show a strong peak at lower energies attributable to compound nucleus emission, and this was evaluated using the Hauser-Feshbach theory. The total cross-section is well described by the sum of compound nucleus and MSD processes, as shown in figure 8. At these energies, the contribution of two-step processes is quite small, but at higher energies the two and three-step processes become increasingly important, especially at the lower outgoing energies. Further analyses have been made of the ^{59}Co(p, α) reaction at 120 MeV, including these higher-order processes. To do this, the computer program was modified to include the $(p, p')(p', \alpha)$ and $(p, n)(n, \alpha)$ two-step processes and the four $(p, N)(N, N')(N', \alpha)$ three-step processes. All these processes contribute incoherently and, as they each include just one nucleon-alpha interaction and one alpha-particle pre-formation factor, they are all multiplied by the same normalisation factor. The calculations fit the data quite well, except for the backward angles at low ejectile energies, due to the omission of higher order processes. The $(p, p')(p', \alpha)$ and $(p, n)(n, \alpha)$ two-step processes dominate at the higher ejectile energies and have similar angular distributions, and the three-step processes become more important as the ejectile energy decreases. As expected, the angular distributions become less forward-peaked as the number of steps increases.

5. Collective Excitations

Some nuclei are deformed in their ground states and can be excited into rotational motion, retaining their shapes. Others are spherical in their ground states and can be excited into vibrational motion. At high excitation energies rotational nuclei can change from oblate (pancake) to prolate (cigar), and some show the extreme deformation known as super deformation, a prolate deformation with a ratio of major to minor axis of two (Hodgson 1987).

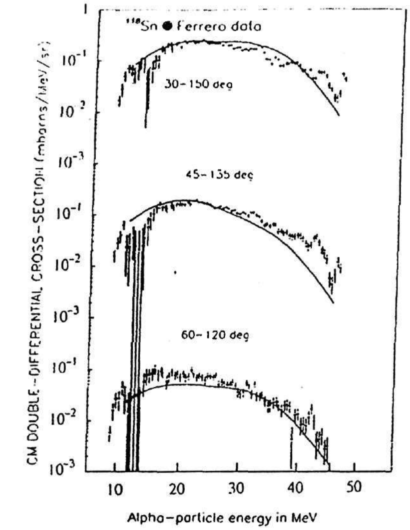

Figure 7. Subtracted double-differential cross-sections for the (p, α) reactions at 30 MeV for ^{118}Sn compared with similarly subtracted zero-range MSD FKK calculations using the alpha-particle knock-out model (Olaniyi, Demetriou and Hodgson, 1995).

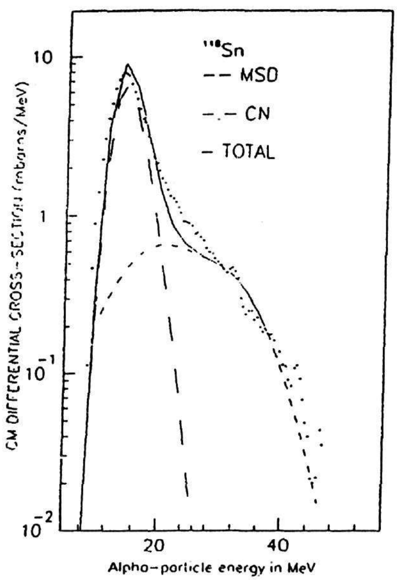

Figure 8. Angle-integrated cross sections for the (p, α) reaction at 44.3 MeV compared with zero-range MSD FKK and compound nucleus calculations (Olaniyi, Demetriou and Hodgson, 1995).

These overall shapes of rotating nuclei are mainly governed by classical forces, and it is found that the curves of energy as a function of angular velocity for different shapes cross each other. At low angular velocities, oblate shapes are energetically favoured, like the Earth, while at high angular velocities a prolate shape is favoured. Unlike classical bodies, however, excited nuclei exist in discrete states, and the correlated motions of the individual nucleons determine their detailed structure.

In contrast to single-particle excitations, the nucleons move collectively in such excitations. The excited states form rotational or vibrational bands and de-excitation usually takes place by gamma emission by successive transitions down a band. Collective nuclei often have many bands based on states of different structure, and transitions can take place, usually with more difficulty from one band to another.

A rotational band may be identified using the quantum-mechanical expression for the energy of a rotator:

$$E_J = \frac{\hbar^2}{2\mathcal{I}} J(J+1) \tag{7}$$

where I is the moment of inertia and J the total angular momentum quantum number. A rotator with ground state 0^+ has a series of states forming a band with $J = 0, 2, 4, \ldots$ and energies given by equation (7). The values of J increase by two because all states must have the same parity $\pi = (-)^J$. The moment I of inertia can be determined from equation (7) and is usually found to be about a third or half the classical rigid-body value, indicating that the nucleons in the spherical core do not participate in the rotation.

Heavy nuclei in the regions away from the magic numbers can be excited to highly excited states by heavy-ion reactions. An off-centre collision between nuclei, for example ^{40}Ca on ^{112}Sn, is the only way to give high angular momentum to the compound system without immediately breaking it up. Studies of the γ-rays emitted from such highly excited nuclei have revealed complicated structures of bands and, about ten years ago, Twin *et al* (1983) found evidence for bands with very high moments of inertia. Analysis of such a superdeformed band in ^{152}Dy indicated a prolate shape with an axis ratio of 2 to 1. Such superdeformed shapes are in accord with theoretical calculations and have been found in several other nuclei. The moment of inertia is very constant along the superdeformed band, indicating a very rigid shape.

Many detailed studies of nuclear structure are being made by Bonatsos and his colleagues (Bonatsos 1988) using the interacting boson model, and this provides a way of integrating the single-particle and collective features. Their most recent study shows that the $\Delta I = 4$ bifurcation observed in superdeformed bands also occurs in the rotational bands of diatomic molecules (Bonatsos *et al*, 1995).

Low-lying collective states may be identified and studied using the cross-sections of inelastic scattering reactions. At higher energies there are broad multipole resonances in the continuum region. The contribution of such collective excitations to the continuum in neutron inelastic scattering has recently been studied by Marcinkowski *et al* (1995ab, 1996). To do this, the cross-sections of (p,n) reactions on several nuclei from 9 to 27 MeV were analysed using the Feshbach-Kerman-Koonin theory. Since the collective contributions to this reaction are small, this establishes the value of the effective nucleon-nucleon interaction strength.

Using the same parameters as the (p,n) analysis, the corresponding (n,n') cross-sections were calculated, and there was a shortfall compared with the data, indicating the presence of collective contributions to the continuum. This was confirmed by calculating the collective contributions using the experimental values of the energies and strengths of the low-lying collective states, and including the contributions of the giant multipole resonances, if any, using the energy-weighted sum rule. The results of this

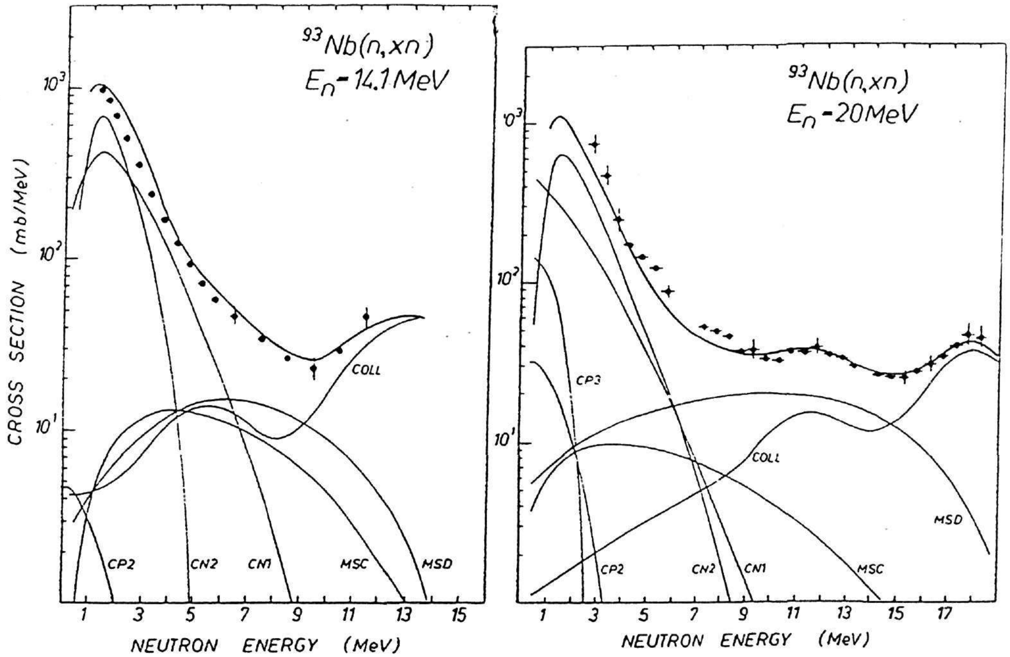

Figure 9. Multistep calculations of the ^{93}Nb (n, n') cross-section at 14.1 and 20 MeV showing the contribution of the collective excitations in the continuum (Marcinkowski *et al*, 1995).

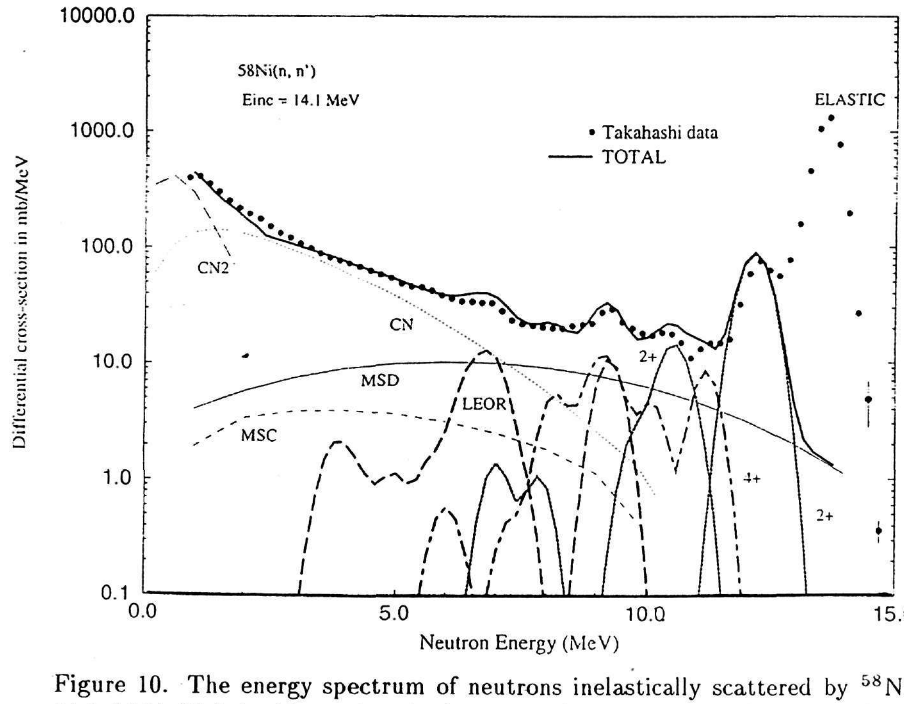

Figure 10. The energy spectrum of neutrons inelastically scattered by ^{58}Ni at 14.1 MeV (Takahashi *et al* , 1992) compared with MSC, MSD and collective cross-sections (Demetriou *et al* , 1996).

calculation, when added to the other contributions, are in good accord with the data, as shown in figure 9. In this way a consistent analysis of (p,n) and (n,n') reactions can be made that includes all the contributing processes. Further studies of the contributions of collective excitations (Demetriou *et al*, 1996) showed that they are able to account for the resonance structure of the (n,n') energy spectra (see figure 10). This structure was also reproduced by Lenske *et al* (1994), using a microscopic model.

6. Conclusion

This brief survey of a few areas of current research in nuclear physics shows the value of nuclear models, particularly those that can be visualised using classical concepts. Notable features of recent work are the increased precision of the experimental measurements and the sophistication of new theories. This is enabling us to inter-relate our nuclear models, and to attain a unified view of nuclear structure and nuclear reactions.

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References

Anagnostatos, G.S. 1985, *Int. J. Theor. Phys.*, **24**, 579; 1988, *Phys. Lett.* **A128**, 266; 1993, Predeal Conference.

Antonov, A.N., Hodgson, P.E. and Petkov, I.Zh., 1993, *Nucleon Correlations in Nuclei* (Berlin: Springer).

Antonov, A.N., Kadrev, D.N. and Hodgson, P.E., 1994, *Prog. Rev.*, **C50**, 164.

Antonov, A.N., Nikolov, E.N., Petkov, I.Zh., Hodgson, P.E. and Lalazissis, G.A., 1992, *Bulg. J. Phys.*, **19**, 11.

Ballentine, L.E., 1970, *Rev. Mod. Phys.*, **42**, 358.

Bonatsos, D., 1988, *The Interacting Boson Model*, (Oxford University Press).

Bonatsos, D., Daskaloyannis, C., Drenska, S.B., Lalazissis, G.A., Minkov, N., Raychev, P. and Roussev, R.P., (1995). (In press).

Bonetti, R., Crespi, F. and Kubo, K.-I., 1989, *Nucl. Phys.*, **A499**, 381.

Brink, D.M. and Castro, J.J., 1973, *Nucl. Phys.*, **A216**, 109.

Brody, T.A., 1993, *The Philosophy Behind Physics*, edited by L. de la Peña and P.E. Hodgson (Berlin: Springer).

Buck, B. and Merchant, A.C., 1989, *J. Phys.*, **G15**, 615.

Buck, B., Dover, C.B. and Vary, J.P., 1975, *Phys. Rev.* **C11**, 1803.

Buck, B., Merchant, A.C. and Perez, S.M., 1990, *Phys. Rev. Lett.* **65**, 2975; 1991a, *J. Phys.* **G17**, 1223; 1991b, *Mod. Phys. Lett.* **A6**, 2453; 1992, *Phys. Rev.* **C45**, 2247.

Chung, W., van Hienen, J., Wildenthal, B.H. and Bennett, C.L., 1978, *Phys. Lett.* **79B**, 381.

Ciofi degli Atti, C., Pace, E. and Salmé, G., 1989, *Nucl. Phys.*, **A497**, 361c.

Delbar, Th., Grégoire, Gh., Paic, C., Ceuleneer, R., Michel, F., Vanderpoorten, R., Budzanowski, A., Dabrowski, H., Freindl, L., Grotowski, K., Micek, S., Planeta, K., Strzalkowski, A. and Eberhard, K.A., 1978, *Phys. Rev.*, **C18**, 1237.

Demetriou, P., Marcinkowski, A. and Hodgson, P.E., 1996, *Nucl. Phys.* **A596**, 67.

Feshbach, H., Kerman, A. and Koonin, S., 1980, *Ann. Phys.*, (New York), **125**, 429.

Fulbright, H.W., Bennett, C.L., Lindgren, R.A., Markham, R.G., McGuire, S.C., Morrison, G.C., Strohbusch, U. and Tōke, J., 1977, *Nucl. Phys.*, **A284**, 329.

Gottfried, K., 1963, *Ann. Phys.* (N.Y.), **21**, 29.

Hodgson, P.E., 1963, *The Optical Model of Elastic Scattering* (Oxford University Press); 1971, *Nuclear Reactions and Nuclear Structure* (Oxford University Press); 1987, *Contemp. Phys.*, **28**, 365; 1990, *Contemp. Phys.*, **31**, 295; 1992, *Proceedings of the International Conference on Nuclear Data for Science and Technology*, Julich, p.768; 1994, *The Nucleon Optical Potential* (Singapore: World Scientific).

Horiuchi, H. 1985, *Prog. Theor. Phys.*, **73**, 1172.

Katō K. and Bandō, H. 1978, *Prog. Theor. Phys.*, **59**, 774.

Leech, J., 1957, *Math. Gaz.*, **41**, 81.

Lenske, H., Wolter, H.H., Herman, M. and Reffo, G., 1994, *Proc. 7th Int. Conf. on Nuclear Reaction Mechanisms* (Ed. E. Gadioli) Ricerca Scientifica and Educazione Permanente, Supplemento No. 100, 110.

Malaguti, F., Ugnzzoni, A., Verondini, E. and Hodgson, P.E., 1982, *Revista Nuova Cim.* **5**, 1.

Marcinkowski, A., Marianski, B., Demetriou, P. and Hodgson, P.E., 1995a, *Phys. Rev.*, **C52**, 2021.

Marcinkowski, A., Demetriou, P. and Hodgson, P.E., 1995b, *J. Phys.*, **G21**, 1089.

Merchant, A.C. and Buck, B., 1989, *Europhys. Lett.*, **8**, 409.

Merchant, A.C., Pál, K.F. and Hodgson, P.E., 1989, *J. Phys.*, **G15**, 601.

Michel, F., Reidemeister, G. and Ohkubo, S., 1986a, *Phys. Rev. Lett.* **57**, 1215; 1986b, *Phys. Rev.* **C34**, 1248; 1988, *Phys. Rev.* **C37**, 292.

Ohkubo, S., 1988, *Phys. Rev.*, **C38**, 2377.

Olaniyi, H.B., Demetriou, P. and Hodgson, P.E., 1995, *J. Phys.*, **G21**, 1089.

Ray, L. and Hodgson, P.E., 1979, *Phys. Rev.*, **C20**, 2403.

Takahashi, A., Gotoh, M., Sasaki, Y. and Sugimoto, H., 1992, *Oktavian Report* A–92–01.

Twin, P.J., Nolan, P.J., Aryaeinejad, R., Love, D.J.G., Nelson, A.H. and Kirwan, A., 1983, *Nucl. Phys.*, **A409**, 343c.

Varga, K., Lovas, R.G. and Liotta, R.J., 1992a, *Phys. Rev. Lett.*, **69**, 37; 1992b, *Nucl. Phys.*, **A550**, 421.

Wada, T. and Horiuchi, H., 1988, *Phys. Rev.* **C38**, 2063.

Wilson, D., 1983, *Rutherford: Simple Genius* (London: Hodder and Stoughton), p.579.

Yamaya, T., Oh-ami, S., Fujiwara, M., Itahashi, T., Katori, K., Tosaki, M., Kato, S., Hatori, S. and Ohkubo, S., 1990, *Phys. Rev.*, **C42**, 1935.

Yamaha, T., Ohkubo, S., Okabe, S. and Fujiwara, M., 1993, *Phys. Rev.* **C47**, 2389.