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Dispersive Correction to the p+16 0 Optical Montel Potential

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ABSTRACT

The optical model potential to p+^{16}O scattering is derived by taking into account the polarization potential induced by the energy dispersion relation. The real part of the potential is derived by the RGM-method with the Volkov or Minnesota potential as a basis for the n-n force. It is shown that the polarization potential effects an adjustment of the parameters of the n-n force due to the constraints imposed by the energy dispersion relation.

I. Introduction

Theoretical investigations of the energy dependence of the optical model potential (OMP) in nuclear reactions have shown that if the imaginary part is not deduced by microscopic considerations but it is "put by hand", it has to obey constraints due to the energy dispersion relation (1-3). In fact this imaginary potential will induce a real polarization potential $\Delta V(E,R)$ to the real part of the OMP which will satisfy the energy dispersion relation.

The effects of this polarization potential have been recently investigated (4) with considerable success in a light reaction such as the α-α system, in which the real part of the OMP was derived microscopicaly by the resonating group method (RGM) in a one-channel approximation. It was shown there that a polarization potential could be constructed throughout the energy range which gave a significant improvement to the α-α scattering phase-shifts up to 60 MeV. Furthermore it was suggested that if one resorts to a single channel approximation this can even be done consistently if the induced polarization potential could be taken into account right at the beginning, i.e., at the nucleon-nucleon force.

It is obvious that if this approach could be extented to heavier nuclei then one would hope to develope a semimicroscopic description of the OMP, which would correspond more closely to the actual interaction, but with much less efforts. In this work we shall turn our attention to proton-nucleus scattering and in particular to p-^{16}O for which the kernel of the real non-local interaction is well known (5). In addition two important and experimentally determinable quantities which are of interest for an optical model analysis the differential cross-sections and the reaction cross-sections are also known. Namely for the above system there exist a self consistent set of measurement of these quantities (6-9) covering the energy range of 20 to 50 MeV.

This energy region is of special interest since it shows deviations from the linear energy dependence $V(E) = \alpha + \beta E$ of the strength of the imaginary part of the optical potential which has been assumed to exist at lower energies. Therefore if one starts with a microscopic calculation of the real part one might be able to achieve a better fit at higher energies.

. The reaction cross-section is particularly useful in this context because it is very sensitive to the total absorption. In addition the angular distribution in the energy range from 20 to 50 MeV might give information about intermediate structure in the energy dependence at such cross-sections.

We shall report in this paper the first in a series of calculations on the interactions of medium energy protons with a heavier nuclei, i.e., Oxygen, using the elastic scattering data covering the center of mass angles from 10 to 165 degrees at energies 20 to 50 MeV. Our real part of the OMP is derived by the RGM-method in the one-channel approximation. Since our aim is to test the applicability of our approach rather than to achieve an exact fit, we shall not use the more elaborate full RGM-kernel (5) but we shall use instead the K-kernel of ref. (10). This kernel is easier to handle and one can derive its local equivalent potential with much less difficulties. It has of course only the direct term and the knock-on exchange term of the total kernel thus it does not take into account contributions arising from nucleon-excange terms. However, this approximation is justified since it has been shown (10) that this particular term plays an important role in determining the essential behavior of the system. Furthermore at the energies which we are considering, we are expecting the elastic scattering to be most dominant and therefore the ommision of the excange terms will not alter our results significantly.

II. Formalism

The method which we shall use here is outlined in ref (4). For clarity we shall only repeat here the main points. The resonating group method has been extensively used to describe the behaviour of nuclear systems from a microscopic viewpoint. In particular for scattering problems it can give accurate results even in the one channel approximation, by which a nucleon-nucleus scattering is reduced to the scattering of a single particle from a one-body potential. This potential is nonlocal, therefore energy dependent, and constitutes the basis of the real part of the OMP. For calculations at energies at which reaction channels are open one has to add a phenomological imaginary potential $iW(E,R)$ which is also energy dependent. Thus, if one takes the equivalent local potential of the non-local RGM, then the total potential is of the form:

$$V(E,R) = V^{ELP}(E,R) + iW(E,R) \tag{1}$$

where $V^{ELP}(E,R)$ is the equivalent local potential to the non local kernel derived by the RGM-method. In the present case the imaginary part $W(E,R)$ is based on a Woods-Saxon form factor

well known in nucleon optical model potentials and contains both terms, a Woods-Saxon form for volume absorption and a derivative Woods-Saxon form for surface absorption.

$$W(E,R) = \frac{f(E)}{(1+\exp(R-Rv)/Av)} + \frac{4.0g(E)\exp((R-Rs)/As)}{(1+\exp(R-Rs)/As)^2}$$

(2)

where $f(E)$ and $g(E)$ are the energy dependent strengths. The choise is sensible since we will try to parametrize data from a wide range of energies and scattering angles and a detailed variation of form factors could complicate the problem unnecessarily.

In our approach we first calculate the RGM-kernel and its Wigner transform for both nucleon-nucleon interactions : The Volkov and the Minnesota forces of ref. (10). They have the basic forms :

Volkov :

$$V_{ij} = - V_0 \exp(-\varkappa r_{ij}^2)\,(w - m P_{ij}^\sigma P_{ij}^\tau + b P_{ij}^\sigma - h P_{ij}^\tau)$$

$$- \frac{1}{2h} V_\lambda \exp(-\lambda r_{ij}^2)\,(\sigma_i + \sigma_j) \cdot (r_i - r_j) \times (p_i - p_j) + \frac{e^2}{4r_{ij}}(1+\tau_{iz})(1+\tau_{jz})$$

$V_0 = 72.98$ MeV $\varkappa = 0.46$ fm^{-2} $w = m = 0.4075$ $b = h = 0.0925$ $a_{RGM} = 0.32$ fm   (3)

Minnesota :

$$V_{ij} = \left[V_R + \frac{1+P_{ij}^\sigma}{2} V_t + \frac{1-P_{ij}^\sigma}{2} V_s \right] \left(\frac{u}{2} - \frac{2-u}{2} P_{ij}^\sigma P_{ij}^\tau \right)$$

$$- \frac{1}{2h} V_\lambda \exp(-\lambda r_{ij}^2)\,(\sigma_i + \sigma_j) \cdot (r_i - r_j) \times (p_i - p_j) + \frac{e^2}{4r_{ij}}(1+\tau_{iz})(1+\tau_{jz})$$

where   with

$V_R = V_{0R} \exp(-\varkappa_R r_{ij}^2),$ $V_{0R} = 200.0$ MeV, $\varkappa_R = 1.487$ fm^{-2}

$V_t = - V_{0t} \exp(-\varkappa_t r_{ij}^2),$ $V_{0t} = 178.0$ MeV, $\varkappa_t = 0.639$ fm^{-2}

$V_s = - V_{0s} \exp(-\varkappa_s r_{ij}^2),$ $V_{0s} = 91.85$ MeV, $\varkappa_s = 0.465$ fm^{-2} (4)

Then by using the expressions given in ref(10) for the k-model and ref.(4) for their Wigner transforms we get the results : for the Volcov potential

Direct nuclear :

$$V^D(R) = -V_0\gamma \, (4w-m+2b-2h) \left[\frac{4a+\varkappa}{a+\varkappa} + \frac{2a\varkappa^2}{(a+\varkappa)^2} R^2\right] \exp\left(\frac{-2\varkappa}{a+\varkappa} R^2\right)$$

(5)

Wigner Transform for the knock-on-exchange part :

$$V^N(q^2,R^2) = -V_0\beta \, (-w+4m-2b+2h) \left[1+2a(\frac{-3}{8B}+R^2+\frac{q^2}{16B^2})\right]$$

$$*(\frac{\pi}{B})^{3/2} \exp(-4A^2 - \frac{q^2}{4B})$$

where $\gamma = (\frac{a}{a+\varkappa})^{3/2}$, $\beta = (\frac{a}{\pi})^{3/2}$, $4A=a$, $4B=a+4\varkappa$

and for the Minnesota potential

Direct Nuclear :

$$V^D(R) = \sum_i C_i \, V_{oi} \, \gamma_i \exp(\frac{-aR^2}{a+\varkappa_i})$$

(6)

$$C_i = \left((\frac{5}{2}u-1), -(\frac{9}{4}u-1), -(\frac{u}{4}+\frac{1}{2})\right)$$

Wigner Transform for the knock-on-exchange part :

$$V^N(q^2,R^2) = \sum_{i=1}^{3} C_i \, V_{oi} \, \gamma_i \left(\frac{4a+\varkappa_i}{a+\varkappa_i} + \frac{2a\varkappa_i^2}{(a+\varkappa_i)^2} R^2\right) \cdot \exp(\frac{-a\varkappa_i R^2}{a+\varkappa_i})$$

(7)

$$C_i = \left((4-\frac{5}{2}u), -(3-\frac{9}{4}u), -(1-\frac{u}{4})\right)$$

Then for each of the angular distributions given below in fig.(1) we try to get the best possible fit by varying the geometric parameters and the energy dependence of the imaginary part. In this way one gets a wide range of geometric parameters $Rv(Rs)$ and $Av(As)$ which confirm our earlier predictions that they are also energy dependent (11). Since in order to be able to derive the energy dispersion relation and the polarization potential these parameters must be taken energy independent one has to make an appropriate choise, such that a reasonable fit for all scattering angles throughout the energy range is possible. With the

geometric parameters so defined one then derives the energy dependence of the imaginary strength and via eq.(19) of ref.(4)

$$V^{pol}(E) = \frac{P}{\pi} \int_{E_{thresh}}^{\infty} \frac{(f(E')+g(E'))dE'}{E'-E}$$

(8)

the strenght of polarization potential $V^{pol}(E)$, where P stands for principal value. The polarization potential $V^{pol}(E,R)$ is then derived from eq.(2) and it is added to the real part of the OMP, which now satisfies the energy despersion relation. The full OMP potential therefore is

$$V_{OMP}(E,R) = V(E,R) + V^{pol}(E,R)$$

(9)

Since the addition of $V^{pol}(E,R)$ will alter the goodness of the fit, one now has to get down to the nucleon nucleon force and restore the same goodness of the fit by
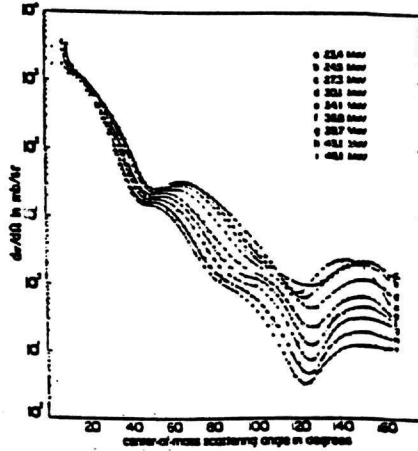

Fig 1: Differential cross section angular distributions for the elastic scattering of protons by ^{16}O at 23.4, 24.5, 27.3, 30.1, 34.1, 36.8, 39.7, 43.1, and 46.1 MeV. The smooth curves are drawn to guide the eye and have no significance. They have been taken from ref. (8).

varying its parameters. This process could be repeated selfconsistently untill convergency is achieved. We shall note here that the introduction of an external energy dependence to the real part of the potential via the addition of the polarization potential will in fact necessitate a further modification of the imaginary strength. The best way to account for this selfconsistently is to terminate this minimization procedure by a simultaneous minimization of the parameters of the nucleon-nucleon force and imaginary strengths. We shall note further that for the results reported here this procedure was repeated only twice since the introduction of the polarization potential introduced only slight variations of the parameters of the nucleon-nucleon force in order to restore the same quality of fits.

III. Results and Discussion

For the p-^{16}O system we use the angular distributions reported in refs.(6-9) for the energies 23.4, 24.5, 27.3, 30.1, 34.1, 36.8, 39.7, 42.1, 43.1, 46.1 and 52.5. MeV. For the analysis of our data we used the least-squares method as a goodness-of-fit estimator to derive the best set of potential parameters. Our computer code contained an automatic search routine for up to nine parameters with the view to minimize the function

$$\Delta = \sum_{totals} (W_k \frac{\sigma_t^{exp} - \sigma_t^{th}}{\Delta \sigma_\tau})^2 + \frac{1}{N} (\frac{d\sigma_i^{exp} - d\sigma_i^{th}}{\Delta \sigma_i})^2 \tag{10}$$

where the subscripts t and i stand for total and angular distribution quantities, $\Delta \sigma$ are their uncertainties and W_k are weight factors.

Initially, attemts were made to get good fits to both $\frac{d\sigma}{d\omega}$ and σ_{tot} for all of the above energies simultaneously by varying all nine parameters. However such a general fit resulted in unphysical values for the geometric parameters and or the potential strengths. Another intriguing difficulty was that when the varied parameters exceeded the number three, the searches converged on several distinct local minima depending on the starting values. Thus it was found necessary to experiment in a discriminating alternation of fixing and varying grouped parameters in order to get optimum values within physically reasonable grounds.

As a general comment we can say that it was difficult to get convergency by a simultaneous variation of potential strengths and associated radii and that convergency was easier achieved by specifying different radii for the different potentials. In addition preliminary calculations have shown that the introduction of the spin-orbit potential of ref.(10) was causing convergency problems and in agreement with ref.(10) it was ommited alltogether.

For each one, of the above energies we contacted searches of two general kinds, i.e., one with the Volkov and one with the Minnesota potential, as the nucleon-nucleon force, in our RGM non-local kernel. The first case produced erratic results for some sets of starting values but nevertheless it had always converged to a satisfactory fashion to $\Delta < 30$. This was achieved however only by allowing the potential strengths to increase about 100 MeV which would seem to be unacceptably high. It was further noticed that the difficulties were arising by the requirement of fixed Rs(Rv) and for all energies, as a constraint imposed by the dispersion relation in order to have only the potential strengths depend on the energy. This in conjuction with the fact that searches converged on several local minima (usually at least with one unacceptable final parameter) depending on the starting values, brought the realisation that there was no remedy against it other than to accept higher values of Δ but in any case at least a factor of five better than those reported in ref.(8) for the l-independent potential search.

A complete set of parameters for the whole energy range together with the minimum Δ is given in table (1) for the Volkov and in table (2) for the Minnesota potential. Since the surface and absorption strengths for the Volkov potential are unphysically high we decided to continue our calculations using only the Minnesota force. The strength for the surface and volume absorptions and the corresponding polarization potential as a function of

Table 1.Imaginary potential strengths $\dot{W}_V$ and W_s, geometric parameters R and A and minimization values Δ for the different energies. The numbers (1) and (2) refer to the different sets for the geometric parameters. The Volkov potential of eq (3) is taken as the basis for the n-n force. Δ is the minimization values of the l-independent potential of ref. (8) for the same energies.

(1) $R_V - R_s = 1.37$ fm, $A_V = A_s = 0.569$ fm				(2) $R_V = 1.3701$ fm, $A_V = 0.568$ fm $R_s = 1.3846$ fm, $A_s = 0.577$ fm			
ECM (MeV)	$W_V(1)$ (MeV)	$W_S(1)$(MeV)	$\Delta(1)$	$W_V(2)$(MeV)	$W_S(2)$(MeV)	$\Delta(2)$	Δ
23.4	27.743	35.294	27.08	27.396	33.898	26.03	136.48
24.5	22.184	33.485	24.21	18.76	32.92	24.09	474.35
27.3	88.465	24.627	39.19	87.604	23.771	36.12	316.25
30.1	71.759	34.99	43.06	76.773	37.249	41.25	394.84
34.1	86.73	38.315	30.84	66.419	37.448	30.81	247.5
36.8	86.73	66.39	44.01	83.05	59.45	43.30	91.08
39.7	84.668	50.23	184.71	104.911	95.46	125.14	46.35
42.1	98.66	75.33	53.4	106.112	99.63	40.12	87.54
43.1	107.35	113.93	56.34	107.35	113.93	50.88	34.59
46.1	110.09	128.65	59.57	110.09	128.68	48.46	33.25
52.5	90.28	71.15	58.68	101.32	105.37	47.51	368.44

energy are given in fig.(2). Some polarization potentials as a function of R for various energies are given in fig.(3). We have set the threshold energy $E_{thresh} = 6.06$ MeV which is the first excited state of ^{16}O. For energies above 60 MeV in eq.(8) we have used the same prescription as in ref.(4). Namely

$$W(E) = W(E_0) \exp(-\alpha(E-E_0)) \qquad E > E_0 = 60 \text{ MeV}$$

Variations of the parameter α between 0.2 and 0.5 did not change our polarization potential more than 1% and we have set this parameter to $\alpha=0.5$. For some of the energies the best fits where achieved for either volume (f.e.30.1MeV) or surface absorption (f.e. 34.1 MeV) but adequate fits for all levels were possible only with a mixture of volume and surface absorption.

With the polarization potentials derived in this way, we have constructed the total final potential of eq.(6) and tried to get the best possible fit by varying the parameters of the nucleon-

nucleon force. Excellent fits were possible if one was to accept such parameters for the nucleon-nucleon force which would produce unreasonable values for the binding energy of the deuteron.

Table 2. The same as Table 1 but for the Minnesota potential of eq (4).

	$R_V = 1.3701$fm, $A_V = 0.568$ fm $R_S = 1.384$ fm, $A_S = 0.577$ fm			
Ecm(MeV)	W_V(MeV)	W_S(MeV)	$\Delta(2)$	Δ
23.4.	6.36	7.105	49.36	136.48
24.5	4.415	4.145	38.15	474.35
27.3	4.156	3.241	40.12	316.25
30.1	7.303	2.022	37.69	394.84
34.1	24.496	11.648	41.96	247.5
36.8	39.769	30.292	31.92	91.08
39.7	19.543	18.493	12.66	46.35
42.1	8.07	7.32	39.62	37.54
43.1	4.136	5.398	36.27	34.59
46.1	4.345	5.735	36.41	83.25
52.5	4.156	4.78	34.53	368.44

To avoid such a problem we have set an upper limit of ±1% of variation of the strengths by keeping the oscillator parameters and the radii unchanged. This, in the worst case could have a variation of 0.2 MeV for the binding energy of the deuteron. The searches were repeated for different values of the excange mixture parameter u. The values so obtained are reported in table 3. They not only restore the initial goodness of the fits, but in some cases they also produce a considerable improvement. Furthermore, given the uncertainties in our data they seem quite satisfactory. The main exception was the 37.9 MeV for which the quality of the fit to the angular distribution was relatively poor, where most of the Δ came from a few points at large angles.

As far as the energy dependence is concerned, a monotonic energy variation is only seen at the lower energy region, i.e. below 30 MeV. Above this energy our results show a departure from such a smooth energy variation. In fact around 37 MeV they show some structure which suggests, effects comming from compound states or single particle resonances or possible doorway states from four-quastiparticles in F^{17}.

Table 3. Final minimization values, $\Delta(f)$, potential strengths and nucleon-nucleon parameters.

Ecm(MeV)	V_{oR} = 200.62 MeV, V_{ot} = 178.232 MeV			
	u = 0.918, V_{os} = 91.712 MeV			
	W_V(MeV)	W_S(MeV)	$\Delta(f)$	Δ
23.4.	2.518	4.405	24.17	136.48
24.5	2.334	4.039	27.65	474.35
27.3	2.068	3.469	37.21	316.25
30.1	2.244	3.824	34.37	394.84
34.1	2.452	4.445	38.11	247.5
36.8	3.419	8.693	40.84	91.08
39.7	4.656	18.065	127.32	46.35
42.1	3.256	9.531	39.15	37.54
43.1	2.718	9.494	40.17	34.59
46.1	2.824	7.367	35.01	33.25
52.5	2.603	6.152	34.03	368.44

A word of caution is also relevant here. Our extracted values are by no means unique. The minimization procedure showed that convergency was strongly dependent on the starting values and that the same quality of the fits, Δ, could be achieved by different parameters.
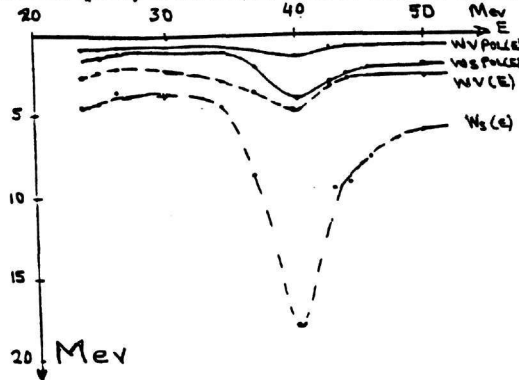

Fig. 2: Final imaginary strengths W_V, W_S and polarization potential strengths WVPOL and WSPOL as a function of energy.

To discriminate between the various different local minima, we have applied the following restrictions : For all energies, we have started each varied parameter at the same initial values. Then after convergency was achieved we have repeated the minimization procedure with starting values below and above the convergency values. Only those physical acceptable

values were accepted which would converge to the same values and would produce the same quality of fit.
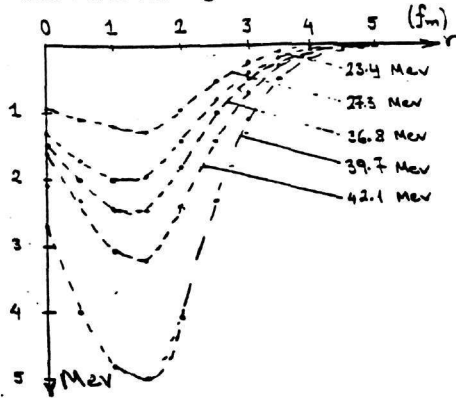

Fig. 3: Polarization potentials VPO$_L$ as a function of R at different energies.

The present study yielded also confirmatory evidence for the need of energy dependence of the geometric parameters (radius and difuseness) of the imaginary potential. In addition they have been found significantly smaller than those usually reported in OMP calculations with Woods-Saxon forms.

Another undesirable feature our calculations have shown was that the required value of the exchange mixture parameter u was somewhat different for different energies the best compromise been the value u=0.918 which was quite closed to u=0.924 reported in ref. (10).

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