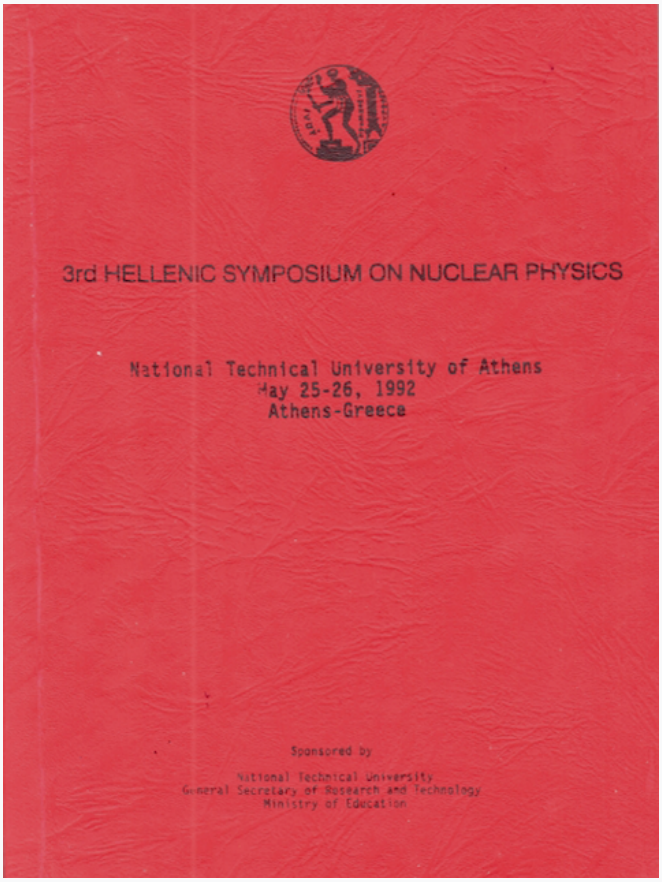
HNPS Advances in Nuclear Physics

Vol 3 (1992)

HNPS1992



The harmonic oscillator energy level spacing for neutrons and protons in nuclei

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doi: 10.12681/hnps.2375

The harmonic oscillator energy level spacing for neutrons and protons in nuclei *

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Abstract

Approximate expressions of $\hbar\omega$ for neutrons and protons separately, as functions of the neutron number N and the proton number Z respectively, are derived. The dependence of $\hbar\omega_n(\hbar\omega_p)$ on N(Z) is established using a rather recently proposed semi-phenomenological density distribution based on the separation energies of the last neutron or proton. The corresponding curves of $\hbar\omega$ show "discontinuities in the slope" at the closed shells throughout the periodic table. The difference $\hbar\omega_n - \hbar\omega_\Lambda$ is also discussed.

1. Introduction

The harmonic oscillator energy level spacing for nucleons $\hbar\omega_N$ as function of the mass number A of nuclei is given by the well known formula $\hbar\omega_N = fA^{-1/3}$, where $f = \frac{5}{4}(\frac{\hbar^2}{m_N r_o^2})(\frac{3}{2})^{1/3} \simeq 41MeV$ ($r_o \simeq 1.2fm$) [1,2]. Various modifications of the above expression have been proposed [3-6]. More recently Daskaloyannis et al. [7] studied in detail the variation of $\hbar\omega_N$ with A for nucleons (i.e without distiguishing neutrons from protons) and obtained improved approximate expressions. The most important point was that those expressions show "discontinuities in the slope" in the curve of $\hbar\omega_N$ as function of A at the doubly closed shells (A=16 and 40). The authors of ref. 7 removed some approximations made in other previous works: they considered the possibility of nuclei with valence nucleons as in [8] without making any approximation in relating the number of the highest filled shell K to the mass number (see also [9]). The centre of mass and finite size corrections were also taken into account in the usual way.

* Presented by C.P. Panos

The purpose of the present paper is to give approximate expressions for $\hbar\omega_n$, $\hbar\omega_p$ for neutrons and protons independently, using theoretical values R_n^{rms}, R_p^{rms} for the rms radii of neutrons and protons respectively, determined from a recently proposed semi-phenomenological density distribution by Gambhir and Patil [10]. This density is related to experiment by means of the separation energy of the last neutron (proton) while the detailed investigation of $\hbar\omega_N$ for nucleons of [7] was based on the Fermi density distribution with parameters determined by fitting to the experimental values of the rms radii. The calculation of $\hbar\omega_n$ was not possible with the formalism of [7] because experimental values of the rms radii of neutrons are not available. In addition, $\hbar\omega_n$ might be useful for an improved study of the difference $\Delta = \hbar\omega_n - \hbar\omega_\Lambda$ instead of $\Delta = \hbar\omega_N - \hbar\omega_\Lambda$, since the Λ hyperon in the nucleus behaves very much like a neutron, a neutron however, with an additional quantum number the strangeness (S=-1), which makes it a distinguishable particle [12,13]. It is also interesting to observe the "discontinuities in the slope" in the curves of $\hbar\omega_n$, $\hbar\omega_p$ as functions of N,Z respectively at the closed shells for nuclei with $A > 40$.

In section 2 the density distribution proposed by Gabhir and Patil is reviewed, while in section 3 the formalism for $\hbar\omega_n$, ($\hbar\omega_p$) is considered together with their expansions in powers of N, (Z). Finally, in section 4 numerical results are given and discussed.

2. The density distribution of Gambhir and Patil

Gambhir and Patil[10] proposed a simple semi-phenomenological density distribution for neutrons and protons of the form:

$$\rho_i(r) = \frac{\rho_i}{1 + \beta_i[1 + (\frac{r}{R+a_i})^2]^{\alpha_i}[e^{\frac{(r-R)}{a_i}} + e^{\frac{-(r+R)}{a_i}}]} \tag{2.1}$$

where i=n or p (for neutrons or protons), R is a measure of the size of the nucleus and a_i and α_i are given in terms of the separation energy E_i of the last particle (neutron or proton) by the equations:

$$a_i = \frac{\hbar}{2\sqrt{2mE_i}} \qquad \alpha_i = \frac{q}{\hbar}\sqrt{\frac{m}{2E_i}} + 1 \tag{2.2}$$

where q=0 for neutrons and q=Z-1 for protons. The above $\rho_i(r)$ has two important features: 1. Its central behaviour is dictated by the fact that it has a vanishing slope

for r=0 which means that its power series expansion contains only terms with even powers of r_i. 2. It shows the correct asymptotic behaviour of the density of the particles in a nucleus given by :

$$\rho(r) = r^{-2\alpha_i} exp(\frac{-r}{a_i}) \qquad for \qquad r \to \infty \tag{2.3}$$

We also note that in order to identify R with the half-density radius, β_i is taken to be :

$$\beta_i = [1 + (\frac{R}{R + a_i})^2]^{-\alpha_i} \tag{2.4}$$

The nuclear size R determines the half-density radius for both neutrons and protons.
In the method of Gambhir and Patil the unknown parameters are ρ_n, ρ_p and R. Two of these are determined from the normalization:

$$\begin{aligned} 4\pi \int \rho_n(r) r^2 dr = N \\ 4\pi \int \rho_p(r) r^2 dr = Z \end{aligned} \tag{2.5}$$

while ρ_n is taken to be the same for all nuclei, namely: $\rho_n \approx 0.09/fm^3$, which gives the observed rms radius for the charge density in ^{208}Pb. This approximation was removed in a next paper [11] but the results showed that ρ_n=constant is a good approximation.
The integrals in (2.5) as well as the integrations for the rms radius have to be calculated numerically. However, the following analytic expressions were found:

$$4\pi \int \rho_i(r) r^2 dr \approx \frac{4\pi R^3 \rho_i}{3}(1 + x_i^2) \tag{2.6}$$

$$< r_i^2 > \approx R^2(0.6 + 1.4x_i^2) \tag{2.7}$$

where

$$x_i = \frac{\pi}{\frac{R}{a_i} + \frac{2\alpha_i R^2}{(R+a_i)^2 + R^2}} \tag{2.8}$$

The results of Gambhir and Patil and our detailed calculations showed that the above approximations are very satisfactory.

3. Formalism for $\hbar\omega$ of neutrons and protons

The average harmonic oscillator shell model square radius for nucleons may be written [7] as follows:

$$\overline{< r^2 >_{K+n}} = \frac{\hbar}{m_N\omega}\frac{4\sum_{p=1}^{K}(p+1/2)N(p)+(K+\frac{3}{2})n}{4\sum_{p=1}^{k}N(p)+n} \tag{3.1}$$

where K is the number of the highest filled shell, n the number of valence nucleons and K is determined by the solution of the equation:

$$4\sum_{p=1}^{K}N(p)+n = 2\sum_{p=1}^{K}(p^2+p)+n = A \tag{3.2}$$

It is found that K satisfies the equation :

$$\frac{2}{3}K(K+1)(K+2)+n = A \tag{3.3}$$

and the corresponding third order equation can be solved exactly to find K as function of A (relation (4) of [7]).

Using expression (3.1) we obtain for $\hbar\omega_N$ of nucleons :

$$\hbar\omega_N = \frac{3}{4}\frac{\hbar^2}{m_N A}[(K+1)(A+\frac{1}{3}n)+\frac{2}{3}n-2][< r^2 > -(< r_p^2 > + < r_n^2 >)]^{-1} \tag{3.4}$$

where $(< r_p^2 > + < r_n^2 >)$=0.659 fm^2 is due to proton and neutron finite size effects and the constant term -2 in the nominator comes from the correction due to the centre of mass motion.

The relation of $\hbar\omega_N$ with A was established [7] by means of a (phenomenological) Fermi density distribution:

$$\rho(r) = \frac{\rho_o}{1+e^{\frac{r-c}{a}}} \tag{3.5}$$

which leads to

$$< r^2 >= \frac{3}{5}(c^2+\frac{7\pi^2a^2}{3}) \tag{3.6}$$

The parameters of the Fermi distribution ρ_o , a were determined in [7] by fitting to values of $\hbar\omega_N$ reproducing the experimental values of the charge rms radii of nuclei: $< r^2 >_{ch}^{1/2}$ or directly to $< r^2 >_{ch}^{1/2}$. The value of the central density ρ_o was taken to be independent of the mass number A.

In the present paper we use theoretical values R_n^{rms} and R_p^{rms} derived from the density distribution of Gabhir and Patil, separately for neutrons and protons instead of using experimental $< r^2 >_{ch}^{1/2}$ as in [7]. The corresponding expressions for $\hbar\omega_n$, $\hbar\omega_p$ (harmonic oscillator energy level spacing for neutrons and protons respectively) are:

$$\hbar\omega_n = \frac{3}{4}(\frac{\hbar^2}{m_n N})\frac{[(K_n+1)(N+\frac{1}{3}n_n)+\frac{2}{3}n_n-\frac{2N}{A}]}{< r_n^2 >}$$

$$\hbar\omega_p = \frac{3}{4}(\frac{\hbar^2}{m_p Z})\frac{[(K_p+1)(Z+\frac{1}{3}n_p)+\frac{2}{3}n_p-\frac{2Z}{A}]}{< r_p^2 >} \qquad (3.7)$$

The above expressions are derived if we consider the harmonic oscillator states of N neutrons (or Z protons) in the nucleus of mass number A=N+Z and a straightforward modification of expression (3.4). K_n (K_p) is the number of the highest filled shell of the neutrons (protons) and n_n (n_p) is the number of the valence neutrons (protons). We also note that K_n (K_p) satisfy the equations:

$$\frac{1}{3}K_n(K_n+1)(K_n+2)+n_n = N$$

$$\frac{1}{3}K_p(K_p+1)(K_p+2)+n_p = Z \qquad (3.8)$$

which can be solved in analogy with (3.3) to give K_n or K_p as functions of N or Z respectively. In expressions (3.7) the terms $\frac{2N}{A}$ and $\frac{2Z}{A}$ are due to the centre of mass correction and introduce an A dependence. It turns out, however, that this dependence is small and has some effect only for very small A while for larger A ($A > 16$) one can practically put $\frac{2N}{A} \simeq \frac{2Z}{A} \simeq 1$.

The harmonic oscillator energy level spacings $\hbar\omega_n$, $\hbar\omega_p$ can be estimated by substituting to the denominators of expressions (3.7) the calculated values of $< r_n^2 >$ and $< r_p^2 >$ from expression (2.7). However, this procedure does not permit a rather systematic study of these quantities and it is not easy for practical use. Therefore, in order to have an estimate

of the average trend of the variation of $\hbar\omega_n$ and $\hbar\omega_p$ with the neutron (N) and proton (Z) numbers respectively, as well as a direct way of estimating these quantities, we proceed as follows: The determination of $\hbar\omega$ for neutrons and protons separately is performed by using as an input an expression for $< r_n^2 >$ and $< r_p^2 >$ of the form :

$$< r_n^2 >= C_{1n} + C_{2n}N^{2/3}$$

$$< r_p^2 >= C_{1p} + C_{2p}Z^{2/3} \qquad (3.9)$$

where the constants $C_{1n}, C_{2n}, C_{1p}, C_{2p}$ are determined by fitting the theoretical values of R_n^{rms}, R_p^{rms} derived from the density of Gambhir and Patil. In [7] $\hbar\omega_N$ was determined from experimental values of $< r^2 >_{ch}^{1/2}$ for nucleons and the determination of $\hbar\omega_n$ was not possible because experimental values for R_n^{rms} are not available. We may also note that our choice for the functional dependence of $< r_n^2 >$ and $< r_p^2 >$ on N and Z respectively, can be partly justified as follows : for neutrons one can combine relation (15) of [10], namely $R \simeq 1.316N^{1/3}$ with expression (2.7) giving a leading term of $N^{2/3}$ for $< r_n^2 >$, while for protons expression (22) of [11] ($R \simeq 1.239Z^{1/3}$) combined with (2.7) gives again a leading term $Z^{2/3}$ for $< r_p^2 >$. One cannot do better than this because x_i depends on the separation energies and its parametrization is difficult.

4. Numerical results and discussion

The calculations of Gabhir and Patil were carried out for a set of 13 nuclei. Their predictions are shown in table 1 of [10]. We repeated the calculation for a larger set of nuclei to cover the periodic table uniformly as much as possible. In table 1 we tabulate our calculated values of the half-density radius R, and the neutron and proton rms radii (R_n^{rms} and R_p^{rms} respectively). One could compare our calculated values for R_n^{rms},R_p^{rms} with the corresponding values derived from Hartree-Fock calculations (see for example [14]) which, however, were carried out for specific nuclei. In addition in the same table we also present values of $\hbar\omega_n$, $\hbar\omega_p$ calculated from (3.7) putting directly $< r_n^2 >^{1/2}=R_n^{rms}$, $< r_p^2 >^{1/2}=R_p^{rms}$.

Next we fitted expressions (3.9) for $< r_n^2 >$, $< r_p^2 >$ to the theoretical values of R_n^{rms}, R_p^{rms} of table 1 to determine the coefficients C_{1n},C_{2n}, C_{1p},C_{2p}. Their values are : C_{1n}=1.6012,

C_{2n}=1.3164, C_{1p}=0.44446, C_{2p}=1.6116.

Nucleus	R_p^{rms} (fm)	R_n^{rms} (fm)	R (fm)	$\hbar\omega_p$ (MeV)	$\hbar\omega_n$ (MeV)
$^{16}_{8}O$	2.74	2.64	2.49	11.92	12.72
$^{24}_{12}Mg$	3.02	2.89	2.93	12.09	12.83
$^{28}_{14}Si$	3.13	3.00	3.11	11.80	12.50
$^{40}_{20}Ca$	3.54	3.32	3.53	10.09	11.07
$^{48}_{22}Ti$	3.58	3.67	3.83	10.05	10.20
$^{52}_{24}Cr$	3.69	3.73	3.95	10.03	10.13
$^{58}_{28}Ni$	3.85	3.79	4.05	9.78	9.92
$^{64}_{30}Zn$	3.99	3.93	4.24	9.06	9.64
$^{90}_{40}Zr$	4.36	4.36	4.88	8.38	8.86
$^{93}_{41}Nb$	4.47	4.54	4.88	7.84	8.36
$^{109}_{47}Ag$	4.66	4.74	5.21	7.98	8.14
$^{116}_{50}Sn$	4.63	4.80	5.34	8.12	7.96
$^{126}_{52}Te$	4.79	4.98	5.55	7.49	7.68
$^{142}_{60}Nd$	4.99	5.09	5.78	7.44	7.62
$^{197}_{79}Au$	5.54	5.73	6.54	6.38	6.76
$^{206}_{81}Tl$	5.55	5.92	6.62	6.42	6.48
$^{208}_{82}Pb$	5.57	5.88	6.67	6.56	6.61

Table 1. **The values of the rms radii R_n^{rms}, R_p^{rms}, and the half-density radius R calculated from expressions (2.6), (2.7). The values of $\hbar\omega_n$, $\hbar\omega_p$ from (3.7) using the neutron and proton radii of this table are also displayed.**

Then $< r_n^2 >$, $< r_p^2 >$ are substituted to expressions (3.7) and $\hbar\omega_n$, $\hbar\omega_p$ are calculated

numerically for various values of N and Z. The dependence of $\hbar\omega_n$, $\hbar\omega_p$ on N and Z respectivelly , according to the results obtained in this paper is shown in figures 1 and 2 (solid lines).

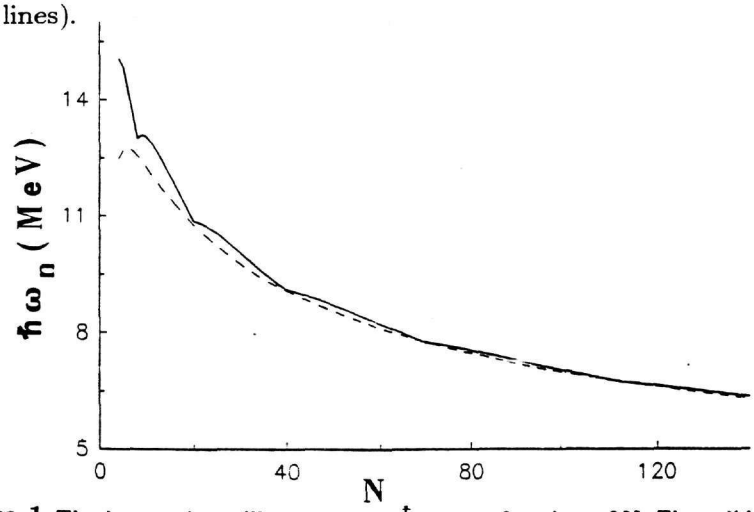

Figure 1 The harmonic oscillator spacing $\hbar\omega_n$ as a function of N. The solid line corresponds to expression (3.7) using relation (3.9), while the dashed one to expression (4.3)

We observe in both curves "discontinuities in the slope" at closed shells (without taking into account the spin-orbit term). This feature was also observed in [7] in the case of nucleons for the closed shells only at A=16 and 40. In the present work we are able to observe the discontinuities in the slope at closed shells for both neutrons and protons across the whole periodic table.

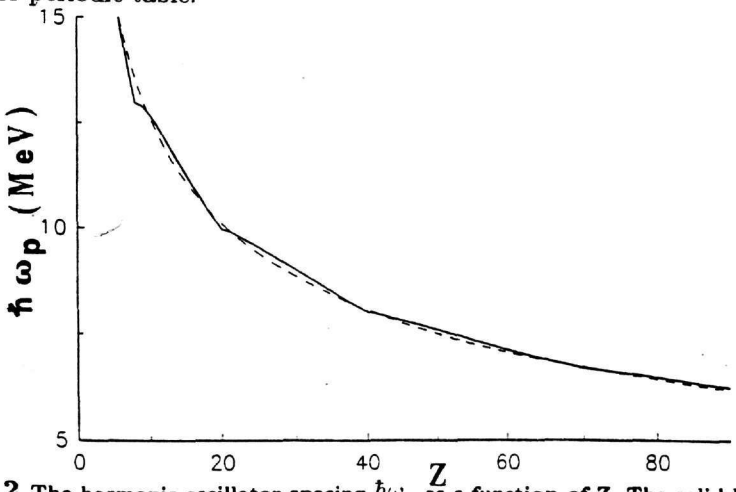

Figure 2 The harmonic oscillator spacing $\hbar\omega_p$ as a function of Z. The solid line corresponds to expression (3.7) using relation (3.9), while the dashed one to expression (4.3)

It is also noted that calculations for $\hbar\omega_n$ are possible because the density of Gambhir and Patil yields theoretical values for R_n^{rms}.

Expressions (3.7) for $\hbar\omega_n$, $\hbar\omega_p$ with $< r_n^2 >$, $< r_p^2 >$ from (3.9) can be expanded in powers of N or Z respectively. The following expansions are obtained:

$$\hbar\omega_n = \frac{3^{4/3}}{4}\frac{\hbar^2}{m_n}\frac{1}{C_{2n}}N^{-1/3}[1 + k_1 N^{-2/3} + k_2 N^{-4/3} + K_3 N^{-5/3} + ...] \qquad (4.1)$$

where

$$k_1 = (\frac{1}{3})^{5/3} - \frac{C_{1n}}{C_{2n}}$$

$$k_2 = 3^{-1/3}(\frac{2n_n}{3} - \frac{2N}{A}) - (\frac{1}{3})^{5/3}\frac{C_{1n}}{C_{2n}} - \frac{C_{1n}^2}{C_{2n}^2} \qquad (4.2)$$

$$k_3 = \frac{2n_n}{3^2}(\frac{1}{3})^{2/3}$$

A similar expression holds for $\hbar\omega_p$ obtained by substituting N with Z and the subscript n with p. The leading term of the expansions is $N^{-1/3}$ or $Z^{-1/3}$ as expected by the well-known asymptotic formula holding for nucleons: $41A^{-1/3}$ (if we set A = 2N or A = 2Z). Keeping the first two terms in equation (4.1) we find:

$$\hbar\omega_n = 34.1N^{-1/3} - 36.0N^{-1}$$

(4.3)

$$\hbar\omega_p = 27.9Z^{-1/3} - 3.2Z^{-1}$$

It is seen in expressions (4.1) and (4.2) that the first two terms of the expansions do not depend on the number of the valence nucleons (n_n,n_p). In figures 1 and 2 are also shown the curves (dashed lines) which correspond to expressions (4.3).

At this point it is also interesting to estimate the difference Δ=$\hbar\omega_n$-$\hbar\omega_\Lambda$. The Λ hyperon is the lightest strange baryon, it is neutral and one could simulate it with a distinct neutron. Therefore it seems more consistent to compare $\hbar\omega_\Lambda$ with $\hbar\omega_n$ and not just with $\hbar\omega_N$ for a nucleon as in [15,16]. Though a reliable determination of Δ is not easy, our estimates indicate that this difference is less than 3 MeV (for $A \geq 16$) and therefore the values of the oscillator spacing of the Λ might not differ very much from those for a neutron. We note

also that in [15,16] the difference $\Delta=\hbar\omega_N$-$\hbar\omega_\Lambda$ was estimated to be less than 2.5 MeV for A > 16.

Nucleus	$\hbar\omega_n$	$\hbar\omega_p$	$\hbar\omega_N$ expr.(7) of [7]	$\hbar\omega_\Lambda$ expr.(33) of [17]	$\hbar\omega_\Lambda$ exprs.(3),(12) of [15]
$^{16}_{8}O$	13.01	12.98	13.06	11.11	11.15
$^{24}_{12}Mg$	12.69	12.15	12.85	9.75	10.09
$^{28}_{14}Si$	12.24	11.56	12.43	9.27	9.59
$^{32}_{16}S$	11.77	10.99	11.97	8.88	9.14
$^{40}_{20}Ca$	10.86	9.98	11.09	8.26	8.36
$^{58}_{28}Ni$	10.06	9.23	10.42	7.32	7.10
$^{66}_{30}Zn$	9.47	9.02	10.02	7.02	6.68
$^{90}_{40}Zr$	8.69	8.03	9.18	6.34	5.74
$^{114}_{50}Sn$	8.03	7.60	8.63	5.87	5.08
$^{138}_{56}Ba$	7.49	7.32	8.06	5.51	4.59
$^{208}_{82}Pb$	6.56	6.43	7.19	4.81	3.67

Table 2. **The values in Mev of $\hbar\omega_n$, $\hbar\omega_p$ and $\hbar\omega_N$ for a number of nuclei. The values of $\hbar\omega_\Lambda$ correspond to hypernuclei with the same A and Z as the nuclei of this table (for details see text).**

In table 2 we present for a number of nuclei values for $\hbar\omega_n$, $\hbar\omega_p$ calculated according to the present approach (expr. (3.7) using (3.9)) and $\hbar\omega_N$ (column 4) calculated from ref.7 where a Fermi distribution was employed (expr (7)). In column 5 we show results for $\hbar\omega_\Lambda$ using expression (33) of [17], which is based on the idea of using a sort of " best approximation in the mean of the Λ-nucleus potential by the harmonic oscillator one ", where a dependence of $\hbar\omega_\Lambda = fA^{-1/3}$ appears. In column 6 we show values for $\hbar\omega_\Lambda$ determined by expressions (3) and (12) of [15], which lead to a dependence $\hbar\omega_\Lambda = fA^{-2/3}$. We recall that the problem of the A dependence of $\hbar\omega_\Lambda$ has attracted interest long ago [15-20] and it has been a matter of controversy [19,20,24]. Recently Dover [25] discussed the A dependence of $\hbar\omega_N$ and $\hbar\omega_\Lambda$

and connected it with the possibility for a indistinguishable Λ within a nucleus (partial quark deconfinement due to s quark).

We should keep in mind, however, that in [7] Δ was estimated using $\hbar\omega_N$ (for nucleons) with parameters determined by fitting the rms radii of nuclei while in the present paper $\hbar\omega_n$ is calculated by employing a density distribution with parameters coming from separation energies of the last neutron or proton. In both cases $\hbar\omega_\Lambda$ (which correspond to hypernuclei with the same mass number A and atomic number Z, as the nuclei in column 1 of table 2) is found by fitting to single particle ground state energies.

We also note that the experimental lowest spacing for a neutron or proton in O^{16} is roughly twice that for the Λ and the situation is similar for Ca^{40} (see also ref. 26). Nethertheless it seems that there are experimental uncertainties in the determination of these spacings, which do not make easy a quantitative comparison between the lowest energy spacing for a Λ and a nucleon.

In conclusion, we note that in the present paper approximate expressions for $\hbar\omega_n$(N), $\hbar\omega_p$(Z) separately, are derived which may be used in practice. Such expressions are proposed for the first time (to our knowledge) apart from ref. 27 where, however, the simplified Moszkowski's procedure was followed (i.e. using a uniform density distribution without taking into account valence nucleons) and no numerical results were reported. Also expressions of the form $\hbar\omega_n(A) = 35A^{-1/3}$, $\hbar\omega_p(A) = 31A^{-1/3}$ (suitable for rather heavy nuclei) were derived in [28] in the framework of the liquid-drop model by least-squares fitting to nuclear ground-state binding energies.

Acknowledgment

One of the authors (G.A.L) would like to thank Professor M.E. Grypeos and the Research Committee of the Aristotle University for financial support under the contract 4019/90.

References

[1] S.A. Moszkowski, In Handbuch der Physik, Vol. 39, (1957) 470

[2] A. De Shalit and H. Feshbach, Theoretical Nuclear Physics, Vol. I, p. 197, (Wiley, New York 1974)

[3] G. Eder, Nuclear Forces-Theoretical Nuclear Physics , p. 197, (MIT Press Massachusetts 1968)

[4] J. Blomqvist and A. Molinari, Nucl. Phys. A106, (1968) 545

[5] G.F. Bertsch, The Practitioners Shell Model, (North-Holland, Amsterdam, 1972)

[6] J. Negele, Phys. Rev. C1, (1970) 1260

[7] C.B. Daskaloyannis, M.E. Grypeos, C.G Koutroulos, S.E. Massen and D.S Saloupis, Phys. Lett. 121B, (1983) 91

[8] W. Hornyak, Nuclear Structure (Academic Press, New York 1975) p. 240

[9] M. R. Gunye and Chindhu S. Warke Phys. Rev. C2 (1970) 2106

[10] Y.K. Gambhir and S.H. Patil, Z. Phys. A-Atoms and Nuclei 321, (1985) 161

[11] Y.K. Gambhir and S.H. Patil, Z. Phys. A-Atomic Nuclei 324 (1986) 9

[12] B. Povh, Ann. Rev. Nucl. Part. Sci. 28, (1978) 1

[13] B. Povh, Comments Nucl. Part. Phys. 9 (1979) 15

[14] R.C. Barrett and D. Jackson, Nuclear sizes and structure, (Clarendon, Oxford, 1977) p. 28

[15] C.B. Daskaloyannis, M.E. Grypeos, C.G Koutroulos, S.E. Massen and D.S Saloupis, Phys. Lett. 134B, (1984) 147

[16] G.A. Lalazissis, M.E. Grypeos, S.E. Massen, Phys. Rev. C37 (1988) 2098

[17] M.E. Grypeos, G.A. Lalazissis, S.E. Massen and C.P. Panos, Il Nuovo Cimento A102 (1989) 445

[18] A. Gal, Ann. Phys. (N.Y.) 49, (1968) 341

[19] A. Gal, Adv. Nucl. Phys. 8, (1975) 1

[20] N. Auerbach and A. Gal, Phys. Lett. 48B, (1974) 22

[21] M. Rayet Ann. Phys. (N.Y) 102, (1976) 226

[22] D.J. Milener, C.B Dover and A. Gal, Phys. Rev. C38 (1988) 2700

[23] F. Fernandez, T. López-Arias and C. Prieto, Z. Phys. A-Atomic Nuclei 334 (1989) 349

[24] A.K. Kerman and H.J. Lipkin, Ann. Phys. (N.Y) 66 (1971) 738

[25] C. Dover, Proceedings of the International Symposium on Medium Energy Physics, Beijing, 1987, ed. by Chian Huan-Ching and Zheng Lin-Sheng (World Scientific, Singapore, 1987) p. 257

[26] L.N. Bogdanova and V.E. Markushin, Sov. J. Part. Nucl. Phys. 15 (1984) 361

[27] M. Bauer and J.J.E. Herrera, J. Phys. G: Nucl. Phys. 4 (1978) 67

[28] P.A. Seeger and W.M. Howard, Nucl. Phys. A238 (1975) 491