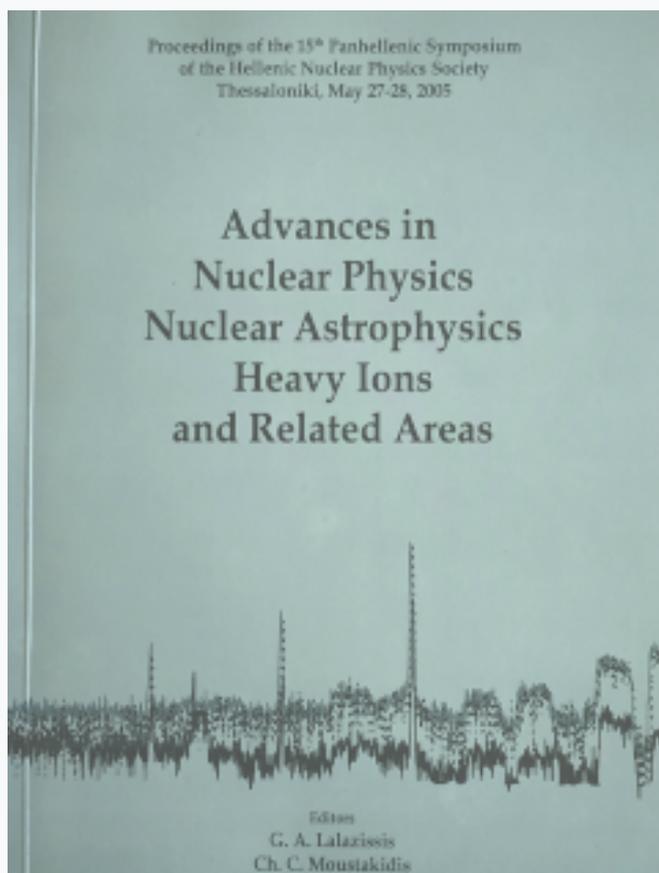


## HNPS Advances in Nuclear Physics

Vol 14 (2005)

HNPS2005



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doi: [10.12681/hnps.2249](https://doi.org/10.12681/hnps.2249)

#### To cite this article:

Fossion, R., & Lalazissis, G. A. (2019). Shape Coexistence In The Neutron-Deficient Lead Isotopes, As Seen From Within Complimentary Models. *HNPS Advances in Nuclear Physics*, 14, 59–64. <https://doi.org/10.12681/hnps.2249>

# Shape Coexistence In The Neutron-Deficient Lead Isotopes, As Seen From Within Complimentary Models

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## Abstract

In the neutron-deficient lead isotopes, at low excitation energies, the nucleus does not have one preferred excitation mode. Instead, three different coexisting "families" of excitation states are found in the excitation spectrum. Theoretical approaches are offered within the mean-field models (both relativistic and non-relativistic) and within the Interacting Boson Model (IBM). Recently, a third approach using boson-coherent states has shown to offer a possible bridge between the former two approaches.

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## 1 Introduction

Ample evidence has been accumulated for the presence of nuclear shape coexistence phenomena throughout the whole table of isotopes, especially at and near closed shells (1; 2). The neutron-deficient lead isotopes in particular, with a closed proton shell at  $Z = 82$ , show very rich excitation spectra. Three "families" of excited states are observed, with different spectroscopic properties, and with a behaviour that strongly depends on the neutron number (see e.g. Ref. (3) and the right-hand panel of Figure 1). The low-lying excited  $0^+$  states have been interpreted within two different frameworks mainly: the shell model and the mean field, respectively.

## 2 Theoretical approaches

### 2.1 Nuclear shell model and the Interacting Boson Model (IBM)

In a shell-model picture, the excited  $0^+$  states are generated by multi-particle multi-hole ( $mp$ - $mh$ ) proton excitations across the closed proton  $Z = 82$  shell gap. The excitation energies of these intruder states are lowered by the residual proton-neutron interaction. The  $mp$ - $mh$  excitations cannot be easily handled in full-scale shell model calculations, in particular for the large model space required for the description of heavy open-shell nuclei. They are, therefore, treated with the help of algebraic models, such as the Interacting Boson Models (IBM) (see Ref. (4)).

An IBM1-mixing calculation has been proposed, that describes the three different intrinsic "shape" configurations, and the interaction (mixing) between the three configurations (see Ref. (5)). In order to reduce the number of parameters that appear in such a configuration-mixing calculation, use is made of the concept of intruder-spin symmetry, relating configurations from neighbouring isotope series with different numbers of particle ( $N_p$ ) and hole ( $N_h$ ) bosons (i.e., fermion pairs), but with a constant total number of bosons ( $N = N_p + N_h$ ). In this way, experimental excitation energies in adjacent Pt and W nuclei are used to fix the essential IBM parameters. The results of this three-configuration mixing calculation can be appreciated in Figure 1 (left-hand panel) and in Ref. (5). In a subsequent publication, also the electromagnetic transitions were studied in the Pb-188 isotope (see Ref. (6))

### 2.2 Mean-field models

In mean-field models (see ref. (7) for a recent review article), the  $0^+$  states observed at low energies are associated with coexisting energy minima which appear for different values of the axial quadrupolemoment. The ground state corresponds to the spherical minimum and the excited  $0^+$  levels to deformed states with an oblate or a prolate shape.

Between the most sophisticated of the relativistic mean-field models is the relativistic Hartree-Bogoliubov (RHB) model, where mean-field and pairing correlations are treated in a unified and self-consistent manner, so that the structure of weakly bound nuclei far from the stability line can be described. Within this framework, a study of the potential energy surfaces (PES) of the neutron-deficient Pb isotopes has been performed as function of the quadrupole deformation, using NL3 + Gogny D1S as the effective interaction (see the plots with the filled diamonds in Figure 2 and Ref. (8)).  $^{182}\text{Pb}$  and  $^{184}\text{Pb}$  have

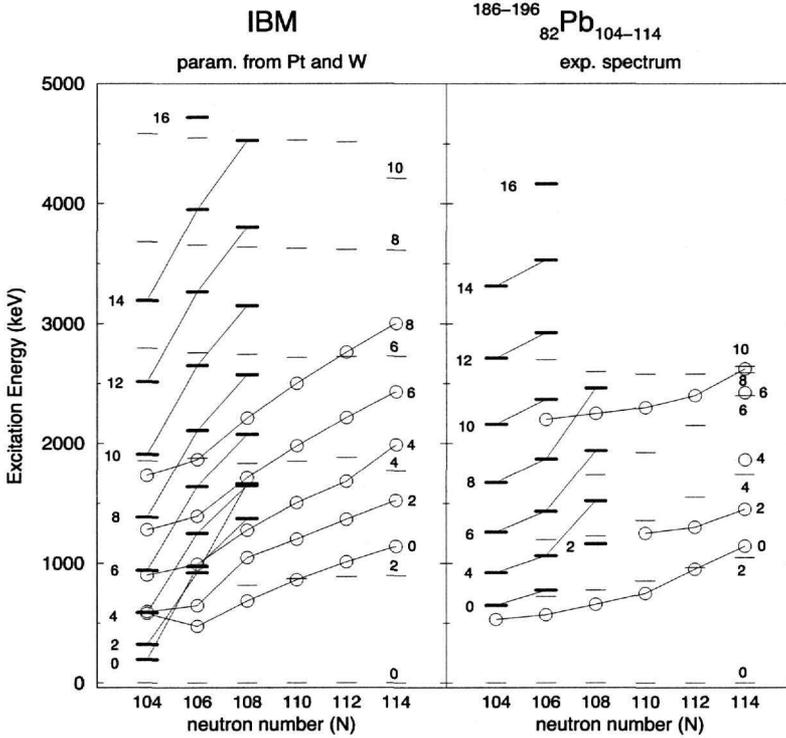


Figure 1. Right panel: the experimental excitation spectrum of the Pb isotopes with  $A = 186$  to  $196$ , containing a regular ground-state spherical band (thin horizontal lines), a 2p-2h intruder band (open circles) and a 4p-4h intruder band (thick horizontal lines). Left panel: A 3-configuration mixing calculation within the IBM1 model, for the Pb isotopes in the same mass region (5).

spherical ground states and, and low-lying oblate and prolate minima, in qualitative agreement with experimental data. With increasing neutron number, however, the oblate minimum is lowered in energy. The nuclei  $^{186-194}\text{Pb}$  have either oblate ground states, or the spherical and oblate minima in the PES surface differ only by some 10 keV in excitation energy. This is in contradiction with the experimental data, which shows that all Pb isotopes maintain a spherical ground state. Only from  $A = 196$ , the calculated Pb ground states become again spherical. It was suggested (8) to re-adjust some the NL3 parameters in such a way that the theoretical  $Z = 82$  shell gaps of all the neutron-deficient Pb isotopes correspond as well as possible to the experimental ones, as the size of the proton shell gaps is seen as the main cause of the shape coexistence phenomenon in the lead isotopes (see section 2.1). Using this adjusted force, the theoretical description of the Pb isotopes is indeed improved, but the description of nuclei in other mass regions gets worse, which is in contrast with the mean-field philosophy. A better approach is to review the NL3 force in a general way, with the wealth on new experimental information on exotic nuclei all over the chart of isotopes, that has become available since the NL3

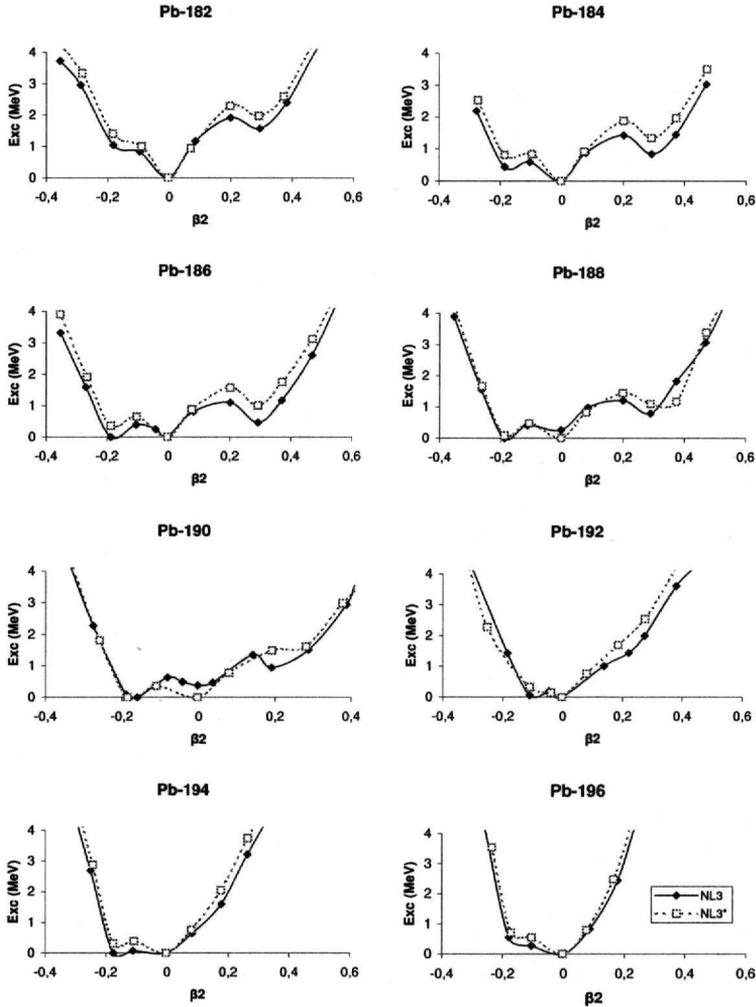


Figure 2. Potential Energy Surfaces (PES) of even- $A$  neutron-deficient Pb isotopes as a function of the quadrupole deformation. The curves correspond to RHB model solutions with constrained quadrupole deformation. The effective interaction is NL3 (filled diamonds) or NL3\* (open squares) + Gogny D1S

force was first presented. A new force: NL3\*, or NL3-revisited, has been fitted to the bulk properties of a number of spherical and exotic nuclei, containing an as large variation as possible in nuclear isospin (see Ref. (9)). Results of the improved description of exotic mass regions by the revised force, NL3\*, will be published in a forthcoming article (see Ref (10)). For the neutron-deficient Pb isotopes, the problem with the unphysical oblate ground states appears to be solved, as all isotopes maintain a spherical ground state now (see Figure 2, plots with the open squares). The isotopes  $^{188,190}\text{Pb}$  have a spherical

minimum, and an oblate minimum that lies some 10 keV higher. These results appear to confirm that a careful fit of the effective forces to the latest available experimental data including a large variation in nuclear isospin, indeed result in a better description of nuclear properties, especially in regions far from the stability line.

However, shape coexistence in the neutron-deficient Pb region cannot be described on the level of mean-field models in a fully satisfactory way. The minima obtained as a function of the quadrupole moment are rather shallow and dynamical effects such as quadrupole vibrations may affect the very existence of these minima. The quadrupole dynamics of the Pb isotopes can however be studied by performing a configuration mixing of mean-field states with different axial quadrupole momentum (see e.g. the Refs. (11) and (12)). The variational configuration mixing with respect to a collective coordinate, the axial quadrupolemoment in this work, removes the contributions to the ground state coming from collective vibrations, and simultaneously is able to provide the excitation spectrum corresponding to this mode. In this way, also the excited bands built on the coexisting minima can be studied.

### *2.3 Phenomenological bandmixing*

Apart from the mean-field and the shell model, a third, purely phenomenological approach has also been used in order to interpret the experimental findings: the shape-mixing picture (13). In this model, the physically observed states are the result of interactions between the several configurations. They result as a superposition of spherical, oblate and prolate configurations, the relative weights in the mixing being determined by a fit to the experimental data.

### *2.4 Matrix coherent states*

Coherent states have been introduced for the geometric interpretation of the (single-configuration) IBM. For any IBM Hamiltonian a potential energy surface (PES) can be generated that depends on the quadrupole-shape variables  $\beta$  and  $\gamma$ , which characterize the deviation from spherical symmetry and from axial symmetry, respectively. The minima of these surfaces are seen to correspond, for example, to the expected shapes associated to the different dynamical symmetries which define the benchmark solutions of the IBM. Transitional regions between dynamical symmetries can be studied in full generality and the shape transitions analyzed for arbitrary Hamiltonians. These studies involve a single  $N$ -boson subspace, which corresponds to the ground shell-model configuration, while particle-hole excitations are not taken into account. In ref. (14)

a general geometric analysis was introduced for the three-configuration mixing study of the Pb isotopes in the IBM framework (see section 2.1). The results are in good agreement with mean-field calculations (as e.g. the calculations presented in ref. (3)), and the framework provides a simple means for studying the evolution of the potential energy surface as a function of (in this case) neutron number.

### 3 Conclusion

The different theoretical approaches that are used, start from physical principles that often are complementary. As such, the results obtained in one model tend to complement the results from other models, possibly leading to the full understanding of the whole problem.

### Acknowledgements

R.F. is grateful to the IWT (Flanders, Belgium) and the European Community (Contract No. 2000-00084) for financial support to the research described in sect. 2.1, and to the I.K.Y. (Greece) for financial support to the research described in sect. 2.2.

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