HNPS Advances in Nuclear Physics

3rd HELLENIC SYMPOSIUM ON NUCLEAR PHYSICS

National Technical University of Athens
May 25-26, 1992
Athens-Greece

Sponsored by
National Technical University
General Secretary of Research and Technology
Ministry of Education
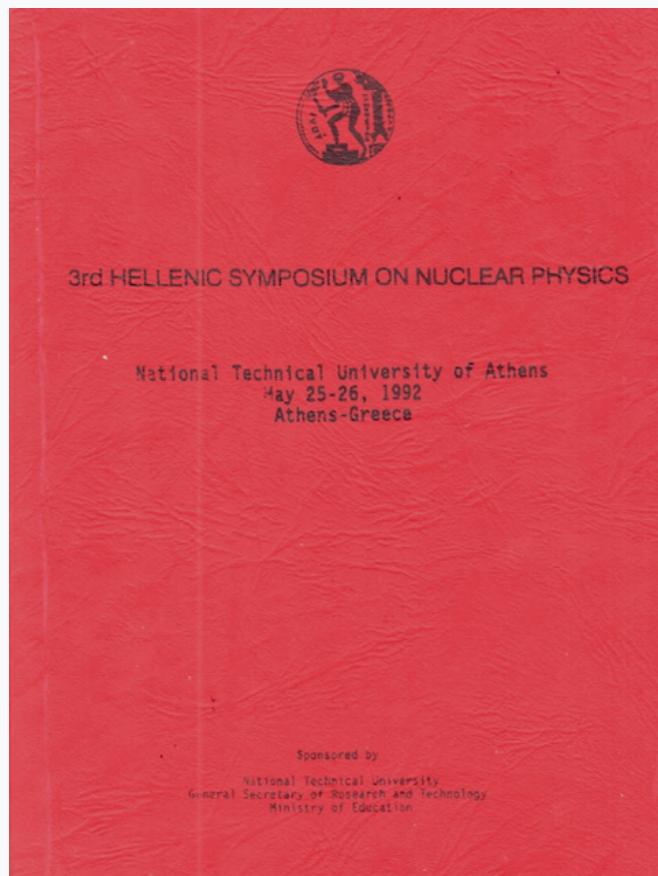
Derivation of the nuclear level density distribution and the partition function

C. Syros, E. Livadas

doi: 10.12681/hnps.2383

DERIVATION OF THE NUCLEAR LEVEL DENSITY DISTRIBUTION AND THE PARTITION FUNCTION

C.Syros and E.Livadas [*]

Laboratory of Nuclear Technology
University of Patras
26110 Patras,Greece

ABSTRACT: *The nuclear density matrix and the partition function have been derived from Generalized Random Quantum Field Theory. The nuclear level density distribution has been deduced as a function of the energy level density. The temperature follows as a functional of the nuclear interaction Hamiltonian.*

1 INTRODUCTION

The purpose of this paper is to present a derivation of the nuclear level density distribution. The density matrix and the thermodynamics of the nucleus will follow from **Random Quantum Field Theory.**

The statistical description of nuclear characteristics has been the subject of intensive investigations since the very beginnings of the nuclear theory. These investigations have been initiated by no less researchers than E.P.Wigner, F.J. Dyson, N.Rosenzweig, L.Landau, M.Gaudin, M.L.Mehta, P.B.Kahn, A.Bohr and many others[1]. They were concerned with the distribution of the eigenvalues of real symmetric, orthogonal and unitary matrices.

_____

(*)
Presented by Livadas postgraduate student at the Laboratory of nuclear Technology.

Of particular importance for Nuclear Physics are the Hermitian matrices as representing the nuclear Hamiltonian. The theory developed by N. Rosenzweig, and N. Rosenzweig and C.E. Porter is of particular interest to us, since it allows the analysis of the eigenvalues distributions of any matrices from a more fundamental point of view[1].

Recently, the problem of the level density and level spacing distributions has been systematically investigated by O. Bohigas, R.U. Haq, and A. Pandey[2] and by O. Bohigas, and H.A. Weidenmüller[3] in the framework of the non-linear systems with chaos properties.

The statistical properties of the above mentioned nuclear characteristics constitute still today an important problem for the Physics of the nucleus, and the present paper aims at presenting some new developments of the density matrix of the nucleus which directly involves the density of the nuclear energy levels.

The statistical properties of the nucleus seen from the point of view of more recent theories is related to the fundamental problem of macroscopic irreversibility of systems whose microscopic behavior is time-reversal invariant.

This will be made clear on the basis of a theory developed recently starting from the second quantized representation of the state vector of the nucleus and the assumption that the Lagrangian density is an infinitely divisible generalized random field.

In sect. 2 the ensemble theories of the nucleus are briefly reviewed in order to compare the results with our theory. In sect. 3 the connection between the level density distribution and the quantum theory is discussed. In sect. 4 the **statistical evolution operator** Σ_e is given and the nuclear density matrix and the partition function of the nucleus are derived.

In sect. 5 the density matrix is derived in dependence on the level density, while in sect. 6 the **partition function** on the nucleus is given. Finally, in sect. 7 some conclusions and a discussion are given.

2 THE ENSEMBLE THEORY OF THE NUCLEAR LEVEL
DENSITY DISTRIBUTION

The importance of the eigenvalues distribution induced many investigations from which the theory developed by C.E.Porter and N.Rosenzweig[1] will be briefly reviewed.

The central idea of this theory is to associate to each given Hamiltonian matrix a measure,which according to the theory of stochastic processes, will be related to the probability of the corresponding energy levels.

Considering a real symmetric Hamiltonian matrix H the metric is defined by

$$ds^2 = Tr(dH \; dH^\dagger),$$ (2.1)

where $\{dH_{\mu\nu}\}$ are the increments of the elements $\{H_{\mu\nu}\}$ of H.

Expression (2.1) written out reads

$$ds^2 = \sum_{\mu,\nu=1}^{N} (dH_{\mu\nu})^2$$

$$= \sum_{\mu=1}^{N} (dH_{\mu\mu})^2 + 2 \sum_{\substack{\mu,\nu=1 \\ \mu<\nu}}^{N} (dH_{\mu\nu})^2.$$ (2.2)

By comparing the expression (2.2) with the line element in a Riemann space

$$ds^2 = \sum_{\mu,\nu=1}^{N} g_{\mu\nu} dx_{\mu} dx_{\nu}.$$ (2.3)

the metric tensor in (2.2) can be determined.

If g is known,then the volume element (the measure) is also given by

$$dV = (det g)^{1/2} dx_1 dx_2 \dots dx_N.$$ (2.4)

Accordingly,the differential probability follows from (2.3)-(2.4) in the form

$$dP = P(H_{11}, H_{12}, \dots H_{N-1 \; N}) 2^{N(N-1)/4} dH_{11} dH_{12} \dots dH_{N-1 \; N}.$$ (2.5)

Expression (2.5) is subsequently diagonalized using three classes of matrices (orthogonal,unitary,symplectic) obtaining thereby a coefficient $P(H_{11}, H_{12}, \dots H_{N-1N})$ in dV which is just the probability densities characterized by the properties of the corresponding transformation matrices.

For an orthogonal transformation the probability density is

$$P(H) = \frac{1}{(4\pi a^2)^{N(N+1)/2}} \, det \left[exp \left(- \frac{(H - E_o I)^2}{4a^2} \right) \right] \quad \begin{array}{l} \text{orthogonal} \\ \text{ensemble} \end{array} \quad (2.6)$$

In a similar manner Porter and Rosenzweig[1] obtained the unitary

$$P(H) = \frac{1}{(4\pi a^2)^{N^{3/2}}} \, exp \left(- \frac{Tr(H - E_o I)^2}{4a^2} \right) \quad \begin{array}{l} \text{unitary} \\ \text{ensemble} \end{array} \quad (2.7)$$

and the symplectic

$$P(H) = \frac{1}{(4\pi a^2)^{(2N^2 - N)/2}} \, exp \left(- \frac{Tr(H - E_o I)^2}{4a^2} \right) \quad \begin{array}{l} \text{symplectic} \\ \text{ensemble} \end{array} \quad (2.8)$$

ensembles.

The usefulness of all above ensembles consists in that they allow to calculate the expectation value of the evolution operator of the nucleus with a view of obtaining, among other things, a kind of Boltzmann exponential factor in the energy level distribution function of the nucleus.

If one considers, for example, the 1×1 Gaussian ensemble

$$P(H) = \frac{1}{(4\pi a^2)^{1/2}} \, exp \left(- \frac{(H - E_o I)^2}{4a^2} \right) \right] \quad \begin{array}{l} \text{Gaussian} \\ \text{ensemble} \end{array} \quad , \quad (2.9)$$

then the expectation value is found

$$\langle exp(-iHt/\hbar) \rangle = exp[-(at/\hbar)^2]. \quad (2.10)$$

This is not of particular interest, because, if one makes the replacement $it/\hbar \rightarrow 1/(kT)$, then one gets a quadratic function of the inverse temperature $(kT)^{-2}$ in the exponent instead of $1/kT$ which is typical for statistical mechanics. The result $(kT)^{-2}$ does not agree with the Boltzmann distribution form.

If, instead, one uses the Lorentzian distribution,

$$P(H) = \frac{1}{2a\pi} \frac{1}{1 + H^2/4a^2} \quad \text{(Lorentzian ensemble)}, \quad (2.11)$$

then one gets the very interesting result

$$\langle exp(-iHt/\hbar) \rangle = exp[-2a|t|/\hbar]. \quad (2.12)$$

By identifying $2a$ with the imaginary part of the interaction potential in the Hamiltonian H one gets the decay intensity of the averaged probability amplitude

$$|\langle exp(-iHt/\hbar) \rangle|^2 = exp[-2W|t|/\hbar] \quad (2.13)$$

and the mean life time of the corresponding state

$$\tau = \hbar/2W. \quad (2.14)$$

The main target of the theory briefly reviewed is to find ways for introducing $it/\hbar \to 1/(kT)$ into the nuclear theory,and to obtain an expression for both the density matrix and the mean life time of the nuclear states.

The justification of the replacement $it/\hbar \to 1/(kT)$ is a major issue in Physics,because it solves the great problem of obtaining irreversibility in Thermodynamics from time reversal invariant equations governing the constituens of matter.

3 THE PHYSICAL CONNECTION BETWEEN THE LEVEL DENSITY DISTRIBUTION AND THE QUANTUM THEORY

The results of sect. 2 are,no doubt,extremely beautiful and mathematically very appealing.However,much effort of interpretation is required to elucidate the physical content of the various kinds of transformations by matrices needed to obtain $P(H)$.

In addition,the replacement $it \to -t$ or,equivalently,

$$it/\hbar \to 1/(kT) \tag{3.1}$$

of the real time by the temperature is a basic counter hypothesis to very important parts of Physics.

For example,if one introduces (3.1) to the Relativity,then obviously many nice and important established physical results are spoiled.We shall not elaborate further on the physical consequences and the general viability of (3.1).

On the other hand,since the results of the ensemble theory for the nuclear level density distribution are meanwhile very well established by nice experiments in many areas of Physics,there is little doubt about the correctness of $P(H)$ as mathematical representations of nuclear energy level densities.

The situation is,however,different with the physical relevance of their derivation.Because it is not obvious that there should be an intrinsic relationship between the Riemann space metric $\{g_{\mu\nu}\}$ and the energy level density distribution function of the atomic nucleus.

It is,therefore,of great interest to find a different way for establishing $P(H)$ or similar,more general expressions for

the level density distribution function which,however,would be in accordance with the basic physical theories as is,e.g., the Quantum Field Theory in Minkowski space.

A way to carry out this program was discovered recently[4] by developping a theory of Statistical Mechanics based on Quantum Field Theory associated with the *Generalized Random Fields*[5].

This new theory not only avoids the physically unclear replacement (3.1),but it also allows to introduce the:

i) $1/k_B T$ in the exponent,

ii)   *mean life time* of states,

iii) **level density** into the distribution function,

iv)   **temperature as a functional** of the interaction
      Hamiltonian.

This is done by deriving the Statistical Mechanics and the Thermodynamics of the nucleus from Quantum Field Theory[6]. Here are given only the main prerequisites for the derivation of the density matrix and the partition function from which the thermodynamics of the nucleus follows.

4 THE DESCRIPTION OF THE NUCLEUS EVOLUTION

The evolution operator as given in QFT is used to derive an evolution operator for the *Generalized Random Quantum Fields*[7]:

$$\mathbf{u}(t,t') = \mathcal{P} \begin{cases} exp[-\hbar^{-1}\int_{t'}^{t} d\tilde{t}H_I(\tilde{t})+\tilde{\Lambda}(q,1)], \textbf{ dissipative} \quad (a) \\ \\ exp[-i\hbar^{-1}\int_{t'}^{t} d\tilde{t}H_I(\tilde{t})+\tilde{\Lambda}(q,2)], \textbf{conservative} \quad (b), \end{cases} \quad (4.1)$$

where

$$\tilde{\Lambda}(q,\sigma) = \Lambda(q,\sigma) + \hbar^{-1}\sum_m H_o.[\tau_{mm-1} + \tau_{mm-1}^{(tr)}]$$
$$= \Lambda(n,\sigma) + \hbar^{-1}H_o.(t-t'),\sigma = 1,2. \quad (4.2)$$

and H_o is the **constant part** of the free field Hamiltonian plus any time-independent energy part equal to its value at the end of the last transition.$\Lambda(n,\sigma)$ takes the following values:

i) **dissipative transitions:**

$\Lambda(n,1)=\pi q$ with, $q=\dfrac{2n-1}{2}$, $n = 0,-2,-4,-6...$ **forwards in time**, (a)

$\Lambda(q,1)=\pi q$ with, $q = \dfrac{2n+1}{2}$, $n = 1,3,5,7...$ **backwards in time**, (b)

ii) **conservative transitions:** (4.3)

$\Lambda(q,2)=\pi q$ with, $q = n$, $n= 0,-2,-4,-6...$ **forwards in time**, (c)

$\Lambda(q,2)=\pi q$ with, $q = n$, $n= 1,3,5,7,9 ...$ **backwards in time**, (d)

where the choice of the signs insures energy renormalization.

The *statistical evolution operator* can be obtained[8] from (4.1).The basic idea is that in order to make Statistical Mechanics one has not to average the state vector of the nucleus itself, but rather to average the time evolution by acting on the state vector by an appropriately averaged evolution operator.

This averaged operator, the *statistical evolution operator* $\mathfrak{X}$, is obtained from the *Generalized Random Quantum Field* evolution operator by geometrically averaging it:

$$\mathfrak{X}(t,t') = \Big[\mathfrak{U}(t_N,t_{N-1})...\mathfrak{U}(t_2,t_1)\mathfrak{U}(t_1,t')\Big]^{1/N} \qquad (4.4)$$

Here N is the number of transitions in the nucleus during the time interval $[t',t]$ with $t'<t_1<...<t_N=t$.

The so defined operator acts on $\Psi(t')$ in the following way:
$\mathfrak{X}_s\Psi(t') = \Psi_A(t)$

$$=\sum_{\alpha=0}^{A'} \frac{1}{\sqrt{\alpha!}} \, e^{-\hat{s}} \prod_{\nu=1}^{\alpha} \int dk_\nu^3 \tilde{c}_m^{\alpha)}(\vec{k}_1,...\vec{k}_\alpha;N)a^+(\vec{k}_\nu)|0\rangle, \qquad (4.5)$$

where

$$\hat{s} = -\mathcal{P}\sum_{\lambda=1}^{N}[\int H_I(t)dt \, / \, \hbar + \tilde{\lambda}^{(\alpha)}]/N. \qquad (4.6)$$

This becomes under the trace operation

$$tr \, \hat{s} = -\sum_{\lambda=1}[H_{Imm-1}^{(\alpha)} \cdot \tau_{mm-1}^{tr(\alpha)}/ \, \hbar + \tilde{\lambda}^{(\alpha)}]/N.$$

and $H_I(t)$ is the **minimal time dependent part** of the interaction Hamiltonian varying inside the time interval of the λ-th transition. The energy eigenvalues included in $\tilde{\lambda}$ are calculated using the **time independent** Schrödinger equation

$$H(\lambda)\tilde{c}_m^{(\alpha)}(\lambda) = E_{mn}^{(\alpha)}(\lambda)\tilde{c}_m^{(\alpha)}(\lambda),$$

where $H(\lambda)$ equals **the maximal constant part** of $H(t)$ for $t\in[t+t_\lambda, \ t+t_\lambda+\tau^{tr(\alpha)}]$.

The transition time $\tau_m^{tr(\alpha)}$ within the α-th nucleon sub-set (e.g.,shell) of the nucleus according to the particle partition in the second-quantisation state vector is found from the **time dependent** Schrödinger equation

$$i\hbar\partial_t \tilde{c}^{(\alpha)}(t) = H(t)\tilde{c}^{(\alpha)}(t), \quad t\in[t+t_\lambda, \ t+t_\lambda+\tau_m^{tr(\alpha)}]$$

by calculating the **transition rate** per unit time

$$\mathbf{w} = 2\pi\hbar^{-1}|c_m^{(\alpha)}(\lambda), \mathcal{T}c_{m-1}^{(\alpha)}(\lambda)|^2\rho_f(E_m) \tag{4.7}$$

Here λ enumerates the transitions in the nucleus, the dependence of m and α on λ is not indicated and $\mathcal{T}$ is the transition matrix[9]. λ disappears in the final results due to $\mathfrak{X}_s$.

The state vector (4.5) describes the time behavior of a set of **hypothetical nucleons one for each α-sub-set averaging the behavior** of the real nucleons in the nucleus.

This form of the state vector will be used to construct the density matrix for both equilibrium and non-equilibrium states of the nucleus.

5 THE NUCLEAR LEVEL DENSITY DISTRIBUTION

We shall now construct the density matrix for both the non-equilibrium and for the equilibrium states. This would require according to the conventional approach, the *ad hoc* introduction as a rule, of a set of probability factors $\{w_n\}$[10], which are, borrowed from the Gibbs or Boltzmann theories[11].

In the present case these factors follow directly from the evolution operator making the theory self-contained.

If we wish to obtain the expression for the density matrix in dependence of the level density we have to consider the nucleus going through a set of states undergoing transitions till it reaches the equilibrium state. This is done by $\mathfrak{X}_s$.

Therefore, the state vector[12] $|\Psi_{\langle m\rangle}(x;t)\rangle$ is considered under the action of $\mathfrak{X}_s$, where $x = \{x_1, x_2, \ldots x_A\}$ and $\{m\}$ is the set of quantum numbers respectively over which the A particles are partitioned.

The nucleus state vector at the initial time is written as

$$|\Psi(x;t')\rangle = |\Psi_{\langle m\rangle}(t')\rangle \tag{5.1}$$

with[12]

$$|\Psi_{\langle m \rangle}(t)\rangle = \sum_{\alpha=1}^{A'} (-1)^{P(\alpha)} P |\Psi_{\langle m \rangle}^{(\alpha)}(t)\rangle,$$

where P is the permutation operator and

$$|\Psi_{\langle m \rangle}^{(\alpha)}\rangle = \frac{1}{\sqrt{\alpha!}} \prod_{\nu=1}^{\alpha} \int dk_{\nu}^{3} c_{m}^{(\alpha)}(\vec{k}_1,\ldots\vec{k}_\alpha; t_\lambda) a^{+}(\vec{k}_\nu)|0\rangle.$$

or $|\Psi_{\langle m \rangle}^{(\alpha)}\rangle = c_{\langle m \rangle}^{(\alpha)}(t)\phi_{m_1}(x_1)\ldots\phi_{m_\alpha}(x_\alpha)$ and $\{\phi(x_i)\}$ one particle
wave functions. It is clear that the quantum number m differs
for different values of the particle number α.

Next, the Schwinger operator $\mathbb{W}(x,x')$ is introduced in the
Schrödinger picture by

$$\mathbb{W}(x,x';t,t') = \sum_{m \in \Omega(\lambda)} |\Psi_{m}(x';t')\rangle\langle\Psi_{m'}(x;t)|, m' \in \Omega(\lambda) \quad (5.2)$$

The dissipative action $\mathfrak{T}_s^d$ of $\mathfrak{T}_s$ on $\mathbb{W}(x,x';t,t')$ is required,
because the conservative $\mathfrak{T}_s^c$ would conserve the energy, and con-
stant energy transitions do not contribute, strictly speaking,
to the evolution of anything. Under the trace operation[11] we get

$$tr\,\mathfrak{T}_s^d = \sum_{m \in \Omega(\lambda)} \sum_{\alpha} exp[-\hbar^{-1}\bar{E}_{mm-1}^{(\alpha)} \bar{\tau}_{mm-1}^{tr(\alpha)} - \tilde{\lambda}_m]. \quad (5.3)$$

Λ takes values according to (4.3a), m' is any of the eigenvalue
quantum numbers of $H(\lambda)$, and

$$\bar{E}_{\Omega(\lambda)}^{(\alpha)} = \Big(\sum_{m \in \Omega(\lambda)} \tau_{mn}^{tr(\alpha)}\Big)^{-1} \sum_{m \in \Omega(\lambda)} E_{mn}^{(\alpha)} \tau_{mn}^{tr(\alpha)}, \quad (5.4)$$

$$\bar{\tau}_{\Omega(\lambda)}^{tr(\alpha)} = N^{-1} \sum_{m \in \Omega(\lambda)} \tau_{mn}^{tr(\alpha)}. \quad (5.5)$$

The set $\Omega(\lambda)$ contains the part of the energy spectrum taken
by the system during the time $t'+t_\lambda$, and the value of the index
s corresponds to $\Omega(\lambda)$.

To make clear the meaning of $\bar{E}_{\Omega(\lambda)}^{(\alpha)}$ and $\bar{\tau}_{\Omega(\lambda)}^{tr(\alpha)}$ it is observed
that, if the system is in a stationary state, then

$$\bar{E}_{\Omega(\lambda)}^{(\alpha)} \rightarrow E_{m}^{(\alpha)} \quad and \quad \bar{\tau}_{\Omega(\lambda)}^{tr(\alpha)} \rightarrow \tau_{mn}^{tr(\alpha)}, \quad (5.6)$$

and the average energy over the transition times of each mem-
ber of the particle sub-set α equals eventually the energy
taken by it in the asymptotic state. Similarly, the average
transition time equals the asymptotic transition time.

The density matrix $\rho(x,x';t)$, is obtained by acting by the
statistical evolution operator, $\mathfrak{T}_s^d$, on the Schwinger operator,

$$\rho(x,x';t) = \mathfrak{Z}^{-1} . \mathfrak{T}_s^d . \mathbb{W}(x,x';t,t). \quad (5.7)$$

By using (5.3) it follows that

$$tr\ \Sigma_s^d \mathbb{B}(x,x';t,t') = tr \sum_{m\in\Omega} \sum_\alpha \Sigma_s^d |\Psi_m^{(\alpha)}(x';t')\rangle\langle\Psi_{m'}^{(\alpha)}(x;t)|$$

$$= \sum_{m\in\Omega(\lambda)} \sum_\alpha \chi(\alpha,m) exp[-\hbar^{-1}\bar{E}_{mm-1}^{(\alpha)}\ \bar{\tau}_{mm-1}^{tr(\alpha)} - \tilde{\lambda}_m] \qquad (5.8)$$

where $\chi(\alpha,m) = |\langle\Psi_m^{(\alpha)}(x,t)|\Psi_m^{(\alpha)}(x,t)\rangle|^2$

In (5.8) the probability factor $exp[-\hbar^{-1}\bar{E}_{mm-1}^{(\alpha)}\ \bar{\tau}_{mm-1}^{tr(\alpha)} - \tilde{\lambda}_m]$ is **automatically renormalized**, since from the exponent $\overline{\bar{E}_{mm-1}^{(\alpha)}\ \bar{\tau}_{mm-1}^{tr(\alpha)}}$ $+\langle H_o\rangle_{mm-1}^{(\alpha)}[\overline{\bar{\tau}_{mm-1} + \tau_{mm-1}^{(tr)}}]$ the term $\pi\hbar.q$ is **subtracted** according to (4.3a) for transitions forward in time.

Now, from (5.8) it is easy to get the energy level density distribution. To this end we find from the reaction theory the transition time $\tau_{mm-1}^{tr(\alpha)}$ as a functional of the interaction energy according to (4.7)

$$\bar{\tau}_{mm-1}^{tr(\alpha)} = \left(2\pi\hbar^{-1}\overline{|c_m^{(\alpha)}(\lambda),\mathcal{J}c_{m-1}^{(\alpha)}(\lambda)|}^2\rho_f(E_m)\right)^{-1}. \qquad (5.9)$$

From (5.8) and (5.9) one finds $3[\rho_f(E)]$ from the density matrix which is the partition function of the nucleus as a function of the nuclear energy level density per unit energy:

$$3[\rho(E)] = tr\ \rho(x,x',t) = tr\Sigma_s^d\mathbb{B}(x,x';t,t) =$$

$$\sum_{m\in\Omega} \sum_\alpha \chi(\alpha,m).exp\left[-\frac{\bar{E}_{mm-1}^{(\alpha)}}{2\pi\ \overline{|c_m^{(\alpha)}(\lambda),\mathcal{J}c_{m-1}^{(\alpha)}(\lambda)|}^2\rho_f(E_m)} - \tilde{\lambda}_m\right] \qquad (5.10)$$

Here $\overline{(c_m^{(\alpha)}(\lambda),\mathcal{J}c_{m-1}^{(\alpha)}(\lambda))}$ is the time averaged matrix element of the perturbation corresponding to the λ-th transition associated with the state m.

Expression (5.10) is the sought **level density distribution** function. Comparison of (2.6)-(2.11) with expression (5.10) shows that the latter is just the energy dependence which follows from the Lorentz ensemble.

6 THE NUCLEAR PARTITION FUNCTION

The nuclear *partition function* is obtained as the normalizing factor 3 of the density matrix $tr\ \rho(x,x';t) = 1$

$$3 = tr\ \Sigma_s^d.\mathbb{B}(x,x';t,t). \qquad (6.1)$$

From (5.2),(5.3),(5.8) and from

$$\langle \Psi_{m'}^{(\alpha)}(x;t') | \psi_m^{(\alpha)}(x;t') \rangle = \delta_{\alpha\alpha} . \delta_{mm} . \sqrt{\chi(\alpha,m)} \quad m',m \in \Omega(\lambda) \qquad (6.2)$$

it follows that

$$tr \, \rho(x,x';t) = \mathfrak{Z}^{-1} \sum_{m\in\Omega(\lambda)} \sum_{\alpha=0}^{A} \chi(\alpha,m) exp[-\hbar^{-1}\bar{E}_{mm-1}^{(\alpha)} . \bar{\tau}_{mm-1}^{tr(\alpha)} - \tilde{\lambda}].$$

From (5.9) and from the normalization $tr \, \rho(x,x';t) = 1$ it follows for the **thermodynamic partition function** $\mathfrak{Z}(T)$

$$\mathfrak{Z}(T) = \sum_{m\in\Omega(\lambda)} \sum_{\alpha=0}^{A} \chi(\alpha,m) . exp\{-\bar{E}_{mm-1}^{(\alpha)}/(k_B T_{mm-1}^{tr(\alpha)}) - \tilde{\lambda}]\}, \qquad (6.3)$$

where

$$T_{mm-1}^{tr(\alpha)} = \hbar\left[\overline{\tau_{mm-1}^{tr(\alpha)} k_B}\right]^{-1}$$

$$= \left(2\pi k_B |\overline{c_m^{(\alpha)}(\lambda), \mathcal{J}c_{m-1}^{(\alpha)}(\lambda)}|^2 \rho_f(E_m)\right)^{-1} \quad (6.4)$$

is the temperature holding in the α-th sub-set of the nucleus during the transition from the state $m-1$ to m. The global nuclear temperature follows if we define a **temperature operator** $\hat{T}$ by

$$tr \, \hat{T}\rho = \mathfrak{Z}^{-1} \sum_{m\in\Omega(\lambda)} \sum_{\alpha=0}^{A} \chi(\alpha,m) . T_{mm-1}^{tr(\alpha)} exp\{-\bar{E}_{mm-1}^{(\alpha)}/(k_B T_{mm-1}^{tr(\alpha)}) - \tilde{\lambda}]\},$$

which, assuming that $T_{mm-1}^{tr(\alpha)} = T$ for all α and m, becomes $tr\hat{T}\rho = T$.

Clearly, the *partition function* of the evolving nucleus within the set of states $\Omega(\lambda)$, describes a non-equilibrium state, if $\Omega(\lambda)$ changes with time. In any case, the partition function involves the actually occupied set of states by the nucleus.

7 DISCUSSION AND CONCLUSIONS

Based on the method of the *Generalized Random Quantum Fields* developed recently the density matrix of the nucleus has been derived. From this the nuclear level density distribution of the nuclear levels has been deduced.

The stochastic behavior of the nucleus follows naturally from the assumption that the Lagrangian density of the field is a generalized, infinitely divisible, random field.

A new form of the evolution operator has been derived which may be conservative or dissipative depending on the kind of postulated quantization of the field action.

It is worth observing that both the conservative and the dissipative evolution operators are defined in the Minkowski space and the Wick rotation is not required for going over to the Eucledian metric. Irreversibility follows automatically.

This made it possible to derive the density matrix and the partition function in the framework of the quantum field theory in the Minkowski space.

Thus, the nuclear temperature is defined as a functional of the nuclear perturbation Hamiltonian.

The density matrix and the partition function follow in a unified way by application of the evolution operator on the second-quantization state vector of the system.

REFERENCES

1 All the mentioned references and many others are contained in C.E.Porter,Statistical Theory of Spectra: Fluctuations, (Academic,New York,1965)

2 O.Bohigas,R.O.Haq,and A.Pandey,Nuclear Data for Science and Technology.Ed.Boeckhoff, (Reidel,Dodrecht,1983),p 809

3 O.Bohigas,and H.A.Weidenmüller,Ann.Nucl.Part.Sci.38(1988) 421

4 C.Syros,Modern Physics Letters,B4(1990)1089

5 I.M.Gel'fand,and N.Ya.Vilenkin,Generalized Functions,Vol 4, (Academic,New York,1964),p 283

6 C.Syros,Praktika Akadimias Athinon,1991.

7 C.Syros,Int.J.of Modern Phys.B,Vol.5 (1991)2909-2934

8 C.Itzykson,and J.B.Zuber,Quantum Field Theory,(MacGraw-Hill, New York,1985),p 176

9 M.L.Goldberger and K.M.Watson,Collision Theory ,R.E.Krieger Publishing Company,N.Y.,1975,p.177

10 N.N.Bogolubov,Lectures on Quantum Statistics Vol I, (Mac-Donald Technical and Scientific,London,1967),p 6

11 L.P.Kadanoff and G.Baym,Quantum Statistical Mechanics Addison-Wesley Publishing Company,Redwood City,Calif.1989,p.2

12 N.N.Bogolubov,ibid.pp 46