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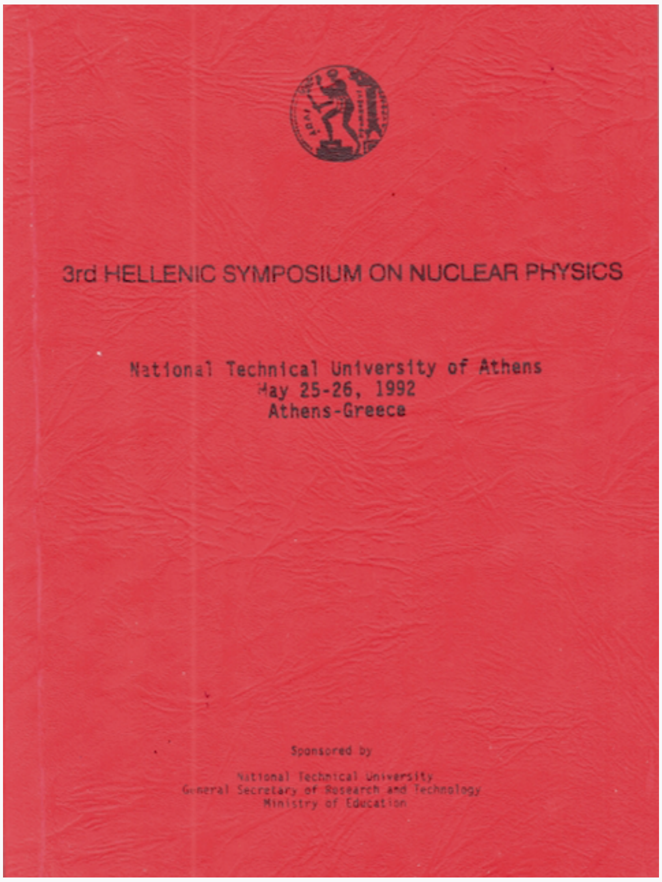

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THE INFLUENCE OF THE NUCLEAR SURFACE DIFFUSENESS ON THE TRANSITION CHARGE DENSITIES.*

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Abstract

Proton partial occupancies of the nuclear surface orbits are used in a modified shell model approach to study isoscalar dipole transition charge densities and form factors for self-conjugate nuclei. The energy-weighted sum-rules of Harakeh-Dieperink for both the transition form factor and transition charge density are modified so as fractional occupation probabilities of the states may be used. The partial occupancies of the surface nlj-levels are determined by fitting to the experimental inelastic scattering data and compared with those found previously in the study of nuclear ground state properties.

1. Introduction

The experimental data in the experiments of inelastic electron, proton and α-particle scattering by nuclei have shown the existence of various giant multipole states in low energies. They have been considered very important, since they are related directly to important quantities such as the effective charge of El transitions. Among them the isoscalar dipole resonances in self-conjugate nuclei at low energies ($1^-, T = 0$ states) have received a special theoretical [1-4] and experimental [5-6] interest since they can be very helpful to clarify the excitation mechanism of these nuclei. Though electric dipole resonances are forbidden by isospin in such nuclei ($T_3 = 0, \Delta T = 0$), it is now very well known that they have been observed in some nuclei like ^{16}O ($E_x = 7.118 MeV$), ^{40}Ca ($E_x = 6.95 MeV$) and others [7-11].

Theoretically, transition charge densities are studied by using nuclear models and sum-rule methods. Particularly the sum rules have played an important role in the analysis of the inelastic electron and hadron scattering by nuclei. In ref. [1] Harakeh

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and Dieperink obtained a sum rule for the form factor describing the isoscalar dipole excitations caused by the hadron-nucleus scattering and which can be interpreted in the long-wavelength limit by means of isospin admixures to the ground state or to the $1^-, T = 0$ states [11,12].

In the mean field approximation to the nuclear many-body problem it is assumed that shells are occupied up to the Fermi level and empty above it and this is a fairly successful model in describing ground state charge densities and in providing the basis for shell model calculations at low excitation energy. However, the basic assumption of closed shells is an approximation only which is not completely supported by recent experiments even for "core nuclei" like ^{40}Ca. One should add dynamical corrections to the static mean field in order to account for the nucleon-nucleon correlations which are predicted to diffuse the Fermi surface and induce a depletion of states below the Fermi level [13-15]. These correlations may in part be understood in terms of partial occupancy of the surface orbits [10,14,15]. It is also given the fact that many nuclear properties do depend on the occupancy of the valence shells. Though absolute occupation probabilities can be determined from the spectral functions obtained from $(e, e'p)$ experiments e.g. at NIKHEF [16], sum rule methods are also used.

In ref. [15], it is given a compact analytical expression for the ground state charge distribution with the aid of which it is constructed a general method such that fractional occupation probabilities for the (surface) orbits can be used. Also compact analytical expressions are obtained by using a Gaussian folding over the point nucleon distribution expressed in terms of spherically symmetric harmonic oscillator wavefunctions. The purpose of the present work is to study the transition charge density $\rho_1(r)$ and the form factor $F_1(q)$ for the $1^-, T = 0$ states of self-conjugate nuclei by using this method. For this aim the isoscalar dipole sum rules of Harakeh and Dieperink [1] are modified such that fractional occupation probabilities of the states may be included. Afterwards, the resulting sum rules are applied in order to determine the properties of the giant dipole state of ^{40}Ca at energy $E_x = 6.95 MeV$ by assuming that the sum rule is saturated by this single collective state.

2. The sum-rule approach

Up to now there have been mainly used three sum-rule methods for the transition charge density of the isoscalar dipole resonances: (i) the difuseness oscillation model [7], (ii) the scaling model [3] and (iii) the doorway dominance model [1,8]. They are all based on spherical ground state densities. In this section we shall present in brief the formalism of the third method first derived by Deal [8] and later extended by Harakeh and Dieperink [1] in order to take into account the center of masss corrections.

2.1 DIPOLE TRANSITION CHARGE DENSITY SUM-RULE

It is well known that the transition of a nucleus from the ground state to an excited state is accompanied by a change in the charge density which is dependend on the structure of the excited state and is described by the nondiagonal matrix element of the charge density operator $\hat{\rho}(\mathbf{r})$. The diagonal matrix element give the ground state charge density. For isoscalar dipole resonances the transition charge density in the sum rule approach is derived with the isoscalar dipole transition operator

$$P^{(1)}(\hat{\mathbf{r}}) = \sum_{i=1}^{A} [r_i^3 - \eta r_i] Y_0^1(\hat{\mathbf{r}}_i) \tag{1}$$

where η is determined by the translational invariance condition

$$\int_0^\infty \rho^{(1)}(r) r^3 dr = 0 \tag{2}$$

This gives $\eta = \frac{5}{3} < r^2 >$ with $< r^2 >$ being the mean square radius of the nucleus in question. The operator $P^{(1)}$ results from the general isoscalar multipole spin independent transition operator $O^{(\lambda)}(q) = \sum_{i=1}^{A} j_\lambda(qr_i) Y^\lambda(\hat{r}_i)$ in the limit of low momentum transfer $q \to 0$. Then the first terms in the expansion of $O^{(\lambda)}$ survive and are proportional to the electric spin independent transition operators $E_\lambda = \sum_{i=1}^{A} r_i^\lambda Y^\lambda(\hat{r}_i)$. In the case of $\lambda = 1$, however, the leading order transition operator is due to the second term in the expansion in qr, but the center of mass (c.m.) motion, proportional to the first term, has to be treated as Harakeh-Dieperink [1] pointed out. Then the transition charge density for an isoscalar dipole state is given by

$$\rho_1(r) = -\frac{\beta_1}{R\sqrt{3}} \Big[3r^2 \frac{d}{dr} + 10r - \frac{5}{3} < r^2 > \frac{d}{dr} + \epsilon \Big(r \frac{d^2}{dr^2} + 4\frac{d}{dr} \Big) \Big] \rho_0(r) \tag{3}$$

where $\rho_0(r)$ is the ground state charge distribution and $R = 1.07A^{1/3}$, is the half density radius of the Fermi mass distribution. The quantity ϵ, significant only for $A \leq 20$, is given by

$$\epsilon = \frac{\hbar^2}{3mA} \Big(\frac{4}{E_2} + \frac{5}{E_0} \Big) \tag{4}$$

where E_0 and E_2 are the monopole and quadrupole resonances, respectively, with energies $E_0 \approx 80.A^{-1/3}MeV$ and $E_2 \approx 63.A^{-1/3}MeV$. The magnidute of β_1 in eq. (3) [1] is fixed by the requirement of the dipole state to exhaust the static isoscalar dipole energy weighted sum rule (EWSR) obtained in the limit $q \to 0$ which is given by

$$m_1 = \frac{\hbar^2 A}{32m\pi}\Big(11 < r^4 > -\frac{25}{3} < r^2 >^2 -10\epsilon< r^2 > \Big) \tag{5}$$

In eqs. (3) and (5) $< r^m >$, m=2,4, represent the mean charge radii of order 2 and 4, respectively (see below).

2.2 DIPOLE TRANSITION FORM FACTOR SUM-RULE

The Fourier transform of eq. (3) leads to the c.m. adjusted sum rule in q space i.e. to the dipole transition form factor

$$F_1(q) = \frac{\beta_1}{2\sqrt{\pi}R}\Big[3q^2\frac{d}{dq}\frac{1}{q}\frac{d}{dq} + 5\frac{d}{dq} + \frac{5}{3} < r^2 > q + \epsilon q^2 \frac{d}{dq}\Big) F_{el}(q) \tag{6}$$

where $F_{el}(q)$ is the elastic form factor.

2.3 INDEPENDENT PARTICLE SHELL MODEL SUM-RULES

The ground state properties $\rho_0(r)$ and $F_{el}(q)$ needed in eqs. (3) and (6), can receive compact analytical forms [15] for harmonic oscillator wavefunctions in closed (sub)shell nuclei, even in the case when nucleon finite size and c.m. motion are taken into account. In this section we will use these expressions in order to obtain tractable analytical forms for the isoscalar dipole sum rules of eqs. (3) and (6).

For point-proton we have written [15] the distribution $\rho_0(r)$ as

$$\rho_0(r) = \frac{1}{\pi^{3/2}b^3} e^{-(r/b)^2}\, \Pi(\frac{r}{b}, Z) \tag{7}$$

where $\Pi(\chi, Z)$ is a polynomial of even powers in χ with simple coefficients (b is the h.o. parameter). The corresponding elastic form factor is obtained by a similar expression as

$$F_{el}(q^2) = \frac{1}{Z} e^{-(qb)^2/4}\, \Phi(qb, Z) \tag{8}$$

with $\Phi(\chi, Z)$ being also an even polynomial in χ similar to $\Pi(\chi, Z)$ but deffering in the values of the coefficients [15]. The mean radii $< r^m >$ obtained by using eq. (7) can be cast in the form

$$< r^m > = b^2 \frac{S_m}{Z} \tag{9}$$

where the sums S_2 and S_4 for some closed (sub)shell nuclei up to Z=50 are the integer and seminteger numbers given in table 1. By using eqs. (7)-(9) the transition dipole charge distribution of eq. (3) can be written as

$$\rho_1(r) = -\frac{\beta_1 e^{-\chi^2}}{\sqrt{3\pi^3}b^2R}\left\{(-6+4\epsilon')\chi^3 + 10\left[1+\frac{1}{3}\frac{S_2(Z)}{Z} - \epsilon'\right]\chi + \right.$$

$$\left.\left[(3-4\epsilon')\chi^2 - \frac{5}{3}\frac{S_2(Z)}{Z} + 4\epsilon'\right]\frac{d}{d\chi} + \epsilon'\chi\frac{d^2}{d\chi^2}\right\}\Pi(\chi, Z), \quad (\chi = r/b) \tag{10}$$

where $\epsilon' = \epsilon/b^2$. Similarly, for the dipole form factor of eq. (6) we obtain

$$F_1(q) = -\frac{\beta_1 b\, e^{-\chi^2/4}}{2\sqrt{\pi}RZ}\left\{\frac{3}{4}\chi^3 - \frac{\epsilon'}{2}\chi^2 + \frac{10S_2(Z) - 15Z}{6Z} + \right.$$

$$\left.[(\epsilon' - 3)\chi^2 + 3\chi + 2]\frac{d}{d\chi} + 3\chi\frac{d^2}{d\chi^2}\right\}\Phi(\chi, Z), \qquad (\chi = qb) \tag{11}$$

Z (N)	Upper j-level	$S_2(Z)$	$S_4(Z)$
2	$0s_{1/2}$	3	7.5
6	$0p_{3/2}$	13	42.5
8	$0p_{1/2}$	18	60.0
14	$0d_{5/2}$	39	154.5
16	$1s_{1/2}$	46	192.0
20	$0d_{3/2}$	60	255.0
28	$0f_{7/2}$	96	453.0
32	$1p_{3/2}$	114	572.0
38	$0f_{5/2}$	141	720.5
40	$1p_{1/2}$	150	780.0
50	$0g_{9/2}$	205	1137.5

Table 1. The sums $S_m(Z)$, m=2,4 giving the moments $< r^m >$ by means of eq.(9).

In obtaining eqs. (10) and (11) we have assumed that the occupation probabilities of the states are unity for states below the Fermi level and zero above it. In the next section we will modify these equations such that the more realistic fractional occupation probabilities for some (mainly surface) levels may be used. In this way we include to some extent configuration mixing and surface correlations in the sum rule description of eqs. (10) and (11).

4. Isoscalar dipole transition sum rules with partial occupancy

By including partial occupation probabilities in eqs. (5), (10) and (11), the quantities $F_{el}(q)$, $\rho_0(r)$ and $< r^m >$ change significantly [10,15] and this causes a change in $F_1(q)$ and $\rho_1(r)$ as well. In order to study this influence we insert in eqs. (10) and (11) expressions for $F_{el}(q)$, $\rho_0(r)$ and $S_m(Z)$ which take into account the diffuseness of the surface of the nucleus under study. We mention that the dipole vibrations, in contrast to the multipole states with $l \geq 2$, encombass almost all the nucleus and they can be affected by the surface diffuseness [10].

The insertion of partial occupation probabilities in the ground state charge distribution $\rho_0(r)$ starts by writing the average ground state density $\rho(r)$ even when there are not closed (sub)shells as

$$\rho(r) = \frac{1}{4\pi} \sum_{(n,l)j} (2j+1)\, a_{nlj} \mid R_{nlj}(r) \mid^2 \tag{12}$$

where a_{nlj} are the proton occupation probabilities for the orbit characterized by the quantum numbers n,l,j. The sum in eq. (12) runs over all the the single particle states. For the "core orbits" below the Fermi level of the nucleus, $a_{nlj} = 1$, i.e. they are equal to the simple shell model predictions, but for the "active surface orbits", $0 < a_{nlj} < 1$.

For a spherically symetric nucleus using harmonic oscillator wavefunctions and assuming partial occupancy for some surface nlj-levels, eq. (12) can be written as

$$\rho(r, Z, a_i) = \frac{e^{-(r/b)^2}}{\pi^{3/2} b^3} \Pi(\frac{r}{b}, Z, a_i), \qquad \Pi(\chi, Z, a_i) = \sum_{\lambda=0}^{N_{space}} f_\lambda\, \chi^{2\lambda}, \tag{13}$$

where $N_{space} = (2n+l)_{max}$, the maximum oscillator quanta of the chosen model space and

$$f_\lambda = \sum_{(n,l)j,\ \lambda \geq l} a_{nlj}\, X^\lambda_{nlj}, \qquad X^\lambda_{nlj} = \frac{\pi^{\frac{1}{2}} n!(2j+1) C^{(\lambda-l)}_{nl}}{2\Gamma(n+l+\frac{3}{2})} \tag{14}$$

($\Gamma(x)$ is the gamma function). The coefficients X^{λ}_{nlj} are simple rational numbers (C^n_n are defined in ref. [15]). Note that for a_{nlj} = 1,0, i.e for independent particle shell mode (IPSM) occupances, f_λ of eq. (14) reduce to those of eq. (3c) of ref. [15].

As an example, we consider partial occupancy for four surface nlj-levels, i.e. thre parametric occupation probabilities for $\rho(r, Z, a_i)$. Then $\Pi(\chi, Z, a_1, a_2, a_3)$ is written as

$$\Pi(\chi, Z, a_1, a_2, a_3) = \Pi(\chi, Z_2)\alpha_1 + \Pi(\chi, Z_1)(\alpha_2 - \alpha_1) +$$

$$\Pi(\chi, Z_c)\left[\frac{Z' - Z}{Z' - Z_c} - \alpha_2 - \alpha_3\right] + \Pi(\chi, Z')\left[\frac{Z - Z_c}{Z' - Z_c} + \alpha_3 - a_4\right] + \Pi(\chi, Z'')\, a_4 \quad (14$$

with $Z_2 < Z_1 < Z_c < Z' < Z''$, for the adjacent Z-closed levels ($Z_c \le Z < Z'$). The occupation probabilities for the four surface levels are: $a_1 = 1 - \alpha_1$, $a_2 = 1 - \alpha_2$, below the Fermi level and $a_3 = (Z - Z_c)/(Z' - Z_1) + \alpha_3$, a_4 for the levels above it, respectively The parameters a_i are not independent and one of them, e.g. a_4, is determined from the constraint, $\sum_{(n,l)j}(2j+1)a_{nlj} = Z$, which they obey. Similar expressions to that o eq. (14) hold also for $\Phi(\chi, a_i)$, giving the elastic form factor ($F_{el}(q, a_i)$) and $S_m(Z, a_i$ giving the mean radial moments ($< r^m >$).

The equations which describe $\rho_1(r)$ and $F_1(q)$ in terms of fractional occupation probabilities of the ground state orbits, are now easily obtained in the sum-rule method. For $A \ge 20$ they are

$$\rho_1(r, Z, a_i) = -\frac{\beta_1 e^{-\chi^2}}{\sqrt{3\pi^3} b^2 R}\left\{ -6\chi^3 + 10(1 + \frac{S_2(Z, a_i)}{3Z})\chi + \left(3\chi^2 - \frac{5}{3}\frac{S_2(Z, a_i)}{Z}\right)\frac{d}{dr}\right\}\Pi(\chi, Z, a_i), \qquad (\chi = \frac{r}{b}) \qquad (16)$$

and

$$F_1(q, Z, a_i) = -\frac{\beta_1 b\, e^{-\chi^2/4}}{2\sqrt{\pi} R Z}\left\{\frac{3}{4}\chi^3 + \frac{5}{3}\frac{S_2(Z, a_i)}{Z} - \frac{5}{2} + (-3\chi^2 + 3\chi + 2)\frac{d}{d\chi} + 3\chi\frac{d^2}{d\chi^2}\right\}\Phi(\chi, Z, a_i), \qquad (\chi = qb) \qquad (17)$$

In the next section we will apply eqs. (16) and (17) in order to study the effect of the diffused nuclear surface on $\rho_1(r)$ and $F_1(q)$ for the nucleus ^{40}Ca (for others see ref. [17]).

5. Results and discussion

As we have stressed before, the transition density of eqs. (10) and (16) is obtained from a sum-rule approach and for this reason is justified only for excitations that exaust a great percentage of the energy-weighted sum-rule. However, the use of sum rules in the description of states which constitute only small parts of the energy-weighted sum-rule, is fairly successful [1-3]. In the present work we study the transition charge density $\rho_1(r)$ and the form factor $F_1(q)$ for the $1^-, T = 0$ state of the self-conjugate nucleus ^{40}Ca at $E_x = 6.95 MeV$, by assuming that a fraction f of the isoscalar energy-weighted sum rule is saturated by this single collective state. We also assumed that the surface nucleons for ^{40}Ca are spread on four subshells: $1s_{\frac{1}{2}}$, $0d_{\frac{3}{2}}$, $0f_{\frac{7}{2}}$ and $1p_{\frac{3}{2}}$ with fractional occupation probabilities a_1, a_2, a_3 and a_4, respectively.

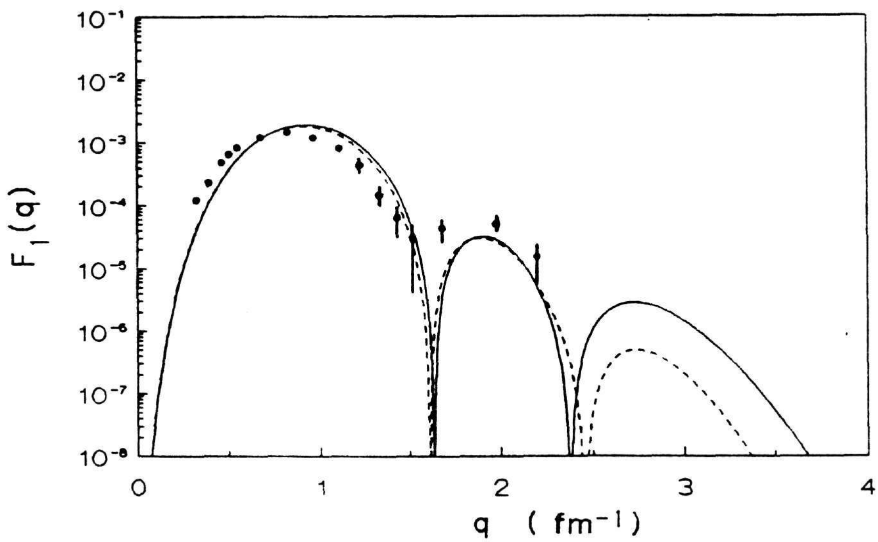

Fig. 1. Transition form factor $F_1(q)$ for the 1^- state of ^{40}Ca. Solid line: IPSM calculations and dashed line: present method with partial occupation probabilities ($a_1 = .70$, $a_2 = .60$, $a_3 = .088$, $a_4 = .150$). Experimental data come from ref. [6].

The values for a_i are determined by fitting eq. (17), see fig. 1, into the experimental dipole charge form factor data [6]. They correspond to a depletion of $\approx 7\%$ of the nuclear Fermi sea for ^{40}Ca. The fit to the data is satisfactory with a portion $f \approx 18\%$ of the isoscalar dipole EWSR exausted by the 1^- state at $6.95 MeV$ for ^{40}Ca. Deal [8] calculated for this state a model independent upper and lower limit of the fraction f to be $6\% \leq f \leq 14\%$. Other authors [4] found $f = 20\%$. Our percentage is in good agreement with these limits.

In table 2 the results for the static energy weighted sum rule m_1 and the corresponding coupling parameter β_1 of the IPSM and by using partial occupation probabilities, are presented. The h.o. parameter is fixed in the present method by the condition that $< r^2 >^{1/2}$ is equal to the experimental value [15].

Model	E_x(MeV)	$< r^2 >^{\frac{1}{2}} (fm)$	$< r^4 >^{\frac{1}{4}} (fm)$	$m_1(e^2fm^6MeV)$	β_1
IPSM	6.95	3.438	3.751	65695	.1945
Present	6.95	3.478	3.804	70267	.1881

Table 2. Specific parameters resulting for ^{40}Ca in the IPSM and present work.

In fig. 2 the dipole charge distribution resulting from eq. (16) with the values of a_i given iin fig. 1 is also shown

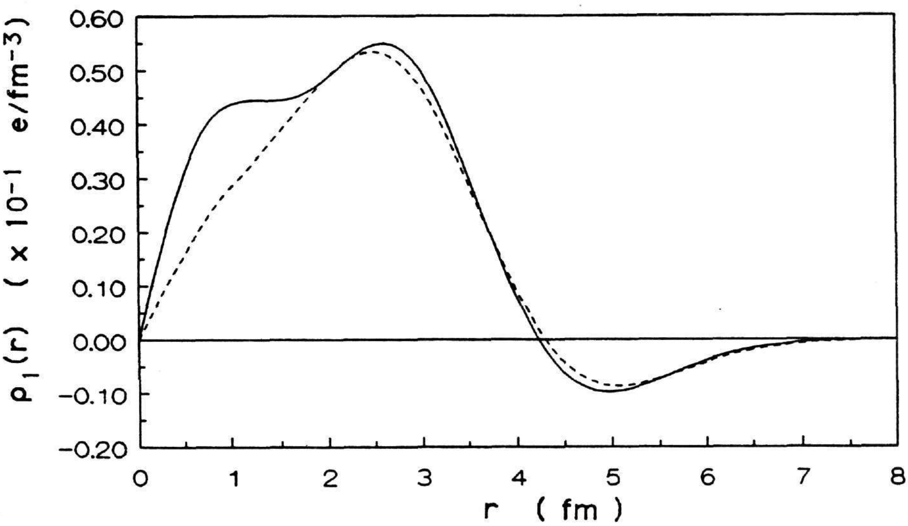

Fig. 2. Transition charge density for the $E1$ ($0 \rightarrow 1^-$) transition in ^{40}Ca nucleus. See explanations of fig. 1.

From figs. (1) and (2) we see that, the use of partial occupancy for the surface states, changes significantly the dipole charge distribution for $r \leq 3fm$ and the dipole form factor for large momentum transfer $q \geq 2fm^{-1}$ and that this change improves the reproducibility of the experimental data. This fact supports the argument of the existence of a diffused Fermi surface for the "core nucleus" ^{40}Ca result consistent with that found for the corresponding ground state properties in this nucleus [10,14,15].

6. Conclusions

In this work we have constructed tractable analytic expressions for isoscalar dipole energy weighted sume-rules based on the Harakeh-Dieperink method and the harmonic oscillator shell model with partial occupancy (a_{nlj}) of the states. For ^{40}Ca have determined a_{nlj} by fitting to the form factor of 1^- state and compared them with those found recently by fitting to the experimental elastic form factor data. We conclude that the surface of ^{40}Ca appears to be diffused and that a more or less unique determination of the occupation probabilities from ground state and excited state properties exists.

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