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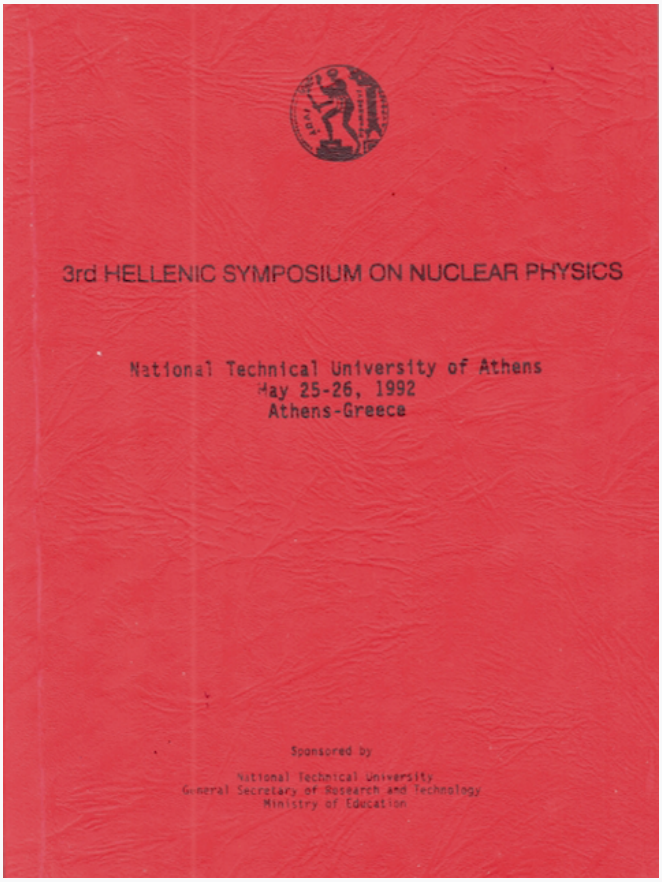

Λ-particle energies from the (π+.K+) associated production reaction on nuclei and the state dependence of the Λ-nucleus potential

G. A. Lalazissis

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Λ-particle energies from the (π^+, K^+) associated production reaction on nuclei and the state dependence of the Λ-nucleus potential

G. A. Lalazissis

Department of Theoretical Physics, Aristotle University of Thessaloniki
54006 Thessaloniki, Greece

Abstract

A Λ-nucleus potential of the symmetrized Woods-Saxon type is mainly used and the Λ-particle energies deduced from the (π^+, K^+) associated production reaction on nuclei are analysed by a least squares fitting. Although a firm quantitative conclusion about the state dependence of the Λ-nucleus potential cannot be drawn on the basis of the existing data from the (π^+, K^+) process, it appears that this dependence is quite weak, as it should be also expected on the basis of other studies. The present analysis indicates that the depth of the symmetrized Woods-Saxon Λ-nucleus potential in the 1p-state is somehow smaller than the depth in the 1s-state, while the range of the potential is slightly longer.

Introduction

The associated production reaction (π^+, K^+) has been very useful in producing bound Λ single particle states in a variety of nuclei [1-5]. The energy levels of this hyperon have been measured by means of this reaction in nuclear targets ranging from ^9Be to ^{89}Y. The peaks observed in the excitation spectra of the (π^+, K^+) reaction in nuclei can be identified with the various orbital angular momentum states s,p,d,f,.. of the Λ hyperon. For the case of the $^{12}C(\pi^+, K^+)^{12}_{\Lambda}C$ reaction where the angular distribution was measured, the idendification of observed peaks with s and p Λ-single particle strength was verified. The basic properties and feasibility of the (π^+, K^+) reaction were theoretically studied firstly by Dover et al [6] and then by Bando and Motoba [7]. We recall that contrary to (K^-, π^-) reaction it has the feature to excite nonsubstitutional stretched-spin states preferentially, since it involves

a substantial momentum transfer of $q \simeq 340$ MeV/c (at $P_\pi = 1040$ MeV/c) and gives the possibility for the study of the medium and heavy hypernuclei.

The usefulness of the new experimental data is that they provide a much better description of Λ binding energies as a function of the mass number A. For the first time we have Λ binding energies for a wide range of mass numbers and in some cases complete sets of bound and first unbound energy levels for nodeless Λ orbitals at fixed A are available [15]. This fact gives the possibility for a better overall fit of the data for the determination of the parameters of properly chosen Λ-nucleus potentials.

In most cases the experimental data are analyzed by considering the Λ particle moving in a local Woods-Saxon potential (see for example [6-13].

$$V(r) = \frac{-D}{1 + e^{(\frac{r-R}{a})}} \tag{1.1}$$

For the potential radius R either the simple expression $R = r_0 A_c^{1/3}$ (A_c being the mass number of the core nucleus) or more complicated ones obtained on the basis of the folding model have been used as in [12] (see also next section) where an overall least squares fit to 1s-state Λ binding energies has been attempted. Dover et al. [6] used the following parametrization for potential (1.1) D=30.7 Mev, $r_0 = 1.1$ fm and $a = 0.6$ fm to predict the A dependence of Λ binding energies. Dover [14], later on, in order to include the A dependence of the radius parameter r_0, proposed in the framework of the folding model the following expression for the radius of the potential (1.1)

$$R = r_0 A_c^{1/3}[1 + \xi(r_0 A_c^{1/3})^{-2}] \tag{1.2}$$

where $\xi = \frac{5}{6} < r^2 >_{\Lambda N}$. The rms radius of the Λ N interaction was taken $< r^2 >_{\Lambda N} \simeq 0.72$ fm, i.e. roughly the two pion exchange range. In this case D = 29.34 MeV, $r_0 = 1.08$ fm and a=0.6 fm. In a recent publication Millener et al. [15] in order to get a further improvement of the fit used a shallower well (D = 28 MeV) which, however, is a little wider, namely the radius parameter r_0 in the expression $R = r_0 A_c^{1/3}$ is now written :

$$r_0 = r_0(A_c) = 1.128 + 0.495 A_c^{-2/3} \tag{1.3}$$

It turns out, however, that for a perfect fit some nonlocality or density dependence of the Λ N interaction is necessery [15].

In all these cases, the depth of the Woods-Saxon well was considered independent of A and l_Λ. This is in agreement with the current belief, that the Λ behaves as a distinguishable particle. Hartree-Fock calculations with Skyrme-type Λ N interactions [16,17] have shown that the depth of the Λ particle well is almost independent of A. The inclusion of ΛNN forces, however, leads to a little deeper wells for light hypernuclei, where their effect is important [17].

The new data from (π^+, K^+) reactions offer also another possibility. Instead of fixing the geometrical parameters from appropriate charge distributions observed for large nuclei or adjusting all the parameters in order to reproduce in both cases the energy of a clear-defined peak of the Λ orbit, they may be used in attempting least squares fittings separately to the available data for Λ binding energies in s_Λ, p_Λ and probably d_Λ orbits and thus, to see for possible state dependence of the parameters.

A symmetrized Woods-Saxon potential has been used for the analysis of Ξ^- hypernuclei [18]. More recently this potential was used in connection with the nuclear case [19] and certain comments were made regarding its form factor, depth and radius. In addition an estimate was attempted of its parameters using various possibilities. The numerical results have shown a remarkable state dependence of the parameters as in the Woods-Saxon case [20].

In this paper we use the symmetrized Woods-Saxon potential to analyse the experimental data of (π^+, K^+) reaction in determining its geometrical parameters in various states of the Λ particle by making an overall least squares fitting to the binding energies of each nodeless single-particle Λ state, mainly in the 1s and 1p state. For the radius of the potential a complicated expression as in [12] is used which is derived in the framework of the folding model and which improves the fit for the lighter hypernuclei.

In section 2 the relevant formalism is presented and in section 3 the numerical results are reported and commented.

The Λ-nucleus potential model

In this paper we follow as in [14] the simple approach approximating the Λ-nucleus interaction with a local potential of the form :

$$V_{\Lambda-A_c}(r) = -V_1 f(r) + V_2 f^2(r) \tag{2.1}$$

using, however, for f(r) the form factor

$$f_{SF}(r) = \frac{\sinh(R/a)}{\cosh(r/a) + \cosh(R/a)}$$
$$= \left[1 + exp(\frac{r-R}{a})\right]^{-1} + \left[1 + exp(\frac{-r-R}{a})\right]^{-1} - 1 \qquad (2.2)$$

where

$$R = r_0(A_c)A_c^{1/3} \qquad (2.3)$$

The radius R characterizes the range of the potential and it is somehow smaller than the "half depth radius", as is the case for the well known Woods-Saxon potential [19]. The potential form factor (2.3) is of symmetrized Fermi type [21-22] and has certain advantages over the one used for the Woods Saxon potential

$$f_{WS} = \left[1 + exp(\frac{r-R}{a})\right]^{-1} \qquad (2.4)$$

Specifically, it has zero slope at the origin, contrary to the f_{WS} form factor, which is not suitable for light hypernuclei. Indeed, for light systems the behaviour of (2.2) inside the hypernucleus is similar to the Gaussian form factor, while at large distances ($r >> R$) it presents regular exponential behaviour. For the medium and heavy hypernuclei practically coincides with that of expression (2.4) for a Woods-Saxon potential.

In this approach, the depth parameter V_1 = D is assumed to be state dependent. Parameter V_2 is taken equal to zero, while the radius parameter r_0 is taken state dependent and also A_c dependent. Finally, the diffuseness parameter a is assumed independent of A_c and is usually fixed to a value, taken from our existing experience. It also turns out, that in the case of the Woods-Saxon hypernuclear potential the surface diffuseness effects influence the coefficients of the third and higher terms of the expansion in powers of A_c for the Λ binding energy B_Λ in the 1s-state [12]. The radius parameter $r_0(A_c)$ is derived by exploiting another advantage in using potential (2.1) The volume integral of this potential is not a transcendental function of the radius R. Thus, one can write (as in the nuclear case [19]) on the basis of the folding model and the well known properties of the convolution [23,24] (and in analogy with the treatment [12] for a Woods-Saxon potential)

$$\frac{4\pi}{3}DR^3(1 + \frac{(\pi a)^2}{R^2}) = A_c|\bar{V}_{\Lambda N}| \qquad (2.5)$$

where $|\bar{V}_{\Lambda N}|$ is the spin-average Λ-N potential. Expression (2.5) (and thus 2.6 and 2.7) have also been derived [12] for a Woods- Saxon potential, but for that potential they hold provided that $e^{-R/a} << 1$, that is apart from the light hypernuclei. The above third order equation is of the same type as the one appearing in the study of the trapezoidall density distribution and can be solved exactly [25], leading to an exact expression of the radius parameter $r_0(A_c)$ in terms of the core mass number A_c :

$$r_0(A_c) = \frac{1}{2^{1/3}} r_0 \left\{ \left[1 + [1 + \frac{4}{27}(\frac{\pi a}{r_0 A_c^{1/3}})^6]^{1/2} \right]^{1/3} + \left[1 - [1 + \frac{4}{27}(\frac{\pi a}{r_0 A_c^{1/3}})^6]^{1/2} \right]^{1/3} \right\}$$

$$= r_0 \left[1 - \frac{1}{3} d^2 A_c^{-2/3} + \frac{1}{81} d^6 A_c^{-2} + \frac{1}{243} d^8 A_c^{-8/3} + ... \right] \qquad (2.6)$$

where

$$r_0 = \left(3|\bar{V}_{\Lambda N}|/4\pi D\right)^{1/3} \quad and \quad d = (\frac{\pi a}{r_0}) \qquad (2.7)$$

Expression (2.6) for the radius parameter is of similar form with those proposed in [14,15]. Both are derived in the framework of the folding model. They have, however, different origin. The A dependence in [14,15] is established by using the expression with the second moments $< r^2 >_V = < r^2 >_\rho + < r^2 >_{\Lambda N}$ (see also [8]), where $< r^2 >_V$ is the m.s radius of the Λ nucleus potential, $< r^2 >_\rho$ the m.s radius of the point nucleon density of the host nucleus and $< r^2 >_{\Lambda N}$ the corresponding one of the Λ N effective interaction usually taken of Yukawa form with 2π, or sometimes, K exchange range. The A dependence in the approach described above is established by evaluating the volume integral of the Λ-nucleus potential and solving equation (2.5) for A_c. The parameter r_0 in (2.6) is expected to be larger than the ones in (1.2), (1.3) since the second term has negative contribution. In addition, the first term of all these expressions, which is independent of A_c leads to the well known expression for the radius R ($R = r_0 A_c^{1/3}$) commonly used for a Woods Saxon potential. It was shown, however, that the use of such an asymptotic form leads to values of the radius R which deviate from those obtained with (2.6), particularly for light hypernuclei. In the region of ${}^{13}_{\Lambda}C$, for example, the relative difference is of about 10-15% [12].

In view of the remarks made above the use of potential (2.1) with an A_c dependent expression for the radius parameter seems desirable in the case of a global fit of the experimental Λ binding energies of both light and heavier hypernuclei.

Numerical results and comments

In this section we shall attempt an estimate of the parameters of the hypernuclear potential. At this point we note that due to (π^+, K^+) associated production reaction on nuclei, experimental data for the Λ binding energies are available for the ground as well as the excited states for a significant range of A ($9 \leq A \leq 89$). Thus, one expects to get a more consistent fit for the parameters, since it is not necessary to use data (in most of the cases for the ground state) from measurements based on different experiments and with different degree of accuracy, as was the case in previous studies [12,26-27].

In this approach we try to determine the geometrical parameters r_0 and a and the depth of the well in the framework of the same fitting procedure, separately for 1s and 1p states. Thus, a number of B_Λ first for the 1s-state and then for the 1p-state are least squared fitted (in a way analogous to that suggested by Bodmer and Rote for the ground state B_Λ and the square well potential [28]) and no other consideration is made. The experimental values for the Λ binding energies are quoted from [2,3,5].

Unfortunately, the existing data are not enough for a reliable determination of all three parameters. We can reduce the number of the parameters by assuming a constant value for the diffusivity a. Thus, setting a=0.6 fm we proceed to the determination of the potential depth D and of the radius parameter r_0 using experimental values with A in the region $16 \leq A \leq 89$, that is, the ones obtained by the associated Λ production through the (π^+, K^+) reaction on the following nuclear targets $^{16}O, ^{28}Si, ^{40}Ca, ^{51}V and\ ^{89}Y$. The obtained best fit values for the 1s-state are : D= 25.67 Mev r_0= 1.43 fm and for the 1p-state: D= 23.70 MeV r_0= 1.44 fm. It is seen that the results indicate that the depth is a little smaller in the 1p-state in comparison with that in 1s-state , the relative difference being 7.7%. It is also seen that the calculated radius parameters lead to a sligthly longer range of the potential in the 1p-state.

It is desirable, however, to have a better estimate for the diffuseness. In view of this, we try another possibility. We use more "experimental" data via an interpolation procedure. Namely, apart from the experimental values of hypernuclei with $A \geq 28$ (displayed in table 1.), "experimental" values of hypernuclei with mass numbers 34, 46, 64, 77 were used. In this case the obtained parameters are: for the 1s-state D= 27.65 MeV, r_0= 1.329 fm, a= 0.65 fm, while for the 1p-state D= 25.10 MeV, r_0= 1.362 fm, a= 0.58 fm. It is seen that we obtain again similar results, the relative difference of the depths being now 9%. It is noted that the decision to include in the fitting procedure hypernuclei with $A \geq 28$ is related

with the fact that for lighter elements some A- dependence of the parameters seems to be necessary. It is also noted that by perfoming a series of fits, including now lighter elements we obtained rather unrealistic values for the diffuseness (for example for the 1p-state a value larger than 0.9 fm is obtained) which might be an indication of the surface effects. Therefore the above mentioned fit (with A$\geq$ 28) seems to be the most reliable among those considered in the case where all three parameters are free and D and a are A_c independent. Another quantity which can be also calculated using the values of the parameters is the volume integral J of the Λ-nucleus potential. This is given by the LHS expression of (2.5), namely:

$$J = \frac{4\pi}{3} D R^3 (1 + \frac{(\pi a)^2}{R^2})$$

This quantity is of interest because it gives a measure of the "total strength" of the Λ-nucleus interaction. Our numerical results show that the difference in the volume integrals of 1s and 1p state is very small, the relative difference $(J_s - J_p)/J_s$ being 6.4% (fixed a) and 2.4% for the two variants of the fitting procedure.

In table 1, as an example, the experimental B_Λ values and the theoretical predictions for the 1s and 1p-state together with the radii of the potential well and the corresponding volume integrals of the Λ-nucleus interaction are displayed for hypernuclei (for which experimental values for the Λ energies are available) used in the fitting procedure ($A \geq 28$).

	$B_{1s}^{exp.}$	$B_{1s}^{theor.}$	R_{1s}	J_{1s}	$B_{1p}^{exp.}$	$B_{1p}^{theor.}$	R_{1p}	J_{1p}
${}^{28}_{\Lambda}Si$	16.00	16.00	3.637	7338	6.90	6.90	3.817	7164
${}^{40}_{\Lambda}Ca$	18.70	18.40	4.197	10600	10.50	10.10	4.380	10344
${}^{51}_{\Lambda}V$	19.90	19.70	4.611	13593	12.00	12.00	4.798	13265
${}^{89}_{\Lambda}Y$	22.10	22.20	5.675	23922	16.00	15.80	5.876	23347

Table 1. Experimental and theoretical 1s and 1p-state Λ energies and the corresponding potential radii and volume integral. The energies are in MeV, the distances in fm, while the volume integrals in MeVfm^3.

For the sake of comparison we used also other hypernuclear potental models, like the square well and the one used in [27], following the same procedure. These potential models suffer,

however, from the fact that they are not proper for an overall fitting with both relatively light and heavier hypernuclei due to their different ranges of validity. The results we obtained show also small differences of the parameters between 1s and 1p-state but they are in contradiction with the ones obtained previously, that is the strength is not smaller in the 1p-state. This, as it was already pointed out, might be due to the fact that the former is better for the heavier elements while the latter for the lighter ones.

In conclusion our analysis of the Λ binding energies obtained from the (π^+, K^+) reaction on nuclei show a very weak state dependence of the depth of the Λ-nucleus potential contrary to the nuclear case where a remarkable state dependence has been observed. This is in agreement with other different studies [14,15]. The existing data do not permit a thorough analysis, but in general the results reported here are consistent with the picture of the Λ as a distinguishable particle.

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