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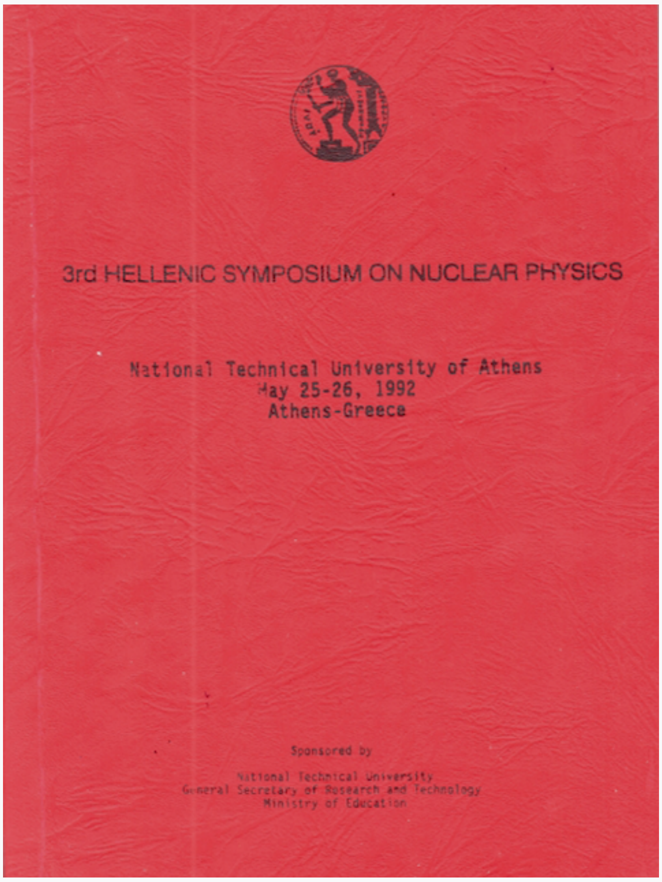

A systematic study of the effect of short range correlations on the occupation numbers of the shell model orbits in light nuclei

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A systematic study of the effect of short range correlations on the occupation numbers of the shell model orbits in light nuclei *

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Abstract

The role of short-range correlations on the depletion of the Fermi sea is studied in light nuclei.The short range correlations are considered in an approximate treatment allowing a systematic study of nuclei in the region $4 \leq A \leq 40$. The "natural orbital" representation is used for the determination of the occupation probabilities of the shell model orbits of the ground state wave function. The depletion of the nuclear Fermi sea appears to be, on the average, about 32%.

1. Introduction

Investigations concerning the limits of validity of the mean field approximation have attracted much interest last years[1]. In this approach all the single-particle states with energy larger than the Fermi energy are completely empty while the other ones are fully occupied and form the Fermi sea. However, reality is far from this simple picture and the nucleon-nucleon interactions give rise to a depletion of the Fermi sea.
In an early theoretical study it was argued [2], that the depletion may be as large as 30%. Later, electron scattering experiments supported this idea [3,4]. More recently high resolution (e,e′p) experiments have shown significant deviations from the mean field picture[1,5,6]. For mass $A \geq 12$ systems a systematic reduction of spectroscopic strength is observed amounting to 40-50% of the full-shell value expected in the mean field approximation to the many-body wave function. This causes a substantial depletion of the quantum

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states, especially those near the Fermi surface [1] and clearly demonstrates that the single particle orbits are partially occupied due to nucleon-nucleon correlations [5]. It is noted, however, that in such experiments only a part of the spectroscopic strength is measured and therefore it is expected that the occupancy of the single particle orbits is larger (e.g. a value of about 80% is reported for the occupation probability of $3S_{1/2}$ in lead [7], while an analysis of (e,e$'$p) data yields a spectroscopic strength of about 50% [8,9]).

The depletion of occupied states can be attributed to two processes. Firstly, there is a coupling of the Hartree-Fock ground state to low-lying collective modes. Secondly, short-range correlations due to hard collisions between nucleons at relative distances smaller than about 0.5 fm may result in a scattering of the nucleons into states of higher energy up to 1 GeV. Calculations for nuclear matter including these short-range correlations have shown [10,11] that the depletion of the otherwise filled orbits is 10-20%.

The "natural orbital" representation [12] offers a unique possibility in keeping the simplicity and visuality of the single-particle description, while at the same time the effects of short-range and tensor correlations (which already have been examined in more sophisticated beyond mean field theories) are taken into account in an effective way, that is, expressing the ground-state wave function in terms of the occupation probabilities of single particle orbits.

The "natural orbital" approach has already been applied in the past for nuclear structure studies [13-15]. Recently this approach was employed [16] within a varionational Jastrow-type correlation method to study quantum liquid drops such as Fermi liquid ^{3}He and Bose liquid ^{4}He.

Jaminon et al. [17-20] paid special attention to the "natural orbital" method. In their approach the radial part of the single particle Woods-Saxon wave functions was identified with the natural orbitals, for states below the Fermi level, while those above the Fermi level were constructed by setting a particular cut-off procedure and suitable boundary conditions. The parameters of the Woods-Saxon potential were determined to fit the energy-level scheme of the self-consistent Hartree-Fock calculations. The occupation numbers were those of a RPA calculation for ^{208}Pb [21] and of nuclear matter calculations where the effects of short-range and tensor correlations were considered [4]. Such an approach permits the study of both density and momentum distribution in lead. For the depletion of the Fermi sea various possibilities were considered corresponding to different sets of occupation numbers. It was concluded that by comparing their results with the empirical values of the momentum distribution the set of the occupation numbers which leads to 11.6% depletion is the most reliable among those considered. Their results are also in semi-quantitative keeping with those of several microscopic calculations performed in light nuclei.

In a series of papers [22-24] correlated charge form-factors $F_{ch}(q)$ and densities of s-p and s-d shell nuclei were calculated by using correlated wave functions of the relative motion and the factor cluster expansion of Ristig et al. [25]. The parameters of the method were calculated by fitting the theoretical values of $F_{ch}(q)$ to the experimental ones for the corresponding nuclei.

The aim of the present paper is to study the effect of short-range correlations on the depletion of the nuclear Fermi sea for nuclei in the region $4 \leq A \leq 40$. The "natural orbital" representation is employed by imposing the condition the correlated proton density distribution (derived from the work mentioned in the previous paragraph) to be equal to that constructed by natural orbitals.

The paper is organized as follows: In sect. 2 we present the method and the relevant formalism for the short-range correlations. In sect. 3 we describe our method and give the corresponding expressions for the occupation probabilities. In sect. 4 we present and discuss the results of our calculations. Finally, sect. 5 summarises our conclusions.

2. The correlated charge form-factors and densities of s-p and s-d shell nuclei

A general expression for the charge form-factor, $F_{ch}(q)$, of light closed shell nuclei was derived [22] using the factor cluster expansion of Ristig et al. [25] as reviewed by Clark [26]. Next this formula was simplified by considering normalised correlated wave functions of the relative motion which are parametrized in the following way:

$$\psi_{nls}(r) = N_{nls}[1 - exp(-\lambda_{nls} r^2/b^2)]\phi_{nl}(r) \tag{2.1}$$

where N_{nls} are the normalisation factors, $\phi_{nl}(r)$ are the harmonic oscillator wavefunctions and $b = \sqrt{2}b_1$ ($b_1 = \sqrt{\hbar/m\omega}$) is the harmonic oscillator (HO) parameter for the relative motion. Thus, an expression for $F_{ch}(q)$ of the ^{16}O nucleus was derived in closed form. The correlation parameters λ_{nls} (state dependent) and the HO parameter b_1 were determined by fitting to the experimental values of $F_{ch}(q)$ of ^{16}O.

In an extension of the above work [23,24] an approximate formula for the two-body term $F_2(q)$ in the cluster expansion of $F(q) = F_1(q) + F_2(q)$ ($F_1(q)$ is the one-body term) was found for the case of ^4He, ^{16}O and ^{40}Ca, which contains b_1 and the correlation parameter λ (state independent). The two body term has the form

$$F_2(q) = \lambda^{-3/2}[A(y)e^{-y} + B(y)e^{\frac{-y}{(1+\lambda)}} + C(y)e^{\frac{-y}{(1+2\lambda)}}]e^{-y} \tag{2.2}$$

where $y = \frac{b_1^2 q^2}{8}$ and $A(y)$, $B(y)$, $C(y)$ are polynomials of zero order for ^4He, second order for ^{16}O and fourth order for ^{40}Ca. The coefficients α_i, β_i, γ_i of these polynomials are shown in table 1 of ref. 24. The coefficients of $A(y), B(y), C(y)$ for the other p shell (or s-d shell) nuclei were determined by making a linear interpolation between the corresponding values of ^4He and ^{16}O (or ^{16}O and ^{40}Ca).
The correlation parameter λ was a free parameter for ^4He, ^{16}O and ^{40}Ca, while for the open p shell or s-d shell nuclei was determined by interpolation using the relation $\lambda = \lambda_0 + \lambda_1 A^{1/3}$.
The method described above offers the possibility of finding the correction to the uncorrelated charge (or proton) density analytically by a Fourier transform of $F_2(q)$. Thus the correlated proton density distribution is written :

$$\rho_{cor}(r) = \rho_1(r) + \rho_2(r) \tag{2.3}$$

where $\rho_1(r)$ is the Fourier transform of $F_1(q)$, which for s-p shell nuclei is :

$$\rho_1(r) = \frac{1}{\pi^{3/2} b_1^3}[1 - \frac{Z-2}{Z}(1 - \frac{2}{3}\frac{r^2}{{b_1}^2})]e^{-r^2/b_1^2} \tag{2.4}$$

while for s-d shell nuclei is:

$$\rho_1(r) = \frac{1}{\pi^{3/2} b_1^3}[\frac{1}{4} - \frac{Z-20}{3Z}\frac{r^2}{{b_1}^2} + \frac{Z-8}{3Z}\frac{r^4}{b_1^4}]e^{-r^2/b_1^2} \tag{2.5}$$

and the two-body term $\rho_2(r)$ is the Fourier transform of $F_2(q)$, which for s-p and s-d shell nuclei has the form :

$$\rho_2(r) = \frac{\lambda^{-3/2}}{2\pi^{3/2}}[I(x)e^{-x} + J(x_1)e^{-x_1} + K(x_2)e^{-x_2}] \tag{2.6}$$

where

$$x = \frac{r^2}{b_1^2}, \qquad x_1 = \frac{r^2}{\delta_1^2}, \qquad x_2 = \frac{r^2}{\delta_2^2} \tag{2.6a}$$

$$\delta_1^2 = \frac{1+\lambda/2}{1+\lambda}b_1^2, \qquad \delta_2^2 = \frac{1+\lambda}{1+2\lambda}b_1^2, \tag{2.6b}$$

and

$$I(x) = \frac{1}{b_1^3}[2\alpha_0 + \frac{3}{2}\alpha_1\ {}_1F_1(-1;\frac{3}{2};x) + \frac{15}{8}\alpha_2\ {}_1F_1(-2;\frac{3}{2};x)+$$
$$+\frac{105}{32}\alpha_3\ {}_1F_1(-3;\frac{3}{2};x) + \frac{945}{128}\alpha_4\ {}_1F_1(-4;\frac{3}{2};x)] \tag{2.7}$$

The expression for $J(x_1)$ results from (2.7) setting $x \to x_1$, $b_1 \to \delta_1$ and $\alpha_k \to (\frac{1+\lambda}{1+\lambda/2})^k\beta_k$, k= 0,1,2,3,4, while the expression for K(x_2) by setting $x \to x_2$, $b_1 \to \delta_2$ and $\alpha_k \to (\frac{1+2\lambda}{1+\lambda})^k\gamma_k$.

Finally, from (2.3) one can find analytic expressions for the various moments of the proton density distribution for the s-p and s-d shell nuclei. The moments have the form :

$$< r^k >_{cor} = < r^k >_1 + < r^k >_2 \tag{2.8}$$

where $< r^k >_1$ and $< r^k >_2$ are the contributions of the one body and two body density respectively. The former can be found from equations (2.4) and (2.5) and for the s-p shell nuclei has the form :

$$< r^k >_1 = \frac{4b_1^k}{\sqrt{\pi}}\left[\frac{1}{Z}\Gamma(\frac{3+k}{2}) + \frac{Z-2}{3Z}\Gamma(\frac{5+k}{2})\right] \tag{2.9a}$$

while for the s-d shell nuclei is written :

$$< r^k >_1 = \frac{4b_1^k}{\sqrt{\pi}}\left[\frac{1}{8}\Gamma(\frac{3+k}{2}) - \frac{Z-20}{6Z}\Gamma(\frac{5+k}{2}) + \frac{Z-8}{6Z}\Gamma(\frac{7+k}{2})\right] \tag{2.9b}$$

The corresponding expression for $< r^k >_2$ can be found from (2.6). If we keep powers of λ up to $\lambda^{-3/2}$, it takes the simple form :

$$< r^k >_2 \simeq C_k b_1^k \lambda^{-3/2} \tag{2.10}$$

The values of C_k (k=1,2,3,4) for ^4He, ^{16}O and ^{40}Ca are given in table 1.

Nucleus	C_1	C_2	C_3	C_4
^4He	1.6323	3.7043	7.2054	13.8909
^{16}O	2.3992	7.1775	17.5076	40.8075
^{40}Ca	3.7509	12.4673	35.0501	92.6091

Table 1. **The values of C_k (k=1,2,3,4) for ^4He, ^{16}O and ^{40}Ca used in expression (2.10)**

3. Theoretical method

The "natural orbitals" $\{\phi_q\}$ are defined [12] as the orthogonal basis which diagonalizes the one-body density matrix :

$$\rho(\mathbf{r},\mathbf{r}') = \sum_q \alpha_q u_q^*(\mathbf{r}) u_q(\mathbf{r}') \tag{3.1}$$

where α_q is the occupation number of the state q ($q \equiv nlj$)
In this representation the density distribution takes the simple form:

$$\rho(\mathbf{r}) = \sum_q \alpha_q |u_q(\mathbf{r})|^2 \tag{3.2}$$

Notice that the summation in (3.2) runs over the q largest occupied state above the Fermi level. In the case of a spherical symmetric system we get :

$$\rho(r) = \frac{1}{4\pi} \sum_{nl} (2j+1) n_q |\phi_q(r)|^2 \tag{3.3}$$

where $n_q = \frac{\alpha_q}{(2j+1)}$ is the occupation probability of the q(=nlj) state.
In our approach we assume that the radial part of the single-particle wave functions $\{R_{nl}\}$ of a harmonic oscillator potential can be identified with the "natural orbitals" $\{\phi_q\}$ in a

way similar to [14,19]. One, however, could consider as "natural orbitals" proper linear combinations of the single-particle wave functions $\{R_{nl}\}$, that is, as approximations to "generalized natural orbitals" [27]. At this point we note that in a previous work of Boffi and Pacati [28] the natural orbitals were expanded in terms of a few harmonic oscillator eigenstates and the expansion coefficients were used as fitting parameters. In the present work, as a first step and for the sake of simplicity of our approach (that is less parameters), we keep only the dominant term of the expansion. This seems reasonable since in the Boffi-Pacati approach the value of the coefficient of the dominant term was found to be large i.e close to unity. In addition one should keep in mind the advantage of harmonic oscillator wave functions to depend only on one parameter, namely the size parameter b_1. On the other hand one could use instead of harmonic oscillator wave fuunctions the ones of a more realistic single-particle potential like the Woods-Saxon one as in [14,17-20]. In such a case, however, the simplicity of the approach and the possibility of obtaing analytic expressions are lost.

Next we assume that the proton density distribution ρ_{cor} (expr. 2.3) in which the effect of short range correlations is taken into account, equals with the density distribution $\rho_{n.o}(r)$, corresponding to the "natural orbital" representation :

$$\rho_{cor}(r) = \rho_{n.o}(r) \tag{3.4}$$

where $\rho_{n.o}(r)$ is given by (3.3) with $\phi_q(r) \equiv R_{nl}(r)$. An alternative proposal is to use instead of $\rho_{cor}(r)$ the experimental density distributions resulting from the corresponding experimental charge form factors $F_{ch}(q)$. Such an approach, however, does not permit a systematic study of s-p and s-d shell nuclei. In addition, in our expression for the ρ_{cor} (which is analytic) the two body correlations enter in an explicit way allowing an estimate of their role to the deviation from integer occupation numbers and to the depletion of the nuclear Fermi sea.

If we choose our harmonic oscillator basis to cover n=3 major shells, (3.3) becomes :

$$\rho_{n.o}(r) = \frac{1}{Z}\frac{1}{(\sqrt{\pi}b_1)^3}[N_1 + 2N_2\frac{r^2}{b_1^2} + 4N_3\frac{r^4}{b_1^4} + 8N_4\frac{r^6}{b_1^6}]exp(-r^2/b_1^2) \tag{3.5}$$

where

$$N_1 = 2n_{1s} + 3n_{2s}$$

$$N_2 = 2n_{1p} - 2n_{2s} + 5n_{2p}$$

$$N_3 = \frac{2}{3}n_{1d} + \frac{1}{3}n_{2s} - 2n_{2p}$$

$$N_4 = \frac{2}{15}n_{1f} + \frac{1}{5}n_{2p} \tag{3.6}$$

It is seen from (3.5) and (3.6) that $\rho_{n.o}(r)$ depends on the four parameters N_i (i=1,...,4) and the size parameter b_1 of the harmonic oscillator potential. The determination of the parameters can be carried out by imposing the condition :

$$< r^k >_{cor} = < r^k >_{n.o} \tag{3.7}$$

that is, the first few moments of $\rho_{cor}(r)$ to become equal to the ones of $\rho_{n.o}(r)$ distribution. The general expression for the moments in the "natural orbital" representation has the form :

$$< r^k >_{n.o} = \frac{2}{Z\sqrt{\pi}} b_1^k [N_1 \Gamma(\frac{k+3}{2}) + 2N_2 \Gamma(\frac{k+5}{2})$$

$$+4N_3 \Gamma(\frac{k+7}{2}) + 8N_4 \Gamma(\frac{k+9}{2})] \tag{3.8}$$

Using (3.7) and (3.8) for k=0,1,...,4 we obtain five equations with five unknown variables N_1, N_2, N_3, N_4 and b_1. The equation for b_1 is :

$$b_1^4 - 2\sqrt{\pi} r_1 b_1^3 + 4 r_2 b_1^2 - \sqrt{\pi} r_3 b_1 + \frac{4}{15} r_4 = 0 \tag{3.9}$$

while the equations for N_i are linear and r_i (i=0,1,...,4) are the moments of $\rho_{cor}(r)$ distribution, that is $r_i = < r^i >_{cor}$.

If we reduce the HO space of the bound states up to n=2 major shells, $\rho_{n.o}$(r), N_1,N_2,N_3,N_4 come out from (3.5),(3.6) by setting n_{2p}=n_{1f} = 0. In such a case N_4 =0 and b_1 satisfies the following equation:

$$4b_1^3 - 6\sqrt{\pi}r_1 b_1^2 + 8r_2 b_1 - \sqrt{\pi}r_3 = 0 \tag{3.10}$$

A further reduction (up to n=1 major shell) leads to only two unknown variables N_1, N_2 while $N_3 = N_4 = 0$ and b_1 satisfies the equation:

$$3b_1^2 - 3\sqrt{\pi}r_1 b_1 + 2r_2 = 0 \tag{3.11}$$

It should be noted that such reductions are necessary for lighter nuclei in order to get physical acceptable values for the occupation probabilities and the size parameter b_1 (see also next section). We also note that $\rho_{n.o}(r)$ (expr. 3.5) is normalized to unity.
It is interesting to investigate the possibility of estimating the occupation probabilities n_{nl} and the depletion of the Fermi sea using this simple method. We note that we have the four equations (3.6) for six variables n_{nl} where N_i are already known in the case with n =3 major shells. In the case with n =2 major shells we have $N_4 = 0$ and equations (3.6) become a system of three linear equations with four variables. Finally, for n =1 major shells N_3=N_4=0 and equations (3.6) lead to a system of two linear equations which allow the determination of n_{1s} and n_{1p} in a unique way. From a first glance, it seems that there are an infinite number of combinations for n_{nl} which satisfy (3.6) in the cases with n =3 and n=2 major shells. However, we can limit the values of n_{nl} to a narrow region by setting plausible conditions on them.
We note that since in the present approach we consider the "natural orbital" representation, the number of particles in the Fermi sea should be expected to be maximum [17-19,29,30]. Taking into account the above remark as well as the fact that the deeper states should have higher occupancy, we impose the conditions :

$$n_{1s_{1/2}} > n_{1p_{3/2}} > n_{1p_{1/2}} > n_{1d_{5/2}} > n_{2s_{1/2}} > n_{1d_{3/2}} > n_{1f_{7/2}} > n_{2p_{3/2}} > n_{1f_{5/2}} > n_{2p_{1/2}} \tag{3.12}$$

where the occupation probabilities n_{nlj} are related to n_{nl} as follows:

$$n_{nl} = \frac{l+1}{2l+1} n_{nl,l+1/2} + \frac{l}{2l+1} n_{nl,l-1/2} \tag{3.13}$$

From the possible solutions of (3.6) satisfying the conditions (3.12) we choose the ones which yield the smallest depletion of the Fermi sea.

4. Numerical results and discussion

The moments r_i i=0,1,...,4 of the correlated proton density distribution $\rho_{cor}(r)$ are calculated numerically using ρ_{cor} coming from ref. 19. Then we solve for b_1 (n.o) eq. (3.9) for nuclei belonging to n=3 major shells, eq. (3.10) for n=2 major shells and eq. (3.11) for n=1. The results for the values of b_1 (n.o) along with b_1 (HO) (uncorrelated) and b_1 (cor) are shown in table 2.

Nucleus	$b_1(HO)$	$b_1(cor)$	$b_1(n.o)$
^4He	1.363	1.215	1.118
^{12}C	1.639	1.529	1.465
^{16}O	1.786	1.679	1.597
^{24}Mg	1.807	1.760	1.714
^{28}Si	1.891	1.821	1.761
^{31}P	1.849	1.746	1.685
^{32}S	1.860	1.793	1.728
^{39}K	1.969	1.866	1.792
^{40}Ca	1.950	1.860	1.785

Table 2. Comparison of b_1(HO), b_1(cor), b_1(n.o) for various nuclei.

We note that though the number of solutions is greater than one, it turns out that in each case from the real solutions of eqs. (3.9, 3.10, 3.11) only one (the smallest) is physically acceptable i.e yields a reasonable value for $\hbar\omega$. It is seen from table 3 that the following inequality holds:

$$b_1(HO) > b_1(cor) > b_1(n.o) \tag{4.1}$$

The values of N_1, N_2, N_3, N_4, which are displayed in table 3, are determined from equations (3.7), (3.8) and the calculated values of r_i.

Nucleus	N_1	N_2	N_3	N_4
^4He	0.9508	0.3497	0.	0.
^{12}C	1.3463	1.0585	0.0985	0.
^{16}O	1.6019	1.2085	0.1848	0.
^{24}Mg	2.4424	0.8267	0.3847	0.0125
^{28}Si	2.6790	0.7140	0.4119	0.0286
^{31}P	2.7964	0.6871	0.4176	0.0369
^{32}S	2.9013	0.6763	0.4162	0.0460
^{39}K	3.1932	0.7316	0.3828	0.0750
^{40}Ca	3.2706	0.7816	0.3587	0.0858

Table 3. The values of N_1, N_2, N_3, N_4 for various nuclei.

At this point we note that using an expression similar to (3.5), in momentum space, one could calculate the corresponding proton momentum distribution n(k). It should be noted, however, that in such a case we do not expect the calculated n(k) to be the proper ones, since it is well known [17-18,31] that the momentum distribution is determined by the non-diagonal elements of the one body density matrix.
Next step is to estimate the occupation probabilities n_{nlj} and the depletion of the Fermi sea. Therefore, eqs. (3.6) are solved for n_{nlj}, taking into account the relation (3.13) and the restriction (3.12) (which is justified in the previous section). In table 4 the occupation probabilities n_{nl} of the shells 1s, 1p, 1d, 2s, 1f and 2p are shown. It is seen that as it should be expected the probabilities for the deeper states are much higher than those near the Fermi surface.

Nucleus	n_{1s}	n_{1p}	n_{1d}	n_{2s}	n_{1f}	n_{2p}
^{4}He	0.48	0.17				
^{12}C	0.67	0.53	0.15			
^{16}O	0.69	0.68	0.24	0.08		
^{24}Mg	0.72	0.70	0.46	0.33	0.07	0.02
^{28}Si	0.70	0.69	0.51	0.42	0.16	0.04
^{31}P	0.70	0.69	0.54	0.47	0.21	0.05
^{32}S	0.70	0.69	0.56	0.50	0.25	0.06
^{39}K	0.68	0.67	0.64	0.61	0.38	0.12
^{40}Ca	0.68	0.67	0.65	0.64	0.43	0.14

Table 4. Occupation probabilities n_{nl} for various nuclei.

In the framework of this approach the following values of the depletion of the nuclear Fermi sea are obtained: ^{4}He 53%, ^{12}C 33%, ^{16}O 32%, ^{24}Mg 30%, ^{28}Si 30%, ^{31}P 32%, ^{32}S 34%, ^{39}K 35%, ^{40}Ca 34%. It is observed that apart from ^{4}He where the depletion is very large, for nuclei with $A \geq 12$ the depletion is nearly constant, that is 32$\pm$2%. This has to be expected taking into account that the phenomenon of nuclear saturation causes a stabilization of the interior matter density distribution and the related momentum distribution [1].
The calculated occupation probabilities do not agree well with those obtained in microscopic calculations in nuclear matter [32,33]. However, one should have in mind the simplicity of our model and the fact that the present approach is pure phenomenological. On the other hand, occupation probabilities in nuclei may differ from those in nuclear matter due to surface effects [30,32]. In addition it was stated that the transition from nuclear

matter to finite nuclei is an open theoretical issue and a fully consistent calculation is clearly called for [34].

In table 5 we present the occupation numbers α_q corresponding to our approach (column 4) and the available "experimental values" (which correspond to nuclei in the region of our interest) quoted from table X of ref. 35. These values correspond to a DWIA analysis of Tokyo, Saclay (e,e'p) [36-39] and CERN (p,2p) [40,41] experiments. The (e,e'p) results seem to be more reliable even for deep shells where the (p,2p) reaction fails to give reasonable numbers, due to too high contributions from multiple collision background [35]. In column 3 we give for comparison the occupation numbers of the independent particle model (IPM). Finally in column 7 the numbers in parentheses correspond to a PWIA analysis of the data of ref. 39. It is seen that our results are close to the "experimental" occupation numbers. It should be noted, however, that there are large uncertainties concerning the absolute determination of the occupation numbers, which do not allow one to assert that the occupation numbers are known from experiments to better than $\sim 20\%$ [35]. In addition the various analyses are not entirely model independent [35].

Nucleus	State	IPM	α_q	CERN (p,2p)	TOKYO (e,e'p)	SACLAY (e,e'p)
^{12}C	1s	2	1.34	2.0	1.34	1.0
	1p	4	2.68	1.3	2.6	2.5
^{16}O	1s	2	1.37			1.6
	1p	6	4.09			3.6
^{28}Si	1s	2	1.41	(>11)		0.9
	1p	6	4.15	2.8		2.9
	1d	6	4.14	4.8		5.5
	2s	-	0.85	0.5		0.4
^{40}Ca	1s	2	1.35	(>40)		1.5
	1p	6	4.03	6.0	10.2 (1.8)	5.7
	1d	10	6.5	5.0	8.9 (2.6)	7.7
	2s	2	1.28	0.5	2.0 (0.9)	1.3

Table 5. **Comparison of occupation numbers α_q (n.o) calculated in the present method with the "experimental values". See comments in text.**

In ref. 42 a theoretical expression for $F_{ch}(q)$ is derived in the framework of a harmonic oscillator shell model which depends on parameters describing the occupancy of specific surface orbits while the rest are considered fully occupied. Numerical calculations are reported only for ^{40}Ca where the values of the parameters are obtained by fitting to experimental values of $F_{ch}(q)$, while the harmonic oscillator parameter is determined separately in order to reproduce the experimental rms radius. In our approach, however, the size parameter is free to vary together with n_{nlj}. In addition we demand that $\rho_{n.o} = \rho_{cor}$, where ρ_{cor} comes from a theoretical closed form expression for $F_{ch}(q)$ derived by assuming a particular simple form for the short range correlations and the value of λ is obtained by fitting to experimental $F_{ch}(q)$. It would be interesting to use other forms for the short range correlations in order to observe the corresponding change of n_{nlj}.

A merit of our approach is that, in some way, it establishes a relationship of fractional occupation probabilities with short range correlations. However, this relationship is not completely clear, because we are not able to distinguish the corrections to $F_{ch}(q)$ for large values of q due to short range correlations from the ones due to meson exchange currents. Another limitation is that we approximate the "natural orbitals" with harmonic oscillator wave functions. In any case, our simple method might be useful to the specialist in order to compare with more sophisticated models.

Finally, we would like to make clear, that this approach is an attempt for a rough estimate of the occupation probabilities and the depletion. Our aim is to give the average trend of the variation of the occupation probabilities and the depletion of the Fermi sea in a rather systematic way in the region $4 \leq A \leq 40$. A more precise estimate of n_{nlj} is beyond the capabilities of this approach, having also in mind that the charge distributions alone are not sufficient to provide a reliable estimate of absolute occupation probabilities of single particle orbits [5]. On the other hand, experimentally precise occupation numbers are still uncertain [5]. Therefore, one hopes that the experiments which are planned in NIKHEF, MAMI and those in CEBAF in the near future will shed more light on this problem.

5. Summary

In this paper we propose a simple method for the introduction of short range correlations in the ground state nuclear wave function for nuclei in the region $4 \leq A \leq 40$. In the present approach we adopt correlations of Jastrow type, characterised by the correlation parameter λ and the harmonic oscillator parameter b_1 which are determined by fitting the correlated charge form factor to the corresponding experimental values.

The "natural orbital" representation is employed for the determination of the occupation

probabilities of the shell model orbits of the ground state wave function by imposing the condition the correlated proton distribution to be equal to the corresponding one calculated with "natural orbitals". In this method the effect of short range correlations is taken into account in an effective way and is absorbed in the values of the calculated occupation numbers and the size parameter b_1. The results show that apart from ^4He the depletion of the nuclear Fermi sea is about 32%.
A firm quantitative comparison of the calculated occupation numbers with the "experimental" ones cannot be done due to the large ambiguities of the existing data.

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