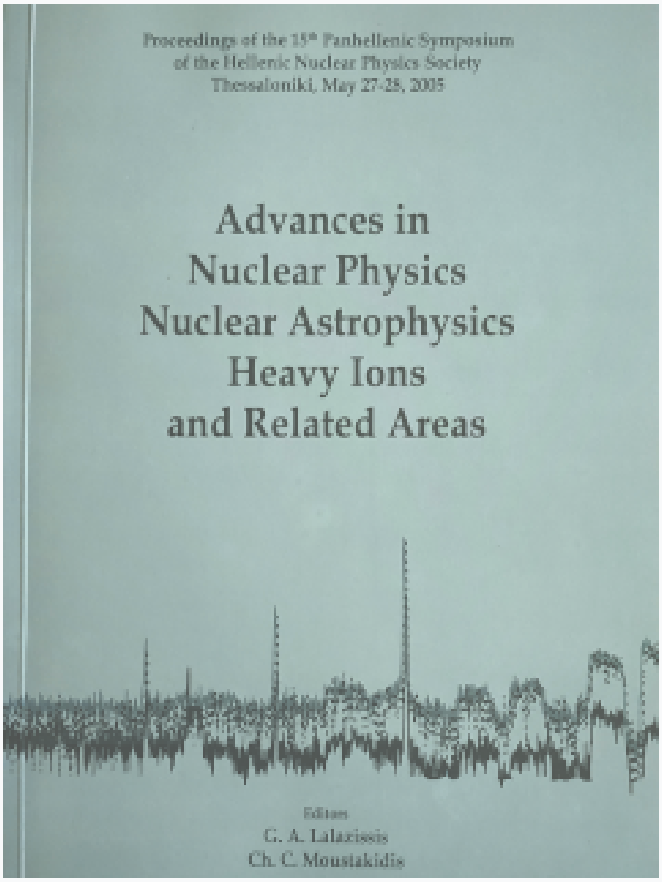
HNPS Advances in Nuclear Physics

Vol 14 (2005)

HNPS2005



Doubly critical character of the N=90 isotones 150Nd, 152Sm, 154Gd, and 156Dy

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doi: 10.12681/hnps.2246

Doubly critical character of the N=90 isotones ^{150}Nd, ^{152}Sm, ^{154}Gd, and ^{156}Dy

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Abstract

The N=90 isotones ^{150}Nd, ^{152}Sm, ^{154}Gd, and ^{156}Dy, which are known to provide the best examples of the X(5) critical point symmetry between quadrupole vibrations [U(5)] and axial quadrupole deformation [SU(3)], are proved to lie on the border between the regions of stable axial octupole deformation and octupole vibrations, described in terms of an Analytic Quadrupole Octupole Axially symmetric (AQOA) model including tunneling effects.

1 Introduction

The N=90 isotones ^{150}Nd [1], ^{152}Sm [2], ^{154}Gd [3,4], and (to a lesser extend) ^{156}Dy [4,5] are known [6] to be the best examples of realization of the critical point symmetry X(5) [7], which corresponds to the transition from vibrational to rotational behaviour in the quadrupole degree of freedom. Using the Analytic Quadrupole Octupole Axially symmetric (AQOA) model [8] we are going to show that these nuclei also correspond to the border between the regions of octupole deformation and octupole vibrations [9].

In Section 2 of the present paper the inclusion of tunneling effects in the AQOA model will be discussed, while in Section 3 numerical results and comparisons to experiment will be given, the conclusions being summarized in Section 4.

2 Analytic Quadrupole Octupole Axially symmetric (AQOA) model with tunneling

The AQOA model has been described in another article in this volume [8], to which the reader is referred. Separation of variables was achieved by assuming the potential to be of the form $u(\tilde{\beta}, \phi) = u(\tilde{\beta}) + u(\tilde{\phi}^{\pm})$, with

$$u(\tilde{\phi}^{\pm}) = \frac{1}{2}c(\phi \mp \phi_0)^2 = \frac{1}{2}c(\tilde{\phi}^{\pm})^2, \qquad \tilde{\phi}^{\pm} = \phi \mp \phi_0, \tag{1}$$

leading in the case of an infinite well potential ($u(\tilde{\beta}) = 0$ if $\tilde{\beta} \leq \tilde{\beta}_W$; $u(\tilde{\beta}) = \infty$ if $\tilde{\beta} > \tilde{\beta}_W$) to Bessel eigenfunctions with

$$\nu = \sqrt{\frac{L(L+1)}{3(1+\sin^2\phi_0)} + \frac{3}{\sin^2 2\phi_0}}. \tag{2}$$

An alternative way of separating variables is to consider potentials of the form $u(\tilde{\beta}, \phi) = u(\tilde{\beta}) + u(\tilde{\phi}^{\pm})/\tilde{\beta}^2$ [10], with $u(\tilde{\phi}^{\pm})$ still given by Eq. (1). In this case the ϕ-equation results [11] in

$$\left[-\frac{\partial^2}{\partial(\tilde{\phi}^{\pm})^2} + u(\tilde{\phi}^{\pm}) - \epsilon_\phi\right]\chi(\tilde{\phi}^{\pm}) = 0, \qquad \epsilon_\phi = \sqrt{2c}\left(n_\phi + \frac{1}{2}\right) \mp \delta_{n_\phi}, \tag{3}$$

where n_ϕ is determined from a transcendental equation [11], while δ is the splitting occuring because of the tunneling between the two wells, depending on ϕ_0 and c. Then the $\tilde{\beta}$-equation reads

$$\left[-\frac{\partial^2}{\partial\tilde{\beta}^2} - \frac{1}{\tilde{\beta}}\frac{\partial}{\partial\tilde{\beta}} + \frac{1}{\tilde{\beta}^2}\left(\frac{L(L+1)}{3(1+\sin^2\phi_0)} + \frac{3}{\sin^2 2\phi_0} + \sqrt{2c}\left(n_\phi + \frac{1}{2}\right) \mp \delta_{n_\phi}\right)\right.$$
$$\left. + u(\tilde{\beta}) - \epsilon_{\tilde{\beta}}(L)\right]\psi_L^{\pm}(\tilde{\beta}) = 0, \tag{4}$$

which, following the same steps as in Ref. [8], leads to Bessel eigenfunctions with

$$\nu = \sqrt{\frac{L(L+1)}{3(1+\sin^2\phi_0)} + \frac{3}{\sin^2 2\phi_0} + \sqrt{2c}\left(n_\phi + \frac{1}{2}\right) \mp \delta_{n_\phi}}. \tag{5}$$

In what follows only the levels with the lowest value of $n_\phi = 0$ are taken into account. Furthermore, in order to keep the number of parameters to a minimum, the term proportional to $\sqrt{2c}$ will be omitted, without much loss of predictive power, as *a posteriori* will be seen.

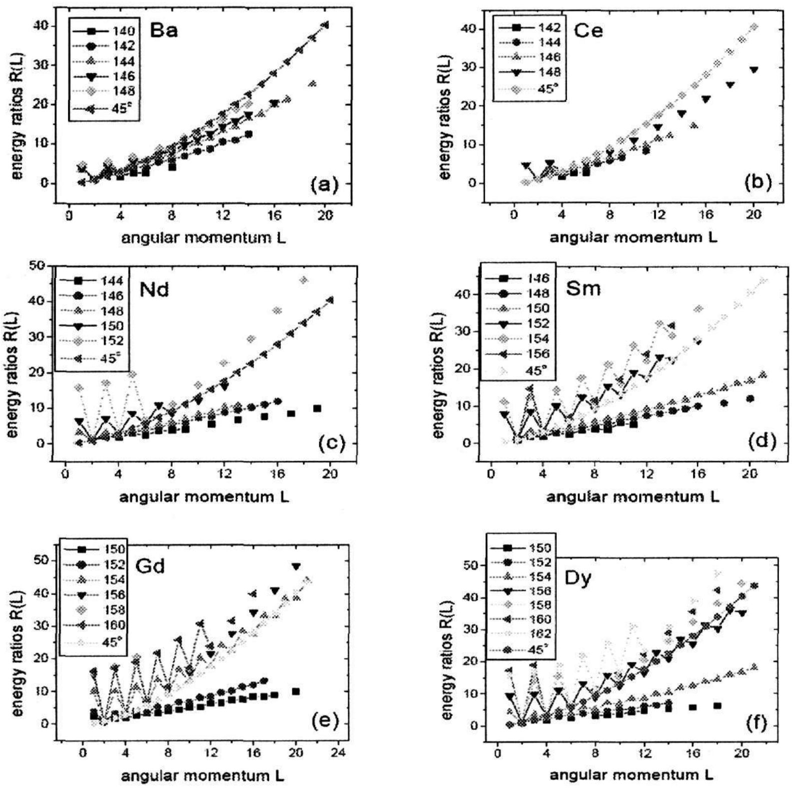

Fig. 1. (a) Energy ratios $R(L) = E(L)/E(2)$ of the ground state band and the associated negative parity band for $^{140-148}$Ba, as a function of the angular momentum. (b) Same for $^{142-148}$Ce. (c) Same for $^{144-152}$Nd. (d) Same for $^{146-156}$Sm. (e) Same for $^{150-160}$Gd. (f) Same for $^{150-162}$Dy. All data are taken from Ref. [12].

3 Numerical results and comparison to experiment

Spectra of the ground state bands and the associated negative parity bands, normalized to the 2_1^+ state of each nucleus, are shown in Fig. 1. It is clear that ^{150}Nd, ^{152}Sm, ^{154}Gd, and ^{156}Dy lie on or near the border between the regions of octupole deformation (corresponding to isotopes lighter than them) and octupole vibrations (occuring in isotopes heavier than them). In the same plot, the theoretical predictions of the AQOA model [8] for $\phi_0 = 45°$ (which corresponds to "equal" presence of quadrupole and octupole deformation) and without tunneling ($\delta = 0$) are shown. It is clear that the even levels of the above mentioned nuclei coincide with the predictions of the model, which is known [8] to provide a good description of nuclei on the border between the regions of octupole vibrations and octupole deformation in the actinide region. In the case of the Ba and Ce isotopes, it is quite clear that ^{148}Ba and ^{148}Ce lie near the border, with not enough data existing for the neutron richer isotopes, which should correspond to octupole vibrations.

The above findings suggest that the N=90 isotones ^{150}Nd, ^{152}Sm, ^{154}Gd, and ^{156}Dy, which are known [6] to be good examples of the X(5) symmetry, corresponding to the shape phase transition between spherical [U(5)] and axially symmetric deformed [SU(3)] nuclei, also lie close to the transition point between octupole deformation and octupole vibrations. In other words, taking into account only the quadrupole degree of freedom, as in the X(5) model, one sees that these nuclei are critical as far as the transition from quadrupole vibrations to axial quadrupole deformation is concerned, while the inclusion of the octupole degree of freedom, as in the AQOA model, leads to the conclusion that the same nuclei are also critical with respect to the transition from octupole vibrations to axial octupole deformation.

For the determination of the parameter values of the AQOA model with tunneling effects appropriate for each nucleus, the following path has been followed. For a given nucleus, there is a large number of (ϕ_0, δ) pairs that reproduce correctly the $R(4) = E(4)/E(2)$ ratio, and, as a consequence, all the levels of the ground state band. Among these pairs, the one leading to the best agreement with the levels of the negative parity band is chosen. The levels of the $s = 2$ band are then calculated with the same set of parameters.

Numerical results for the spectra of ^{150}Nd, ^{152}Sm, ^{154}Gd, and ^{156}Dy are compared to the experimental data, as well as to the X(5) predictions in Table 1. The following comments are now in place.

1) Ground state bands are reproduced very accurately, the AQOA predictions being closer to experiment than the parameter-free X(5) predictions.

2) Negative parity bands are reproduced quite well, the largest discrepancies occuring for the lowest levels (underpredicted) and the highest levels (overpredicted). This indicates that staggering is not decreasing as a function of angular momentum as fast (exponentially) as it should. It is known [13] that an accurate description of the parity splitting $(E(L^-) - E(L^+))$ is provided by a two-parameter exponential expression, corresponding to an L-dependent barrier in the double-minimum potential in β_3. This description is outside the realm of present AQOA model.

3) The AQOA predictions for the $s = 2$ bands are a little higher than the X(5) predictions, and in addition present the same problem as the X(5) predictions, namely the overprediction of the spacing among the levels by almost a factor of two. It is known that this problem can be resolved by replacing the infinite well potential by a potential with linear sloped walls [14].

In summary, the AQOA model preserves and improves the good description

Table 1
Spectra of the ground state band, the first excited band ($s = 2$), and the negative parity band provided by the AQOA and X(5) models, compared to experimental data [12] for the $N = 90$ X(5) nuclei. All levels are normalized to the energy of the 2_1^+ state. The (ϕ_0, δ) parameter values are $(18.0^o, 6.5)$, $(16.0^o, 7.3)$, $(15.5^o, 7.8)$, and $(15.8^o, 8.5)$ respectively.

L_s^π	^{150}Nd exp	^{150}Nd th	^{152}Sm exp	^{152}Sm th	^{154}Gd exp	^{154}Gd th	^{156}Dy exp	^{156}Dy th	X(5)
0_1^+	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2_1^+	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
4_1^+	2.930	2.929	3.009	3.010	3.015	3.017	2.934	2.934	2.904
6_1^+	5.533	5.502	5.804	5.748	5.831	5.772	5.592	5.515	5.430
8_1^+	8.676	8.614	9.241	9.093	9.299	9.139	8.823	8.641	8.483
10_1^+	12.280	12.224	13.214	12.991	13.302	13.066	12.521	12.268	12.027
12_1^+	16.274	16.309	17.642	17.414	17.752	17.523	16.592	16.375	16.041
14_1^+			22.467	22.345	22.567	22.494	20.961	20.948	20.514
16_1^+			27.607	27.775	27.663	27.967	25.574	25.980	25.437
18_1^+					33.210	33.934	30.327	31.462	30.804
20_1^+					38.859	40.390	35.269	37.392	36.611
0_2^+	5.187	6.248	5.622	7.214	5.531	7.305	4.904	6.227	5.649
2_2^+	6.533	8.048	6.655	8.973	6.626	9.060	6.015	8.023	7.450
4_2^+	8.738	11.342	8.400	12.356	8.512	12.449	7.899	11.317	10.689
1_1^-	6.551	5.790	7.911	6.574	10.086	6.947	9.387	7.095	
3_1^-	7.180	6.853	8.549	7.696	10.170	8.067	9.932	8.118	
5_1^-	8.671	8.678	10.030	9.637	11.409	10.006	11.078	9.892	
7_1^-	11.002	11.173	12.363	12.306	13.603	12.680	13.138	12.340	
9_1^-			15.430	15.630	16.580	16.015	15.871	15.394	
11_1^-			19.108	19.554	20.170	19.958	19.137	19.008	
13_1^-			23.265	24.043	24.224	24.473	22.895	23.146	
15_1^-					28.594	29.532	26.999	27.784	
17_1^-					33.331	35.118	31.437	32.905	
19_1^-					38.478	41.217	36.138	38.498	

of X(5) for the positive parity states, while in addition provides a good description of the negative parity states, which lie outside the realm of the X(5) model.

4 Discussion

In conclusion, we have shown that the N=90 isotones ^{150}Nd, ^{152}Sm, ^{154}Gd, and ^{156}Dy, which are critical with respect to the shape phase transition from quadrupole vibrations [U(5)] to axial quadrupole deformation [SU(3)], and as a consequence are described well by the X(5) model, are also critical with respect to the transition from octupole vibrations to axial octupole deformation, described by the AQOA model. The calculation of intraband and interband $B(EL)$ transitions is in progress.

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