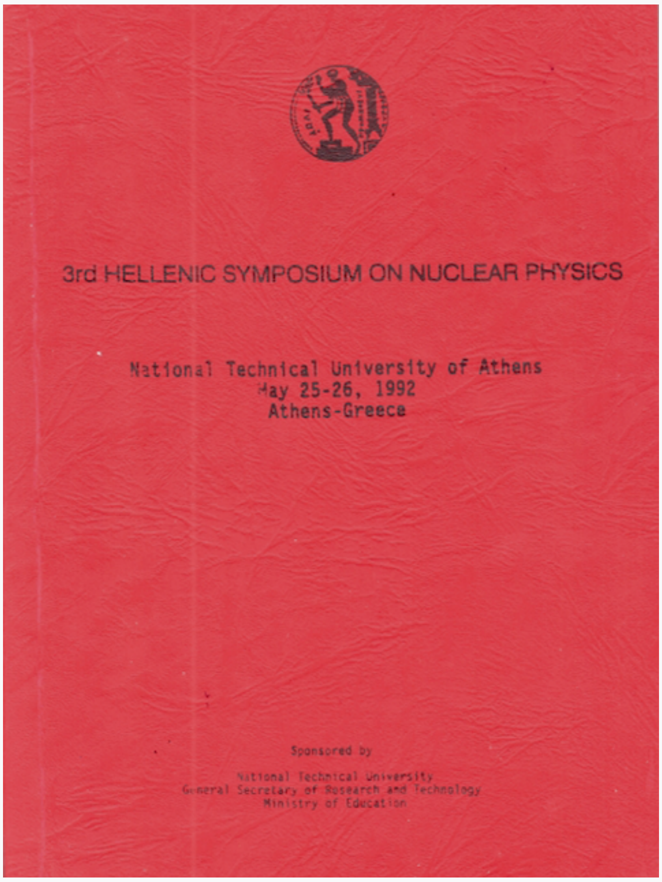
HNPS Advances in Nuclear Physics

Vol 3 (1992)

HNPS1992



The (K-, π+)Σ- - hypernuclear production in flight

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doi: 10.12681/hnps.2374

THE (K^-,π^+) Σ^--HYPERNUCLEAR PRODUCTION IN-FLIGHT †

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Abstract: The in-flight (K^-,π^+) Σ^--hypernuclear spectra are studied, using a simple interaction model with a square well central part and a delta function spin–orbit Σ–nuclear–core interaction. A comparison is made between the theoretical results and the in-flight (K^-,π^+) experimental data for ${}^{12}_{\Sigma}C$, ${}^{16}_{\Sigma}O$ and ${}^{6}_{\Sigma}Li$. A shallow potential, with a central potential depth of $V_c = (-5 - i15)MeV$ and a spin-orbit depth of $V_{so} = 15MeV$, gives a satisfactory representation of the CERN and BNL in-flight data.

The Σ–hypernuclei are produced by the $(K^-,\pi^\pm)$ interaction via the strangeness exchange reaction:

$$K^- + {}^A Z \rightarrow {}^A_\Sigma Z' + \pi$$

using various techniques and kinematics. Details on the experimental and theoretical status of the Σ-hypernuclei can be found in the recent review articles [1-4].

The existing Σ-hypernuclei are: ${}^4_\Sigma He$, ${}^6_\Sigma Li$, ${}^7_\Sigma Li$, ${}^9_\Sigma Be$, ${}^{12}_\Sigma C$, ${}^{16}_\Sigma O$. These data were taken from experiments performed at CERN [5-7], at BNL [8-10] and at KEK [11-19]. The experimental data are classified in three categories, depending on the kinematics of the $(K,\pi)_\Sigma$ production.

i. The target nucleus is bombarded by a kaon beam with $p_k = 713 - 720$ MeV/c; this energy range implies a large momentum transfer of about $q = 130 - 150 MeV/c$ to the nuclear core. This technique was called ***in-flight*** production. The first experiments at CERN have marked the discovery of the first Σ-hypernucleus ${}^9_\Sigma Be$ [5]. The experiments at BNL [8] lead to the discovery of the Σ-hypernuclei ${}^6_\Sigma Li$ and ${}^{16}_\Sigma O$. More recently, additional experiments at BNL [9, 10] have explored the ${}^7_\Sigma Li$ and ${}^{12}_\Sigma C$ hypernuclei.

ii. The in-flight kinematic was used again, with a new kaon beamline having $p_k = 400 - 450$ MeV/c at CERN. In this case, a small momentum transfer of about $q = 50 - 65$

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MeV/c enables the production of substitutional states. The hypernuclei ${}^{12}_{\Sigma}C$ [6] and ${}^{16}_{\Sigma}O$ [7] were studied by this method.

iii. The *at rest* method has been used at KEK [11-19]. The kaons, after being stopped in front of the target, they rotate on atomic orbits, cascading on the nuclear core and they are captured by nucleons forming a Σ-hypernucleus. During this interaction, a large momentum transfer of the order of $q = 170 MeV/c$ enables the production of the spin-orbit partners of the substitutional states. In these experiments the hypernuclei ${}^{12}_{\Sigma}C$ [11, 12] and ${}^{4}_{\Sigma}He$ [17, 18] have been produced.

The early estimates of the data have revealed narrow width resonances, of the order of 5 to 10 MeV above the Σ-energy threshold [5, 6, 11, 12]. These narrow resonant structures were not confirmed by the subsequent experiments [16], although similar resonances have reappeared recently in the study of ${}^{4}_{\Sigma}He$ [17, 18], where the resonant peak is located below the energy threshold. The general features of the pion spectrum are also attributed to the quasi-free background [20, 21]. The generated spectrum by the quasifree Σ-production is rather similar to that one which is calculated by assuming that the Σ particle interacts very loosely with the nuclear core [2].

Morimatsu and Yazaki [22] introduced the Green function method in the study of the hypernuclear production. They have studied the partial wave spectrum and the general features of the pion spectrum were described qualitatively [23]. Hausmann *et al.* [24-27] used a coupled-channel approach with a real potential, which includes the resonant and the quasifree part in a unified way. Žofka [28] and Wünsch- Žofka [29, 30] used the continuum shell model (CSM), which incorporates both quasifree and resonant reaction mechanisms on the same footing. Bandō, Motoba and Žofka [31] used a variation of the Kapur-Peierls method with a complex potential, which also combines the quasifree and the resonant hypernuclear production. The Kapur–Peierls method can be regarded as an approximate calculation of the Green function. Therefore the Green function method [22, 23] and the Kapur–Peierls method should give similar results. Halderson and Philpott [32] and Halderson [33] used the recoil corrected continuum shell model and the hyperon-nucleon Gaussian interaction of Bandō and Yamamoto [34] in order to describe the Σ-hypernuclear production. In this method, bound states, resonances and quasifree scattering are consistently included in the calculation.

All these methods are computer time consuming methods and after extensive computations satisfactory results are predicted. The extraction however, of simple qualitative results seems to be quite a difficult task. A simpler model, facilitating the calculations and the qualitative study of the $(K^-, \pi^{\pm})$ spectrum, could be a useful tool for the initial study of the Σ-hypernuclear interactions, if the proposed model simultaneously represented the experimental data sufficiently. For more detailed calculations one should turn back to the more exact methods, which are described in the already cited literature.

In this paper a square well potential with a delta spin-orbit interaction is proposed as a simplified model, appropriate for the Σ-hypernuclear interaction. We should underline that the square well potential has been already used in the study of the Λ-hypernuclei, giving quite accurate representations of the experimental data and a simplified first order overview of the Λ–nuclear core interaction [35]. The Green function of the proposed potential can be explicitly calculated. Therefore the Green function method, which is analysed in references [22] and [23], can be more easily used. This method takes also into account the resonant and the continuum processes. Thus, it is appropriate for the description of the Σ-hypernuclear production. In this method, the observed spectrum is proportional to the ***production strength*** (response function) S(E), given by the formula [22]:

$$S(E) \;=\; -\frac{1}{\pi} Im F(E) \tag{1}$$

The averaged strength function F(E) over the nuclear spin orientations is given by the formula [23]:

$$F(E) \;=\; \sum_{\ell,j} F_{\ell,j}(E)$$

$$where: F_{\ell,j}(E) \;=\; (2j+1)(2j_N+1)\sum_{\{L\}}(2L+1)\begin{pmatrix} j_N & j & L \\ -1/2 & 1/2 & 0 \end{pmatrix}^2 f^L_{\ell,j}(E)$$

where $\{L\}$ denotes the summation over the permitted values of L such that $\ell_N+\ell+L$ =even.

$$f^L_{\ell,j}(E) = \sum_M \int_0^\infty dr \int_0^\infty dr' \; [f^M_L(r)]^* \, G_{\ell,j}(E;r,r') f^M_L(r') \tag{2}$$

and the weight function $f^M_L(r)$ is:

$$f^M_L(r) = u_{\ell_N,j_N}(r) \int d\Omega \; Y^M_L(\hat{r}) \, [\chi_\pi(\bar{r})]^* \, \chi_K(\bar{r})$$

The $G_{\ell,j}(E;r,r')$ is the radial part of the Green function of the hyperon in the optical hyperon-nucleus potential corresponding to the $[\ell,j]$ Σ-configuration. The $u_{\ell_N,j_N}(r)$ is the radial part of the nucleon wave function corresponding to the $[\ell_N,j_N]^{-1}$ hole configuration.

The χ_π (or χ_K)is the pion (or the kaon) wave function in the pion-nucleus (or the kaon-nucleus) optical potential.

The Green function in eq. (2) corresponds to the Schrödinger equation

$$\frac{d^2u}{dr^2} + \left\{ \left(\frac{2m}{\hbar^2} \right) [E - V(r)] - \frac{\ell(\ell+1)}{r^2} \right\} u = 0 \tag{3}$$

where V(r) is the optical potential, containing a central part and a spin orbit part. For the shake of simplicity, we consider the case of the Σ^o hypernucleus, so that no Coulomb interaction appears.

$$V(r) = V_c w(r) + V_{so}(\bar{l} \cdot \bar{s}) {r_o}^2 \frac{1}{r} \frac{dw(r)}{dr}$$

The potential depths V_c and V_{so} are assumed to be complex. The imaginary part of the potential simulates the Σ to Λ conversion. The function w(r) is the form factor of the potential. In the solvable model, adopted in this paper, this is assumed to be of rectangular shape:

$$w(r) = \begin{cases} 1 & \text{if } r < R\,; \\ 0 & \text{if } r \geq R\,; \end{cases} \qquad R = r_o(A-1)^{1/3} \tag{4}$$

and

$$\frac{dw(r)}{dr} = -\delta(r - R)$$

The Green function $G_{l,j}(E; r, r')$, corresponding to the Schrödinger equation (3), satisfies the equation:

$$G = G_c + G_c U_{so} G$$

where $U_{so} = -V_{so}(\bar{l} \cdot \bar{s}) r_o^2 (1/R) \delta(r - R)$ is the spin-orbit potential and G_c is the Green function corresponding to the central potential. For simplicity, all the indices relative to angular momentum ℓ are omitted. This equation can be solved exactly and after a little algebra we find:

$$G_{l,j}(E, r, r') = G_c(E; r, r') - \frac{V_{so}(\bar{l} \cdot \bar{s}) r_o^2 (1/R) G_c(E; r, R) G_c(E; R, r')}{1 + V_{so}(\bar{l} \cdot \bar{s}) r_o^2 (1/R) G_c(E; R, R)}$$

We can also verify that:

$$G_{l,j}(E, r, r') = -\frac{2m}{\hbar^2} \frac{\phi(r_<)\psi^{(+)}(r_<)}{W[\phi(t), \psi^{(+)}(t)]_{t=R}}$$

The functions $\phi(r)$ and $\psi^{(+)}(r)$ are the regular and Jost solutions of the Schrödinger equation (3). The factor $G_{\ell,j}(E;r,r)$ depends only on the hyperon–nucleus potential.

In this paper, we are interested only in the (K^-,π^+) reactions, as it happens with most of the authors [22-31]. Only Halderson and Philpott [32,33], have included the (K^-,π^-) spectra in their calculations. Also, Yamada and Ikeda have calculated the $(K^-,\pi^-)^9_\Sigma Be$ spectrum , giving a coupled-isotriplet interpretation of the $^9_\Sigma Be$ hypernuclear state [36]. The reason that the (K^-,π^+) reaction is preferred, is that there is only one process for this reaction:

$$K^- + p \rightarrow \Sigma^- + \pi^+$$

On the contrary, there are two possible processes for the case of the (K^-,π^-) reaction:

$$K^- + p \rightarrow \Sigma^+ + \pi^-$$

$$K^- + n \rightarrow \Sigma^o + \pi^-$$

In this case, there is a possible dependence of the residual interaction from isospin, which mixes Σ^+ and Σ^o states by the charge exchange reaction $\Sigma^+ n \leftrightarrow \Sigma^o p$ [24,27,36-38].

In this paper the appropriate potential, representing the $^{12}_\Sigma C$ data, was searched and subsequently this potential was tested in order to reproduce the gross features of the $^{16}_\Sigma O$ data. The observed experimental data are normalized to give the integrated cross section equal to unity,

$$\int_{E_{min}}^{E_{max}} \frac{d^2\sigma}{d\Omega dE} dE = 1, \tag{7}$$

where $[E_{min}, E_{max}]$ is the energy range of the available experimental data; the same normalization is applied to the calculated strength functions. With this normalization, the theoretical predictions are scaled in order to give the same integrated cross sections as those obtained experimentally.

The calculations are given using the free wave pion (χ_π) and kaon (χ_K) wave functions. Hausmann and his collaborators have used DWIA calculations, where the distorted wave functions for the incoming kaon and the outgoing pion are generated from a non-local meson-nucleus optical potential [25]. From the comparison between calculations with distorted waves and with plane waves they concluded that the distortion effects do not bring qualitative changes in the shape of the spectra [25]. This comparison between PW and DW calculations shows that the use of PW can give a satisfactory qualitative representation of the in-flight data [25,26].

For these simplified calculations the Coulomb interaction on the Σ^- is omitted. The parameter r_o in eqn.(4) is taken to be 1.31 fm (see ref. [22]). The calculated spectra of the Σ-hypernuclear production do not depend strongly on the value of r_o, while the pole structure of the Σ-atomic wave functions is very sensitive on the value of r_o [27]. The model for the nucleon interaction is the simplest one. We consider the nucleon in a p–orbit in the harmonic oscillator model, the harmonic oscillator parameter $b = \sqrt{\hbar/m\omega}$ is taken to be b = 1.64 fm for ^{12}C and b = 1.76 fm for ^{16}O [39]. The spin–orbit of the nucleon modifies the values of the Clebsch–Gordan coefficients in our formulae. The nucleon wave function for ^6Li for the p-shell is taken by the formula:

$$\Psi_{01} \simeq r^{2\lambda_1 - 1} e^{-r^2/2b^2}$$

$$where : \lambda_1 = \frac{1}{4}[1 + \sqrt{9 + 8mA/\hbar^2}]$$

In these formulae, m is the nucleon mass and the constants A and b have the values $A = 2.06$ and $b = 1.967$ for ^6Li, see ref.[40].

After several trials, the potential of the value $V_c = (-5 - i15)MeV$ and $V_{so} = 15MeV$ gives almost satisfactory results [41] .

In figures 1 and 2, the (K^-, π^+) spectra are drawn for $^{12}_{\Sigma}C$ at $p_k = 715MeV/c$ for the two values of the angle $\theta = 4^o, 12^o$ from the BNL data [10]. The representation of the experimental data is fairly satisfactory. In the BNL (K^-, π^+) experimental data for ^{12}C , there is a large enhancement in about $2MeV$. So, in these spectra, the existence of peaks with narrow widths was not confirmed. The shape of the spectra between the two different values of the angle changes slightly. This, according to the authors, is due to the larger momentum transfer [10]. The partial wave analysis shows that the $s_{1/2}$ and $p_{3/2}$ waves dominate. The total spectrum has the shape of the $p_{3/2}$ wave.

In figure 3 the (K^-, π^+) spectrum is drawn for $^{12}_{\Sigma}C$, at $p_k = 450MeV/c$ for the value of the angle $\theta = 0^o$ from the CERN data [6]. There is a quite satisfactory correspondence between the data and our predictions. In the (K^-, π^+) experimental spectrum there is a peak at about $279MeV$ in the $M_{HY} - M_A$ scale, about $3MeV$ above the threshold. This peak is ascribed to the $(p_{3/2}, p_{3/2}^{-1})_{\Sigma^- p}$ configuration. This peak is also obvious in the theoretical spectrum.

So, the BNL data agree with the CERN data, even though there is a large difference in the momentum transfer between them. The BNL data do not agree with the early at rest data of KEK for ^{12}C [11], which showed the existence of two narrow peaks, which, however, did not show up in the next KEK experiment [16].

In figure 4 the potential determined by the $^{12}_{\Sigma}C$ data is used for the prediction of the (K^-,π^+) CERN–data [7] for $^{16}_{\Sigma}O$ at $p_k = 450MeV/c$ at the angle $\theta = 0^o$. The calculated spectra reproduce the general features of the experimental data, but the coincidence is not so satisfactory as in the carbon case. One must have in mind that the proposed model is a very simple one and compare our results with the results of the complicated models [24-27]. We must also point out that the experimental data have large uncertainties. In the (K^-,π^+) experimental spectrum , two peaks were observed: the first peak is observed in about 277 MeV and corresponds to the $(p_{3/2}, p^{-1}_{3/2})_{\Sigma^- p}$ configuration , while the second peak is observed in 284 MeV and corresponds to the $(p_{1/2}, p^{-1}_{1/2})_{\Sigma^- p}$ configuration [7]. This two peak structure is not seen in our calculations.

In figure 5 the (K^-,π^+) spectrum is drawn for $^{6}_{\Sigma}Li$ at $p_k = 713MeV/c$ for the angle $\theta = 13^o$ from the BNL data [8]. The representation of the experimental data is quite satisfactory. It is clear that there are no peaks. In the partial wave analysis only the $j = l + 1/2$ waves contribute to the total spectrum and the waves with large l are stronger than the others.

Theoretical calculations of the experimental spectra of Σ- hypernuclei, where there is a direct comparison between the theoretical and experimental values have been done by Hausmann et al and by Halderson.

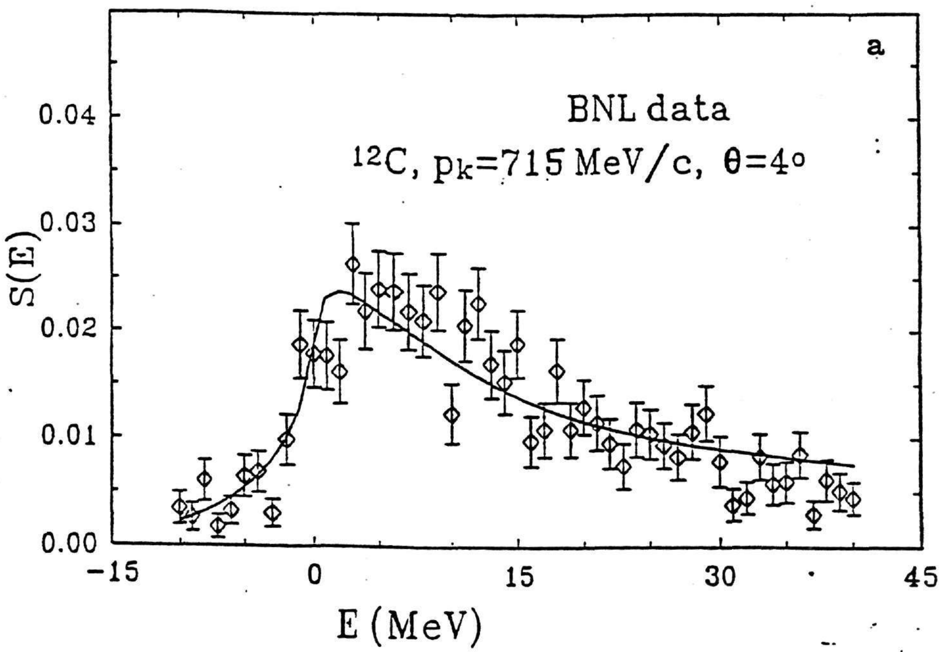
Hausmann et al have produced nearly all of the (K^-,π^+) data with a coupled-channel potential with parameters [25-27]:

$$V_\Sigma = -5MeV, V_{\Sigma\Lambda} = V_{\Lambda\Sigma} = 5MeV$$

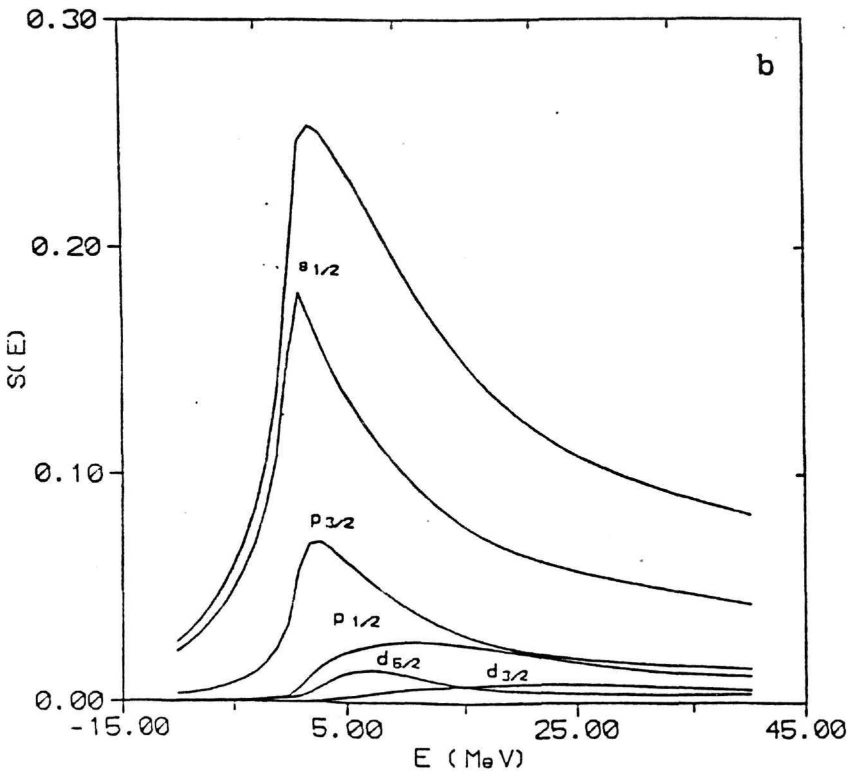
They obtain a satisfactory agreement between the calculated and the experimental spectrum for $^{16}_{\Sigma}O$ [7] , while the calculated spectrum for $^{12}_{\Sigma}C$ [6] does not fit the data satisfactorily as in the oxygen case. The theoretical spectrum for $^{16}_{\Sigma}O$ has a broad peak and not two small ones , like the experimental spectrum (see figure 7 of ref.[27]). This is in agreement with our calculations.

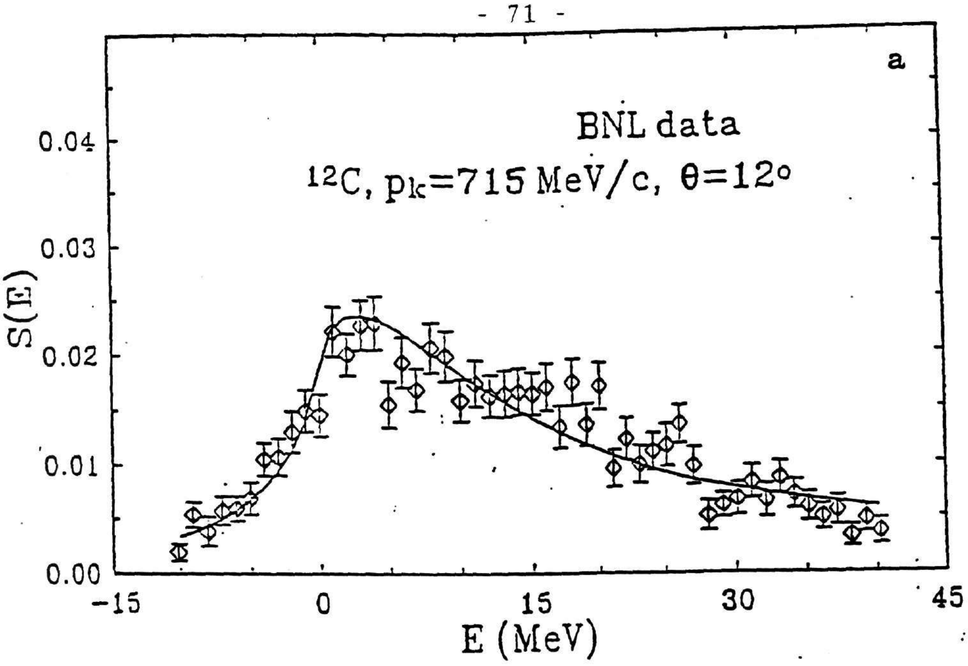
The common point between our calculations is the value of the central potential Σ-nucleus: $V^{\Sigma}_{C} = -5MeV$. This potential is weak (smaller than 10 MeV), which is in agreement with the studies of other authors [28].

Halderson also gives a very good representation of the CERN data for $^{12}_{\Sigma}C$.

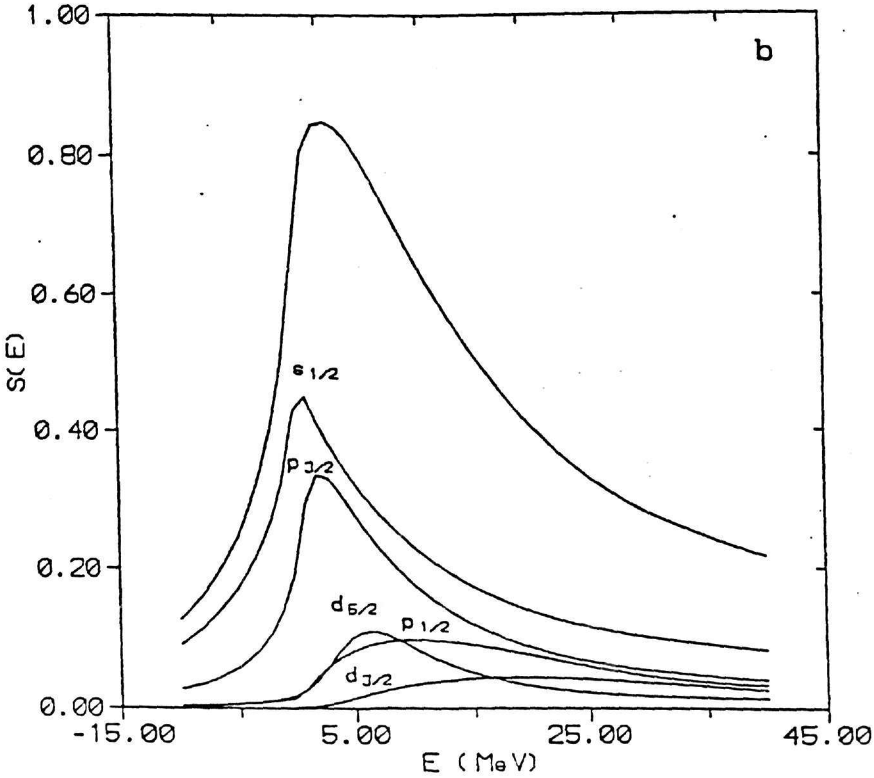


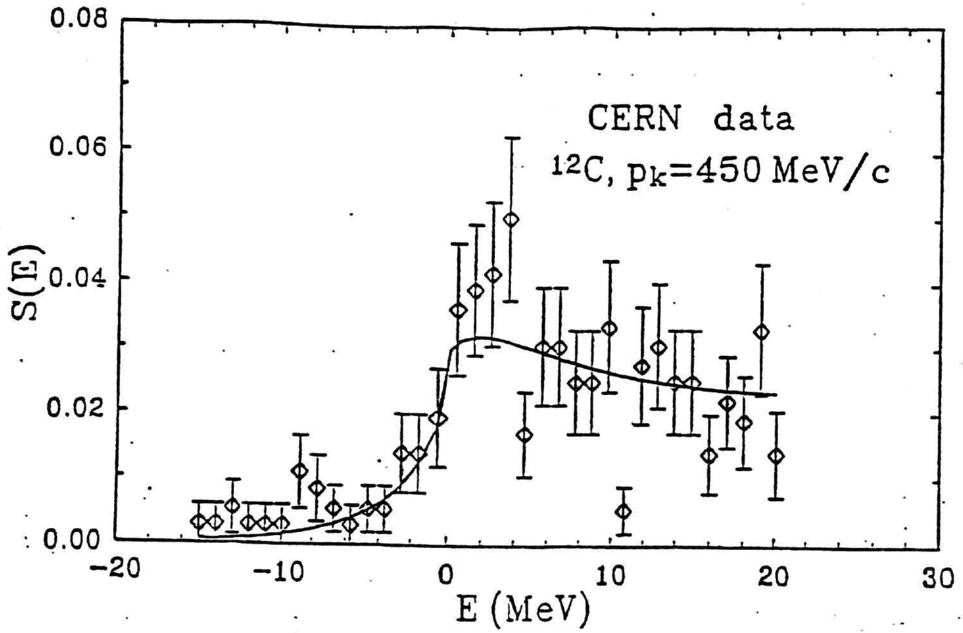
ig. 1. The calculated spectrum for $V_c = (-5 - i15) MeV$ and $V_{so} = 15 MeV$ for the in-flight BNL experiment for ${}^{12}_{\Sigma}C(K^-, \pi^+)$ and $\theta = 4°$. In part (a) the spectrum is compared with the experimental data of ref.[10]. In part (b) the contributions from the different Σ^--states are shown.



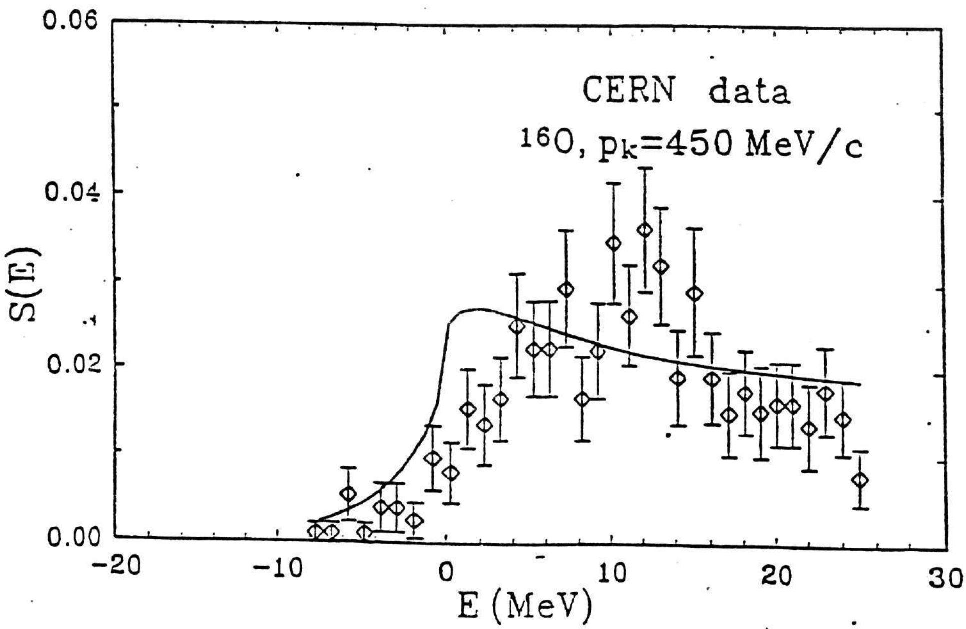


g. 2. The calculated spectrum for $V_c = (-5 - i15)MeV$ and $V_{so} = 15MeV$ for the in-flight BNL experiment for $^{12}_{\Sigma}C(K^-, \pi^+)$ and $\theta = 12°$. In part (a) the spectrum is compared with the experimental data of ref.[10]. In part (b) the contributions from the different Σ^--states are shown.





3. The calculated spectrum for $V_c = (-5 - i15) MeV$ and $V_{so} = 15 MeV$ compared with the in-flight CERN experimental data of ref.[6] , for $^{12}_{\Sigma}C(K^-, \pi^+)$ and $\theta = 0°$.



4. The calculated spectrum for $V_c = (-5 - i15) MeV$ and $V_{so} = 15 MeV$ compared with the in-flight CERN experimental data of ref.[7] , for $^{16}_{\Sigma}O(K^-, \pi^+)$ and $\theta = 0°$.

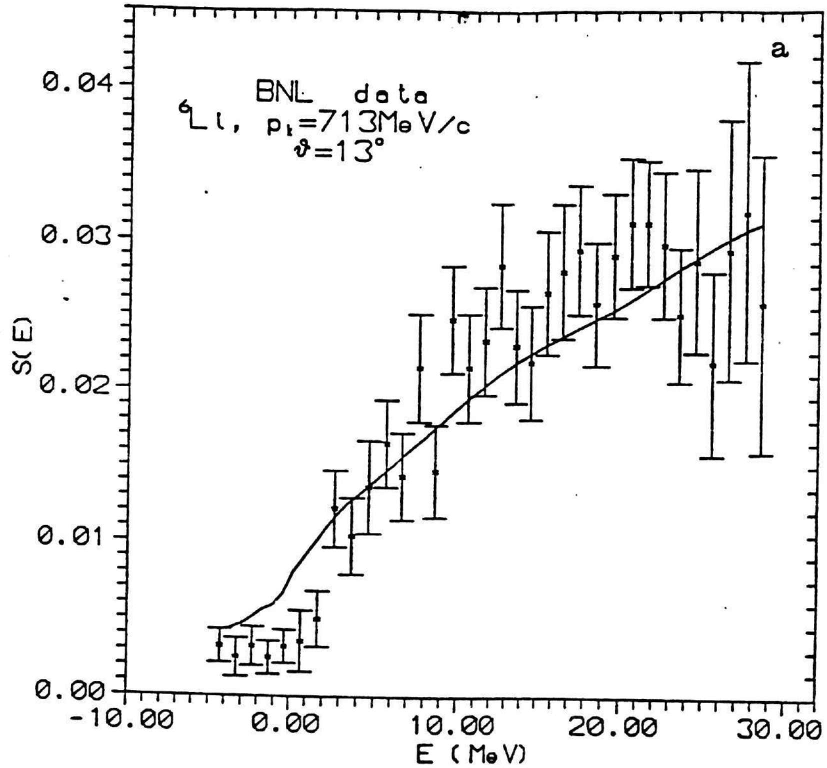
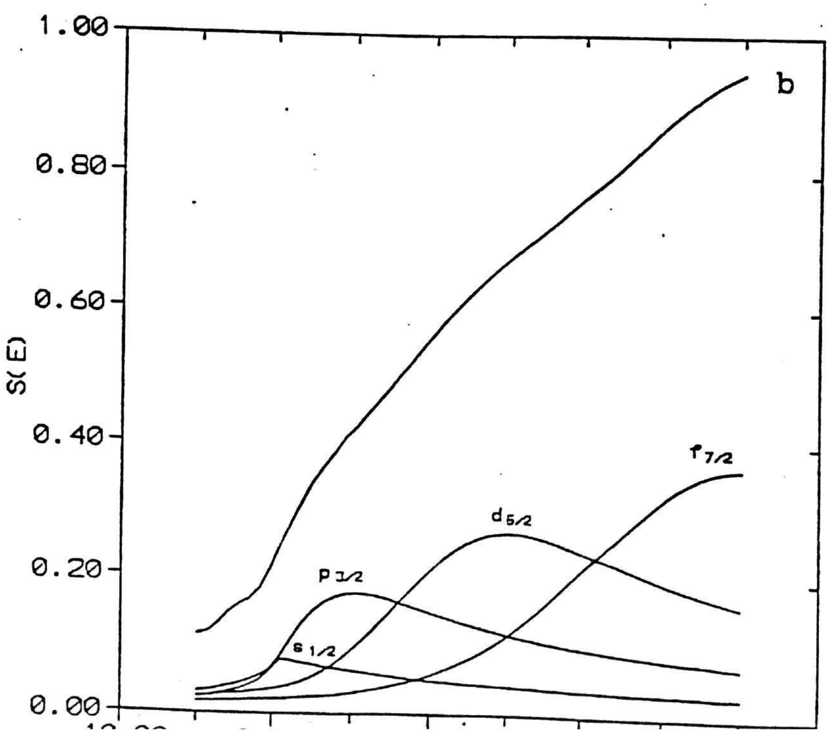


Fig. 5. The calculated spectrum for $V_c = (-5 - i15)MeV$ and $V_{so} = 15MeV$ for the in-flight BNL experiment for ${}^6_\Sigma Li(K^-, \pi^+)$ and $\theta = 13°$. In part (a) the spectrum is compared with the experimental data of ref.[8]. In part (b) the contributions from the different Σ^--states are shown.



Halderson and Philpott [32] and Halderson [33] concluded that the in-flight experiments are better, because they reduce the quasifree background, and an eventual resonance structure can be seen in this kind of spectra.

The differences between ${}^{12}_{\Sigma}C$, ${}^{16}_{\Sigma}O$ and ${}^{6}_{\Sigma}Li$ might indicate that the parameters of the Σ–nuclear core potential could be dependent on the mass number of the nuclear core , while in our calculations the potential parameters are assumed to be independent of A. It is known that in nuclear physics apart from the above mentioned A-dependence (mainly for rather small A), there is also a state dependence of the potential parameters [42]. The existing uncertainties on the experimental data do not permit a detailed study of this possibility in our case. Also, for ${}^{16}_{\Sigma}O$ only the $p_{1/2}$ nucleon orbits are assumed to contribute to the plane wave approximation .

To summarize, we have seen, in the context of the Green function method, that a simplified model of square well potential with depth $V_c = (-5 - i15)MeV$ and a delta function spin–orbit part $V_{so} = 15MeV$ represents fairly well the (K^-, π^+) in-flight data of BNL and CERN for ${}^{12}_{\Sigma}C$, ${}^{16}_{\Sigma}O$ and ${}^{6}_{\Sigma}Li$. The calculated cross sections are not adjusted arbitrarily, but the normalization (7) is used. This potential has a small real part and moderate imaginary and spin-orbit parts; this is in agreement with other models [25-27]. It is obvious that the proposed simplified model leads to the same conclusions drawn on the basis of sophisticated models, which involve however more complicated calculations.

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