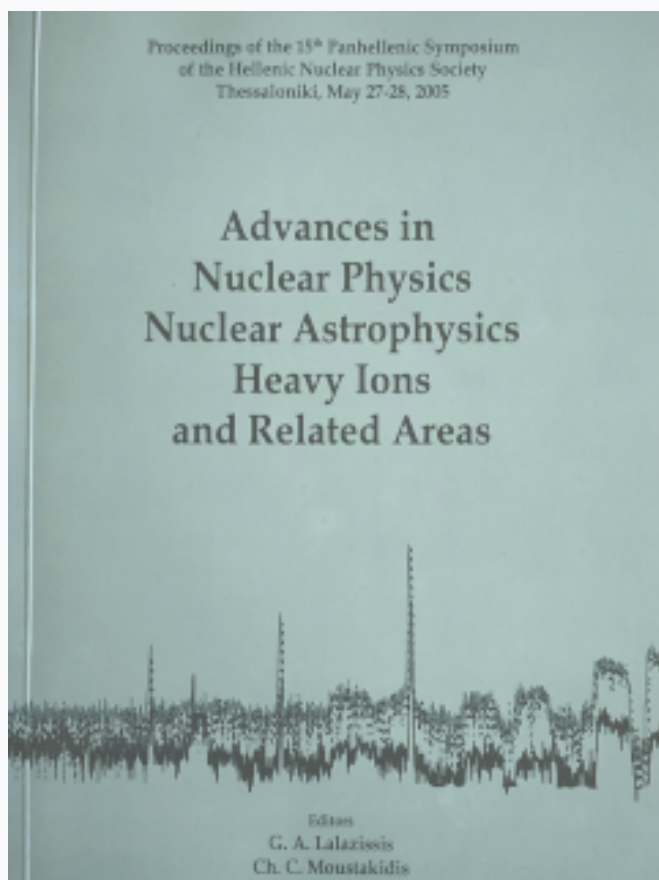


## HNPS Advances in Nuclear Physics

Vol 14 (2005)

HNPS2005



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doi: [10.12681/hnps.2270](https://doi.org/10.12681/hnps.2270)

#### To cite this article:

Petridou, T. (2019). A study on the ( $\pi^-$ ,  $K^+$ ) reaction production of  $\Sigma$ -hypernuclei. *HNPS Advances in Nuclear Physics*, 14, 185–190. <https://doi.org/10.12681/hnps.2270>

# A study on the $(\pi^-, K^+)$ reaction production of $\Sigma$ -hypernuclei

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## Abstract

The  $(\pi^-, K^+)$  reaction production was used recently to produce medium to heavy  $\Sigma$ -hypernuclei with good statistics. The observed spectra do not contain peaks and have a wide energy range. These spectra were studied within the Green function method with a simplified model. The investigation showed that a strong repulsive potential with medium imaginary and spin-orbit parts describes well the spectra.

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## 1 Introduction

The  $\Sigma$ -nucleus interaction, even though is studied for more than 25 years [1,2] is still unclear due to the limited experimental data coming from three sources: First, the hyperon-nucleon scattering data described by  $\Sigma N$  potentials based on one boson exchange models which allow many parameters. Second, the  $\Sigma^-$  atomic X-ray transitions which also involve potentials with many parameters inside the nucleus [3,4]. Third and most important, the  $\Sigma$ -hypernuclei:  ${}^4_{\Sigma}He$ ,  ${}^6_{\Sigma}Li$ ,  ${}^7_{\Sigma}Li$ ,  ${}^9_{\Sigma}Be$ ,  ${}^{12}_{\Sigma}C$ ,  ${}^{16}_{\Sigma}O$  were produced via the  $(K^-, \pi^\pm)$  reactions but no definite conclusion was reached about the nature of the  $\Sigma$ -nucleus potential [1,2].

The only  $\Sigma$ -hypernuclear bound state confirmed so far is the one of  $(K^-, \pi^-)$   ${}^4_{\Sigma}He$  and is due to the Lane potential [2]. There are many contradictory data and theories [1,2] concerning the existence of peaks in  $\Sigma$ -hypernuclei. Many models put constraints to the potential: shallow attractive or repulsive, see [5–8] and refs therein.

This is also connected to the *neutron stars* evolution since  $\Sigma^-$  are the hyperons that seem to appear first in dense nuclear matter; in this case, the sign of the

$\Sigma$ -nucleus potential (repulsive or attractive) is significant [3,4].

So, recently the production of  $(\pi^-, K^+)$  *medium to heavy*  $\Sigma$  *hypernuclei* has started in KEK to study the  $\Sigma$ -nucleus potential (sign and strength) [9,10]. This method has been proved successful in  $\Lambda$ -hypernuclei and is complementary to the other one.

## 2 The model

The method used for the calculation of the spectra is the *Green function method (GFM)* [5,7], where the response function or production strength is given by the formula:

$$S(E) = -\frac{1}{\pi} \text{Im} F(E) \quad (1)$$

The strength function  $F(E)$  contains  $f_L^M(r)$ :

$$f_{\ell,j}^L(E) = \sum_M \int_0^\infty dr \int_0^\infty dr' [f_L^M(r)]^* G_{\ell,j}(E; r, r') f_L^M(r') \quad (2)$$

The weight function  $f_L^M(r)$  contains the nucleon and meson wavefunctions  $u_{\ell N j N}(r)$ ,  $\chi_\pi$  and  $\chi_K$ . The Green function is:

$$G_{\ell,j}(E, r, r') = -\frac{2m}{\hbar^2} \frac{\phi(r_<)\psi^{(+)}(r_>)}{W[\phi(t), \psi^{(+)}(t)]_{t=R}} \quad (3)$$

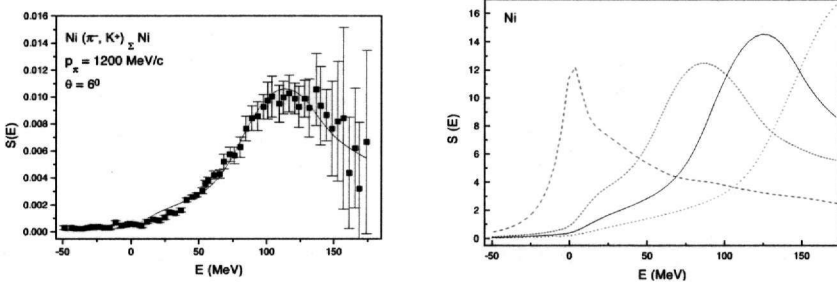


Fig. 1. a. The calculated spectrum for  $V_c = (70, -i15) \text{ MeV}$ ,  $V_{so} = 15 \text{ MeV}$  for the KEK experiment for  $\Sigma Ni(\pi^-, K^+)$  together with the experimental spectrum [10]. b. The calculated spectra for the KEK experiment [10] for  $\Sigma Ni(\pi^-, K^+)$  when  $\text{Re}V_c$  takes the values: -5, 50, 80, 100 MeV from the left to the right respectively and  $\text{Im}V_c = -15 \text{ MeV}$ ,  $V_{so} = 15 \text{ MeV}$ .

The functions  $\phi(r)$  and  $\psi^{(+)}(r)$  are the regular and Jost solutions of the Schrödinger equation. The Green function depends only on the hyperon-nucleus potential given by the formula:

$$V(r) = V_c w(r) + V_{so}(\vec{l} \cdot \vec{s}) r_o^2 (1/r) [dw(r)/dr] \quad (4)$$

The potential depths  $V_c$  and  $V_{so}$  are generally complex. The  $ImV_c$  simulates the  $\Sigma$  to  $\Lambda$  conversion. In this work, a density  $w(r)$  of rectangular shape is used for the nucleon distribution.

The meson wavefunctions are described with plane waves. For the nucleon interaction the nucleon is considered in the outer (valence) shell in the harmonic oscillator model with  $b = \sqrt{\hbar/m\omega}$  being  $A$  depended [11]. The Coulomb interaction on the  $\Sigma^-$  is omitted. The radius parameter  $r_o$  of  $R = r_o(A - 1)^{1/3}$  is 1.31 fm.

A normalization is used for the comparison between the *experimental* and the *theoretical* values; the theoretical predictions are scaled in order to give the same integrated cross sections as those obtained experimentally.

### 3 Description of the $(\pi^-, K^+)$ $\Sigma$ hypernuclear spectra

A new series of experiments has started in KEK the last 5 years in order to enrich the experimental situation of  $\Sigma$ -hypernuclei and to clarify the  $\Sigma$ -nucleus potential [9,10]. They produced  $(\pi^-, K^+)$  spectra for medium to heavy nuclei with the Superconducting Kaon Spectrometer which has been proved successful to  $\Lambda$ -hypernuclei via the elementary interaction:  $p + \pi^- \rightarrow K^+ + \Sigma^-$ . They derived  $\Sigma$ -hypernuclei for C, Si, Ni, In and Bi with good statistics and

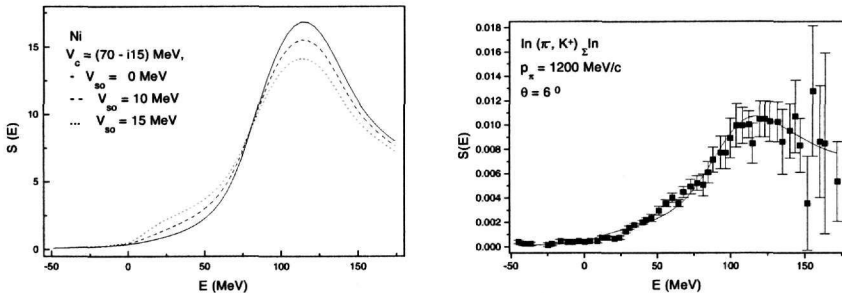


Fig. 2. a. The calculated spectra for  $V_c = (70, -i15)$  MeV and  $V_{so} = 0, 10, 15$  MeV for the KEK experiment for Ni [10].

b. The calculated spectrum for  $V_c = (80, -i20)$  MeV,  $V_{so} = 15$  MeV for the KEK experiment for  $\Sigma In(\pi^-, K^+)$  together with the experimental spectrum [10].

increasing resolution (3.3, 4.4, 4.8 and 5.2 respectively starting from Si). The observed spectra do not contain any peaks and have a wide energy range which is contrary to  $\Lambda$ -hypernuclei. What is interesting in the results is the  $A$  dependence of the spectra. When  $A$  increases, the spectrum is stronger and the maximum is shifted to the right.

They also did theoretical calculations in the context of the GFM using a Woods-Saxon potential and the eikonal distortions. They found that in the case of Si, the potential with parameters  $V_c = (150 - i15) \text{ MeV}$ ,  $V_{so} = 0 \text{ MeV}$  reproduced best the data. In the case of Ni and Bi they found the potential of  $ReV_c = 90 \text{ MeV}$  and  $ImV_c = -10, -5 \text{ MeV}$  respectively, but the representation is not good. It is good only at small energies.

Using the proposed model a similar investigation was made taking many potential values. The case of Si was examined in [8]. It was found that the potential  $V_c = (70 - i15) \text{ MeV}$ ,  $V_{so} = 12 \text{ MeV}$  represents well the data.

First, the case of Ni is examined. In fig. 1a the best spectrum for the potential  $V_c = (70, -i15) \text{ MeV}$ ,  $V_{so} = 15 \text{ MeV}$  is shown. The fit with the data [10] is quite satisfactory. In fig. 1b the  $ReV_c$  is varied from a shallow attractive to a strong repulsive potential: -5, 50, 80, 120 MeV. Then the spectrum is shifted to larger energies and the maximum becomes stronger. The change of the spectrum with  $ImV_c$  and  $V_{so}$  is also studied. In fig. 2a the variation with  $V_{so}$  is shown; its increase broadens and weakens the spectrum. The variation with  $r_o$  is also studied; a smaller radius parameter  $r_o$  increases the cross section in small energies.

Next, the case of In is examined. In fig. 2b the best spectrum for the potential  $V_c = (80, -i20) \text{ MeV}$ ,  $V_{so} = 15 \text{ MeV}$  is shown that fits well the data [10]. In fig. 3a the  $ReV_c$  is varied again from a shallow attractive to a strong repulsive potential taking the values -5, 50, 80, 100 MeV. Then the broad enhancement

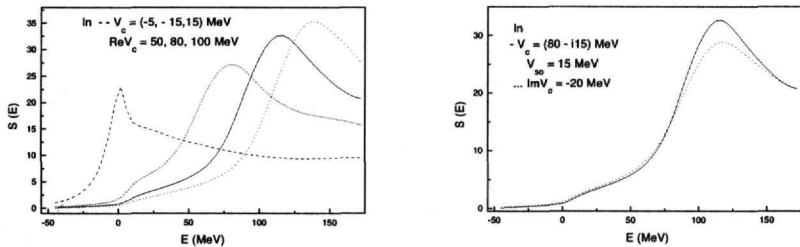


Fig. 3. a. The calculated spectra for the KEK experiment [10] for  $\Sigma In(\pi^-, K^+)$  when  $ReV_c$  takes the values: -5, 50, 80, 100 MeV from the left to the right respectively and  $ImV_c = -15 \text{ MeV}$ ,  $V_{so} = 15 \text{ MeV}$ . b. The calculated spectra for  $V_c = (80, -i15) \text{ MeV}$ ,  $V_{so} = 15 \text{ MeV}$  and also  $ImV_c = 20 \text{ MeV}$  for the KEK experiment for In [10].

is shifted to larger energies and the maximum becomes greater. The change that the variation of  $ImV_c$  and  $V_{so}$  brings to the spectrum is studied again; In fig. 3b the variation with  $ImV_c$  is shown; its decrease broadens and weakens the spectrum.

Last, the case of Bi is examined. In fig. 4a the best spectrum for the potential parameters  $V_c = (80, -i23) \text{ MeV}$ ,  $V_{so} = 25 \text{ MeV}$  is shown which is successful in representing the data [10]. In fig. 4b the  $ReV_c$  is varied taking the values -5, 50, 70, 90 MeV. The same behaviour is valid as for the previous elements. The first spectrum is smaller. The change that the variation of  $ImV_c$  and  $V_{so}$  brings to the spectrum was also studied with the same characteristics.

The case of C is also studied. A very preliminary investigation showed that the potential  $V_c = (30 - i15) \text{ MeV}$ ,  $V_{so} = 0 \text{ MeV}$  can represent the data. Also that  $V_{so}$  changes strongly the spectrum; when it is augmented, some peaks arise.

Finally, a comparison with the  $(K^-, \pi^\pm)$  spectra can be done. In the  $(\pi^-, K^+)$  spectra there is wider energy scale and more peaks appear due to  $V_{so}$ . The results are in agreement with the other theoretical predictions [12–14].

#### 4 Conclusions

The GFM with this model is successful to describe the data for  $(\pi^-, K^+)$  spectra for medium to heavy  $\Sigma$ -hypernuclei.

The *best potential* to represent the experimental spectra from Si to Bi is a strong repulsive one with medium imaginary and spin-orbit parts. More specif-

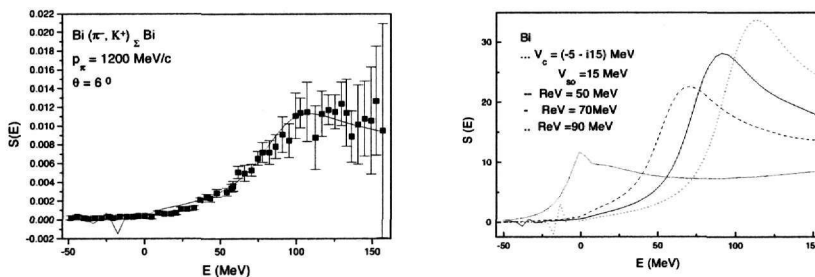


Fig. 4. a. The calculated spectrum for  $V_c = (80, -i23) \text{ MeV}$ ,  $V_{so} = 25 \text{ MeV}$  for the KEK experiment for  $\Sigma Bi(\pi^-, K^+)$  together with the experimental spectrum [10]. b. The calculated spectra for the KEK experiment [10] for  $\Sigma Bi(\pi^-, K^+)$  when  $ReV_c$  takes the values: -5, 50, 70, 90 MeV from the left to the right respectively and  $ImV_c = -15 \text{ MeV}$ ,  $V_{so} = 15 \text{ MeV}$ .

ically: i) The  $ReV_c$  must be strong repulsive for the best representation of the spectrum (stronger for larger  $A$ ) since a shallow attractive potential is inappropriate. The increase of  $ReV_c$  shifts the broad enhancement to the right and increases the maximum; ii) The  $ImV_c$  plays some role. When it is smaller, the spectrum is broadened and weakened (better for large  $A$ ). iii) The  $V_{so}$  is important. When it increases, there is broadening and weakening of the spectrum (good for great  $A$ ) and also some peaks appear in small energies (more in small  $A$ ).

Some of the *perspectives* of this work are: amelioration of the formalism, inclusion of the *Coulomb potential* (important for large  $A$ ), inclusion of the *Lane potential* which is important for heavier nuclei with large  $N-Z$  (inversely dependent on  $A$ ) and more thorough investigation for each element, including study of  $A$  and  $Z$  dependence.

**Acknowledgements.** I would like to thank Prof. Daskaloyannis, Prof. Grypeos and Prof. Koutroulos for useful informations.

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