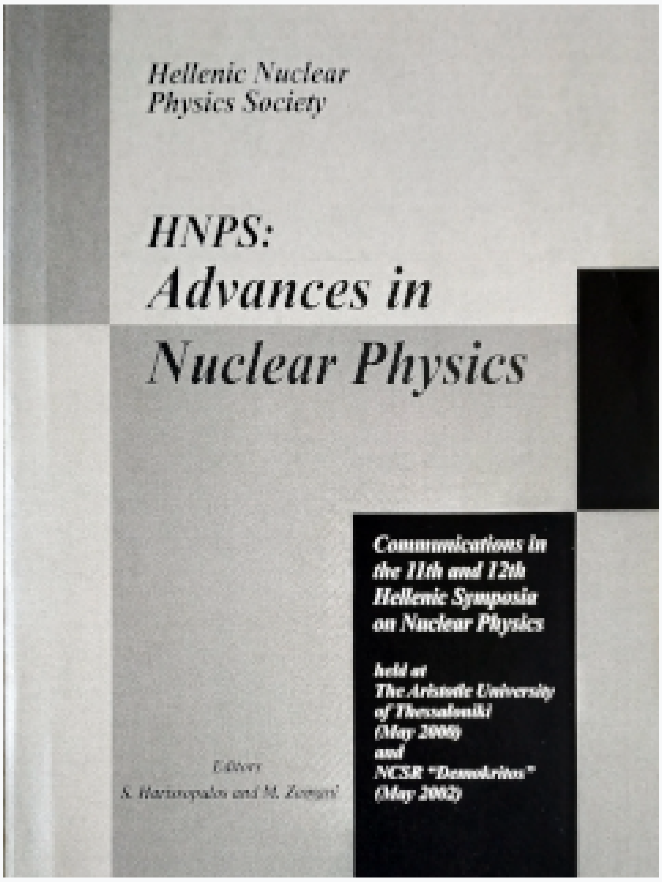
Annual Symposium of the Hellenic Nuclear Physics Society

Τόμ. 11 (2002)

HNPS2000 and HNPS2002



One-body density matrix and momentum distribution in s-p and s-d shell nuclei

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doi: 10.12681/hnps.2199

One-body density matrix and momentum distribution in s-p and s-d shell nuclei

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Analytical expressions of the one- and two- body terms in the cluster expansion of the one-body density matrix and momentum distribution of the s-p and s-d shell nuclei with $N = Z$ are derived. They depend on the harmonic oscillator parameter b and the parameter β which originates from the Jastrow correlation function. These parameters have been determined by least squares fit to the experimental charge form factors. The inclusion of short-range correlations increases the high momentum component of the momentum distribution, $n(\mathbf{k})$ for all nuclei we have considered while there is an A dependence of $n(\mathbf{k})$ both at small values of k and the high momentum component. The A dependence of the high momentum component of $n(\mathbf{k})$ becomes quite small when the nuclei ^{24}Mg, ^{28}Si and ^{32}S are treated as $1d$-$2s$ shell nuclei having the occupation probability of the $2s$-state as an extra free parameter in the fit to the form factors.

1 INTRODUCTION

The momentum distribution (MD) is of interest in many research subjects of modern physics, including those referring to helium, electronic, nuclear, and quark systems [1–3]. In the last two decades, there has been significant effort for the determination of the MD in nuclear matter and finite nucleon systems [4–17]. MD is related to the cross sections of various kinds of nuclear reactions. Specifically, the interaction of particles with nuclei at high energies, such as (p,2p), (e,e′p), and (e,e′) reactions, the nuclear photo-effect, meson absorption by nuclei, the inclusive proton production in proton-nucleus collisions, and even phenomena at low energies such as giant multipole resonances, give significant information about the nucleon MD. The experimental evidence obtained from inclusive and exclusive electron scattering on nuclei established the existence of a high-momentum component for momenta $k > 2$ fm^{-1} [18–21]. It has been shown that, in principle, mean field theories can not describe correctly MD and density distribution simultaneously [9] and the main features of MD depend little on the effective mean field considered [10]. The reason is that MD is sensitive to short-range and tensor nucleon-nucleon correlations which are not included in the mean field theories. Thus, theoretical approaches, which take into account short range correlations (SRC) due to the character of the nucleon-nucleon forces at small distances, are necessary to be developed.

Zabolitzky and Ey [4], employing the coupled-cluster (or $\exp(S)$) method for the microscopic evaluation of nuclear MD for the ground states of ^{4}He and ^{16}O and using various realistic NN-potentials, showed that the contribution of correlations dominates for momenta beyond 2 fm^{-1}. Bohigas and Stringari [6] and Dal Ri et al [7] evaluated the effect of SRC's on the one- and two- body densities by developing a low order approximation (LOA) in the framework of Jastrow formalism. They showed that one-body quantities provide an adequate test for the presence of SRC's in nuclei, which indicates that the independent-particle wave functions cannot reproduce simultaneously the form factor and the MD of a correlated system and also the effect of SRC's strongly modify the MD by introducing an important contribution in the region $k > 2$ fm^{-1}. Stoitsov et al [12] generalised the model of Jastrow correlations within the LOA, to heavier nuclei as ^{16}O, ^{36}Ar, ^{40}Ca. Their analytical expressions for the MD show the high momentum tail. They found that there is an A dependence of MD for small values of k, while for large values of k the slope of $\log n(k)$ versus k is roughly the same for the above three nuclei as well as for ^{4}He. MD for the nuclei ^{4}He, ^{16}O and ^{40}Ca was also calculated by Traini and Orlandini [8] within a phenomenological model in which dynamical short-range and tensor correlations effects were included. They showed that SRC increase the high momentum component considerably while the tensor correlations do not affect the MD appreciably [8,22]. In heavy nuclei, the local density approximation was used [11] for the study of the effect of SRC's in MD and the predictions were in agreement with the results of microscopic calculations in nuclear matter and in light nuclei.

In the various approaches, the MD of the closed shell nuclei ^{4}He, ^{16}O and ^{40}Ca as well as of ^{208}Pb and nuclear matter is usually studied. There is no systematic study of the one body density matrix (OBDM) and MD which include both the case of closed and open shell nuclei. This would be helpful in the calculations of the overlap integrals and reactions in that region of nuclei if one wants to go beyond the mean field theories [23]. For that reason, in the present work, we attempt to find some general expressions for the OBDM $\rho(\mathbf{r}, \mathbf{r}')$ and MD $n(\mathbf{k})$ which could be used both for closed and open shell nuclei. This work is a continuation of our previous study [24] on the form factors and densities of the s-p and s-d shell nuclei. The expression of $\rho(\mathbf{r}, \mathbf{r}')$ was found, first, using the factor cluster expansion of Clark and co-workers [25–27] and Jastrow correlation function which introduces SRC for closed shell nuclei and then was extrapolated to the case of $N = Z$ open shell nuclei. $n(\mathbf{k})$ was found by Fourier transform of $\rho(\mathbf{r}, \mathbf{r}')$. These expressions are functionals of the harmonic oscillator (HO) orbitals and depend on the HO parameter b and the correlation parameter β. The values of the parameters b and β have been determined by fit of the theoretical $F_{ch}(q)$, derived with the same cluster expansion, to the experimental one [24,28]. It is found that the high-momentum tail of the MD of all the nuclei we have considered appears for $k > 2$ fm^{-1} and also there is an A dependence of the values of $n(k)$ for 2 fm^{-1} $< k <$ 5 fm^{-1}. This A dependence of MD was first investigated considering ^{24}Mg, ^{28}Si and ^{32}S as $1d$ shell nuclei. Next we treated the above nuclei as $1d$-$2s$ shell nuclei having the occupation probability of the $2s$ state as an extra free parameter in the fit of the form factors. The A dependence is quite small in the second case.

The paper is organised as follows. In Sec. II the general expressions of the correlated OBDM and MD are derived using a Jastrow correlation function. In Sec. III the analytical expressions of the above quantities for the s-p and s-d shell nuclei, in the case of the HO orbitals, are given. Numerical results are reported and discussed in Sec. IV.

2 CORRELATED ONE-BODY DENSITY MATRIX AND MOMENTUM DISTRIBUTION

A nucleus with A nucleons is described by the wave function $\Psi(\mathbf{r}_1, \mathbf{r}_2, ..., \mathbf{r}_A)$ which depends on $3A$ coordinates as well as on spins and isospins. The evaluation of the single particle characteristics of the system needs the one-body density matrix [29,30]

$$\rho(\mathbf{r}, \mathbf{r}') = \int \Psi^*(\mathbf{r}, \mathbf{r}_2, ..., \mathbf{r}_A)\ \Psi(\mathbf{r}', \mathbf{r}_2, \cdots, \mathbf{r}_A)\ d\mathbf{r}_2 \cdots d\mathbf{r}_A\ , \tag{1}$$

where the integration is carried out over the radius vectors $\mathbf{r}_2, \cdots, \mathbf{r}_A$ and summation over spin and isospin variables is implied. $\rho(\mathbf{r}, \mathbf{r}')$ can also be represented by the form

$$\rho(\mathbf{r}, \mathbf{r}') = N\langle\Psi|\mathbf{O}_{\mathbf{rr}'}|\Psi'\rangle = N\langle\mathbf{O}_{\mathbf{rr}'}\rangle\ , \tag{2}$$

where $\Psi' = \Psi(\mathbf{r}'_1, \mathbf{r}'_2, ..., \mathbf{r}'_A)$ and $N = \langle\Psi|\Psi\rangle^{-1}$ is the normalization factor. The one-body "density operator" $\mathbf{O}_{\mathbf{rr}'}$, has the form

$$\mathbf{O}_{\mathbf{rr}'} = \sum_{i=1}^{A} \delta(\mathbf{r}_i - \mathbf{r})\delta(\mathbf{r}'_i - \mathbf{r}') \prod_{j\neq i}^{A} \delta(\mathbf{r}_j - \mathbf{r}'_j)\ . \tag{3}$$

In the case where the nuclear wave function Ψ can be expressed as a Slater determinant depending on the SP wave functions $\phi_i(\mathbf{r})$ we have

$$\rho_{SD}(\mathbf{r}, \mathbf{r}') = \sum_{i=1}^{A} \phi_i^*(\mathbf{r})\phi_i(\mathbf{r}')\ . \tag{4}$$

The diagonal elements of the OBDM give the density distribution $\rho(\mathbf{r},\mathbf{r}) = \rho(\mathbf{r})$, while the MD is given by the Fourier transform of $\rho(\mathbf{r},\mathbf{r}')$,

$$n(\mathbf{k}) = \frac{1}{(2\pi)^3} \int \exp[i\mathbf{k}(\mathbf{r}-\mathbf{r}')]\ \rho(\mathbf{r},\mathbf{r}')\ d\mathbf{r}\ d\mathbf{r}'\ . \tag{5}$$

In the case of a Slater determinant, MD takes the form

$$n_{SD}(\mathbf{k}) = \sum_{i=1}^{A} \tilde{\phi}_i^*(\mathbf{k})\tilde{\phi}_i(\mathbf{k})\ , \quad \tilde{\phi}_i(\mathbf{k}) = \frac{1}{(2\pi)^{3/2}} \int \phi_i(\mathbf{r}) \exp[i\mathbf{k}\mathbf{r}]d\mathbf{r}. \tag{6}$$

2.1 One-body density matrix

If we denote the model operator, which introduces SRC, by $\mathcal{F}$, an eigenstate Φ of the model system corresponds to an eigenstate $\Psi = \mathcal{F}\Phi$ of the true system. Several restrictions can be made on the model operator $\mathcal{F}$ and it is required that $\mathcal{F}$ be translationally invariant and symmetrical in its arguments $1 \cdots i \cdots A$ and possesses the cluster property [27,31]. In order to evaluate the correlated one-body density matrix $\rho_{cor}(\mathbf{r},\mathbf{r}')$, we consider, first, the generalized integral

$$I(\alpha) = \langle\Psi| \exp[\alpha I(0)\mathbf{O}_{\mathbf{r}\mathbf{r}'}]|\Psi'\rangle\ , \tag{7}$$

corresponding to the one-body "density operator" $\mathbf{O}_{\mathbf{r}\mathbf{r}'}$ (given by (3)), from which we have

$$\langle\mathbf{O}_{\mathbf{r}\mathbf{r}'}\rangle = \left[\frac{\partial \ln I(\alpha)}{\partial\alpha}\right]_{\alpha=0} . \tag{8}$$

For the cluster analysis of equation (8), we consider the sub-product integrals [25–27], for the sub-systems of the A-nucleons system corresponding to the density operators $\mathbf{O}_{\mathbf{r}\mathbf{r}'}(1)$,$\mathbf{O}_{\mathbf{r}\mathbf{r}'}(2)$. The factor cluster decomposition of these integrals, following the factor cluster expansion of Ristig,Ter Low, and Clark [25–27], gives

$$\langle\mathbf{O}_{\mathbf{r}\mathbf{r}'}\rangle = \langle\mathbf{O}_{\mathbf{r}\mathbf{r}'}\rangle_1 + \langle\mathbf{O}_{\mathbf{r}\mathbf{r}'}\rangle_2 + \cdots + \langle\mathbf{O}_{\mathbf{r}\mathbf{r}'}\rangle_A\ . \tag{9}$$

Three- and many-body terms will be neglected in the present analysis. Thus, in the two-body approximation, $\rho_{cor}(\mathbf{r},\mathbf{r}')$, defined by Eq. (2), is written

$$\rho_{cor}(\mathbf{r},\mathbf{r}') \approx N[\langle\mathbf{O}_{\mathbf{r}\mathbf{r}'}\rangle_1 + \langle\mathbf{O}_{\mathbf{r}\mathbf{r}'}\rangle_{22} - \langle\mathbf{O}_{\mathbf{r}\mathbf{r}'}\rangle_{21}]\ , \tag{10}$$

where $\langle \mathbf{O}_{\mathbf{rr}'} \rangle_1 = \rho_{SD}(\mathbf{r}, \mathbf{r}')$, the uncorrelated OBDM associated with the Slater determinant and

$$\langle \mathbf{O}_{\mathbf{rr}'} \rangle_{22} = \sum_{i<j}^{A} \langle ij \mid \mathcal{F}^{\dagger}(r_{12}) \mathbf{O}_{\mathbf{rr}'}(2) \mathcal{F}(r'_{12}) \mid i'j' \rangle_{\mathrm{a}} \,. \tag{11}$$

The term $\langle \mathbf{O}_{\mathbf{rr}'} \rangle_{21}$ is as the term $\langle \mathbf{O}_{\mathbf{rr}'} \rangle_{22}$ without the operator $\mathcal{F}^{\dagger}(r_{12})$. If the two-body operator $\mathcal{F}^{\dagger}(r_{12})$ is taken to be the Jastrow correlation function [32] $f(r_{ij}) = 1 - \exp[-\beta(\mathbf{r}_i - \mathbf{r}_j)^2]$ then

$$\mathcal{F}^{\dagger}(r_{12}) \mathbf{O}_{\mathbf{rr}'}(2) \mathcal{F}(r'_{12}) = \mathbf{O}_{\mathbf{rr}'}(2) \left[1 - \mathrm{g}_1(\mathbf{r}, \mathbf{r}_2) - \mathrm{g}_2(\mathbf{r}', \mathbf{r}_2) + \mathrm{g}_3(\mathbf{r}, \mathbf{r}', \mathbf{r}_2)\right], \tag{12}$$

where

$$\begin{aligned} \mathrm{g}_1(\mathbf{r}, \mathbf{r}_2) &= \exp[-\beta(r^2 + r_2^2)] \exp[2\beta \mathbf{r}\mathbf{r}_2], \quad \mathrm{g}_2(\mathbf{r}', \mathbf{r}_2) = \mathrm{g}_1(\mathbf{r}', \mathbf{r}_2), \\ \mathrm{g}_3(\mathbf{r}, \mathbf{r}', \mathbf{r}_2) &= \exp[-\beta(r^2 + r'^2)] \exp[-2\beta r_2^2] \exp[2\beta(\mathbf{r} + \mathbf{r}')\mathbf{r}_2] \,, \end{aligned} \tag{13}$$

and $\rho_{cor}(\mathbf{r}, \mathbf{r}')$ takes the form

$$\rho_{cor}(\mathbf{r}, \mathbf{r}') \approx N[\langle \mathbf{O}_{\mathbf{rr}'} \rangle_1 - O_{22}(\mathbf{r}, \mathbf{r}', \mathrm{g}_1) - O_{22}(\mathbf{r}, \mathbf{r}', \mathrm{g}_2) + O_{22}(\mathbf{r}, \mathbf{r}', \mathrm{g}_3)] \,. \tag{14}$$

where

$$\begin{aligned} O_{22}(\mathbf{r}, \mathbf{r}', \mathrm{g}_\ell) &= \sum_{i<j}^{A} \langle ij \mid \mathbf{O}_{\mathbf{rr}'}(2) \mathrm{g}_\ell(\mathbf{r}, \mathbf{r}', \mathbf{r}_2) \mid i'j' \rangle_{\mathrm{a}} \\ &= \int \mathrm{g}_l(\mathbf{r}, \mathbf{r}', \mathrm{g}_2)[\rho_{SD}(\mathbf{r}, \mathbf{r}') \rho_{SD}(\mathbf{r}_2, \mathbf{r}_2) - \rho_{SD}(\mathbf{r}, \mathbf{r}_2) \rho_{SD}(\mathbf{r}_2, \mathbf{r}')] d\mathbf{r}_2 \,. \end{aligned} \tag{15}$$

In the above expression of $\rho_{cor}(\mathbf{r}, \mathbf{r}')$, the one-body contribution to the OBDM is well known and is given by the equation

$$\langle \mathbf{O}_{\mathbf{rr}'} \rangle_1 = \rho_{SD}(\mathbf{r}, \mathbf{r}') = \frac{1}{\pi} \sum_{nl} \eta_{nl} (2l+1) \phi^*_{nl}(r) \phi_{nl}(r') P_l(\cos \omega_{rr'}) \,, \tag{16}$$

where η_{nl} are the occupation probabilities of the states nl (0 or 1 in the case of closed shell nuclei) and $\phi_{nl}(r)$ is the radial part of the SP wave function and $\omega_{rr'}$ the angle between the vectors $\mathbf{r}$ and $\mathbf{r}'$. The term $O_{22}(\mathbf{r}, \mathbf{r}', \mathrm{g}_\ell)$, performing the spin-isospin summation and the angular integration, takes the general form

$$\begin{aligned} O_{22}(\mathbf{r}, \mathbf{r}', \mathrm{g}_\ell) &= 4 \sum_{n_i l_i, n_j l_j} \eta_{n_i l_i} \eta_{n_j l_j} (2l_i + 1)(2l_j + 1) \\ &\times \left[4A^{n_i l_i n_j l_j, 0}_{n_i l_i n_j l_j}(\mathbf{r}, \mathbf{r}', \mathrm{g}_\ell) - \sum_{k=0}^{l_i + l_j} \langle l_i 0 l_j 0 \mid k0 \rangle^2 A^{n_j l_j n_i l_i, k}_{n_i l_i n_j l_j}(\mathbf{r}, \mathbf{r}', \mathrm{g}_\ell) \right], \quad \ell = 1, 2, 3, \end{aligned} \tag{17}$$

where

$$A^{n_3l_3n_4l_4,k}_{n_1l_1n_2l_2}(\mathbf{r},\mathbf{r}',\mathrm{g}_1) = \frac{1}{4\pi}\phi^*_{n_1l_1}(r)\ \phi_{n_3l_3}(r')\ \exp[-\beta r^2]\ P_{l_3}(\cos\omega_{rr'})$$
$$\times\int_0^\infty \phi^*_{n_2l_2}(r_2)\ \phi_{n_4l_4}(r_2)\ \exp[-\beta r_2^2]\ i_k(2\beta r r_2)\ r_2^2\ \mathrm{d}r_2, \tag{18}$$

and the matrix element $A^{n_3l_3n_4l_4,k}_{n_1l_1n_2l_2}(\mathbf{r},\mathbf{r}',\mathrm{g}_2)$ can be found from (18) replacing $\mathbf{r}\longleftrightarrow\mathbf{r}'$ and $n_1l_1\longleftrightarrow n_3l_3$ while the matrix element corresponding to the factor g_3 is

$$A^{n_3l_3n_4l_4,k}_{n_1l_1n_2l_2}(\mathbf{r},\mathbf{r}',\mathrm{g}_3) = \frac{1}{4\pi}\phi^*_{n_1l_1}(r)\ \phi_{n_3l_3}(r')\ \exp[-\beta(r^2+r'^2)]\ \Omega^k_{l_1l_3}(\omega_{rr'})\ \times$$
$$\int_0^\infty \phi^*_{n_2l_2}(r_2)\ \phi_{n_4l_4}(r_2)\ \exp[-2\beta r_2^2]\ i_k(2\beta|\mathbf{r}+\mathbf{r}'|r_2)\ r_2^2\ \mathrm{d}r_2, \tag{19}$$

In Eqs. (18) and (19) the modified spherical Bessel function, $i_k(z)$, comes from the expansion of the exponential function $\exp[2\beta\mathbf{x}_1\mathbf{x}_2]$ of the factors g_ℓ in spherical harmonics, while the factor $\Omega^k_{l_1l_3}(\omega_{rr'})$ depends on the directions of r and $\mathbf{r}'$. The expression of the term $O_{22}(\mathbf{r},\mathbf{r}',\mathrm{g}_\ell)$ depends on the SP wave functions and so it is suitable to be used for analytical calculations with the HO orbitals and in principle for numerical calculations with more realistic SP orbitals. Expressions (16) and (17) were derived for the closed shell nuclei with $N=Z$, where η_{nl} is 0 or 1. For the open shell nuclei (with $N=Z$) we use the same expressions, where now $0\le\eta_{nl}\le 1$. In this way the mass dependence of the correlation parameter β and the OBDM or MD can be studied. Finally, using the known values of the Clebsch-Gordan coefficients, Eq. (17), for the case of *s*-*p* and *s*-*d* shell nuclei, takes the form

$$\begin{aligned}
O_{22}(\mathbf{r},\mathbf{r}',\mathrm{g}_\ell) = 4\Big[\ & 3A^{0000,0}_{0000}\eta^2_{1s} + \Big[33A^{0101,0}_{0101} - 6A^{0101,2}_{0101}\Big]\eta^2_{1p} + 3A^{1010,0}_{1010}\eta^2_{2s}\\
&+\Big[95A^{0202,0}_{0202} - \frac{50}{7}A^{0202,2}_{0202} - \frac{90}{7}A^{0202,4}_{0202}\Big]\eta^2_{1d} + \Big[12A^{0001,0}_{0001} + 12A^{0100,0}_{0100}\\
&-3A^{0100,1}_{0001} - 3A^{0001,1}_{0100}\Big]\eta_{1s}\eta_{1p} + \Big[20A^{0002,0}_{0002} + 20A^{0200,0}_{0200} - 5A^{0200,2}_{0002}\\
&-5A^{0002,2}_{0200}\Big]\eta_{1s}\eta_{1d} + \Big[4A^{0010,0}_{0010} + 4A^{1000,0}_{1000} - A^{1000,0}_{0010} - A^{0010,0}_{1000}\Big]\eta_{1s}\eta_{2s}\\
&+\Big[60A^{0102,0}_{0102} + 60A^{0201,0}_{0201} - 6A^{0201,1}_{0102} - 6A^{0102,1}_{0201} - 9A^{0201,3}_{0102} - 9A^{0102,3}_{0201}\Big]\eta_{1p}\eta_{1d}\\
&+\Big[12A^{0110,0}_{0110} + 12A^{1001,0}_{1001} - 3A^{1001,1}_{0110} - 3A^{0110,1}_{1001}\Big]\eta_{1p}\eta_{2s}\\
&+\Big[20A^{0210,0}_{0210} + 20A^{1002,0}_{1002} - 5A^{1002,2}_{0210} - 5A^{0210,2}_{1002}\Big]\eta_{1d}\eta_{2s}\Big]\,.
\end{aligned}\tag{20}$$

It should be noted that Eqs. (17) and (20) are also valid for the cluster expansion of the density distribution and the form factor as it has been found in ref. [24] and also in the cluster expansion of the MD. The only difference is the expressions of the matrix elements A.

2.2 Momentum distribution

The MD for the above mentioned nuclei can be found either by following the same cluster expansion or by taking the Fourier transform of $\rho(\mathbf{r},\mathbf{r}')$ given by (14). In both cases the correlated momentum distribution takes the form

$$n_{cor}(\mathbf{k}) \approx N\left[\langle\tilde{\mathbf{O}}_{\mathbf{k}}\rangle_1 - 2\tilde{O}_{22}(\mathbf{k},\mathbf{g}_1) + \tilde{O}_{22}(\mathbf{k},\mathbf{g}_3)\right] , \qquad (21)$$

where $\langle\tilde{\mathbf{O}}_{\mathbf{k}}\rangle_1 = n_{SD}(\mathbf{k})$ given by Eq. (6) and the term $\tilde{O}_{22}(\mathbf{k},\mathbf{g}_\ell)$, as in the case of OBDM, is given again by the right-hand side of Eqs. (17) and (20) replacing the matrix elements $A^{n_3l_3n_4l_4,k}_{n_1l_1n_2l_2}(\mathbf{r},\mathbf{r}',\mathbf{g}_\ell)$, defined by Eqs. (18) and (19), by the Fourier transform of them, that is by the matrix elements

$$\tilde{A}^{n_3l_3n_4l_4,k}_{n_1l_1n_2l_2}(\mathbf{k},\mathbf{g}_\ell) = \frac{1}{(2\pi)^3}\int A^{n_3l_3n_4l_4,k}_{n_1l_1n_2l_2}(\mathbf{r},\mathbf{r}',\mathbf{g}_\ell)\exp[i\mathbf{k}(\mathbf{r}-\mathbf{r}')]d\mathbf{r}d\mathbf{r}', \quad \ell = 1,3 \ . \qquad (22)$$

As in the case of the OBDM, expression (21) is suitable for the study of the MD for the s-p and s-d shell nuclei and also for the study of the mass dependence of the kinetic energy of these nuclei. The mean value of the kinetic energy has the form

$$\langle\mathbf{T}\rangle = N[\langle\mathbf{T}\rangle_1 - 2T_{22}(\mathbf{g}_1) + T_{22}(\mathbf{g}_3)] , \qquad (23)$$

where

$$\langle\mathbf{T}\rangle_1 = \frac{\hbar^2}{2m}\int k^2 n_{SD}(\mathbf{k})d\mathbf{k} , \quad T_{22}(\mathbf{g}_\ell) = \frac{\hbar^2}{2m}\int k^2\tilde{O}_{22}(\mathbf{k},\mathbf{g}_\ell)d\mathbf{k} , \quad \ell = 1,3 \ . \qquad (24)$$

3 ANALYTICAL EXPRESSIONS

In the case of the HO wave functions, with radial part in coordinate and momentum space,

$$\begin{aligned} \phi_{nl}(r) &= N_{nl}b^{-3/2}r_b^l L_n^{l+\frac{1}{2}}\left(r_b^2\right)e^{-r_b^2/2} , \quad r_b = r/b , \\ \tilde{\phi}_{nl}(k) &= i^l(-1)^{n+l}N_{nl}b^{3/2}k_b^l L_n^{l+\frac{1}{2}}\left(k_b^2\right)\ e^{-k_b^2/2} , \quad k_b = kb , \end{aligned} \qquad (25)$$

where $N_{nl} = (2n!/\Gamma(n+l+3/2))^{1/2}$, analytical expressions of the one-body terms, $\langle\mathbf{O}_{\mathbf{rr}'}\rangle_1$ and $\langle\tilde{\mathbf{O}}_{\mathbf{k}}\rangle_1$ as well as of the matrix elements $A^{n_3l_3n_4l_4,k}_{n_1l_1n_2l_2}(\mathbf{r},\mathbf{r}',\mathbf{g}_\ell)$

and $\tilde{A}^{n_3 l_3 n_4 l_4, k}_{n_1 l_1 n_2 l_2}(\mathbf{k}, \mathrm{g}_\ell)$, which have been defined in Sec. II, can be found. From these expressions, the analytical expressions of the terms $O_{22}(\mathbf{r}, \mathbf{r}', \mathrm{g}_\ell)$ and $\tilde{O}_{22}(\mathbf{k}, \mathrm{g}_\ell)$, defined by Eq. (20), can also be found.

The expressions of the one-body terms, $\langle \mathbf{O}_{\mathbf{r}\mathbf{r}'}\rangle_1$ and $\langle \tilde{\mathbf{O}}_{\mathbf{k}}\rangle_1$, have the forms

$$\langle \mathbf{O}_{\mathbf{r}\mathbf{r}'}\rangle_1 = \rho_{SD}(\mathbf{r}, \mathbf{r}') = \frac{2}{\pi^{3/2} b^3} \Big[2\eta_{1s} + 3\eta_{2s} - 2\eta_{2s}(r_b^2 + r_b'^2) + 4\eta_{1p} r_b r_b' \cos\omega_{rr'}$$

$$+ \frac{4}{3}[\eta_{2s} + \eta_{1d}(3\cos^2\omega_{rr'} - 1)] r_b^2 r_b'^2 \Big] \exp[-(r_b^2 + r_b'^2)/2], \tag{26}$$

$$\langle \tilde{\mathbf{O}}_{\mathbf{k}}\rangle_1 = n_{SD}(\mathbf{k}) = \frac{2b^3}{\pi^{3/2}} \exp[-k_b^2] \sum_{k=0}^{2} C_{2k} k_b^{2k}, \tag{27}$$

where the coefficients C_{2k} are: $C_0 = 2\eta_{1s} + 3\eta_{2s}$, $C_2 = 4(\eta_{1p} - \eta_{2s})$, $C_4 = \frac{4}{3}(2\eta_{1d} + \eta_{2s})$.

The analytical expressions of the matrix element $A^{n_3 l_3 n_4 l_4, k}_{n_1 l_1 n_2 l_2}(\mathbf{r}, \mathbf{r}', \mathrm{g}_\ell)$ ($\ell = 1, 3$) have the form

$$A^{n_3 l_3 n_4 l_4, k}_{n_1 l_1 n_2 l_2}(\mathbf{r}, \mathbf{r}', \mathrm{g}_1) = B_0\, b^{-3} y^k\, r_b^{k+l_1} r_b'^{l_3}\, L_{n_1}^{l_1+\frac{1}{2}}(r_b^2)\, L_{n_3}^{l_3+\frac{1}{2}}(r_b'^2) P_{l_3}(\cos\omega_{rr'})$$
$$\times \exp\left[-\frac{1+3y}{2(1+y)} r_b^2 - \frac{1}{2} r_b'^2\right] \sum_{w_2=0}^{n_2} \sum_{w_4=0}^{n_4} B_{w_2 w_4}(y)\, L^{k+\frac{1}{2}}_{\frac{1}{2}(l_2+l_4-k)+w_2+w_4}\left(\frac{-y^2}{1+y} r_b^2\right) \tag{28}$$

and

$$A^{n_3 l_3 n_4 l_4, k}_{n_1 l_1 n_2 l_2}(\mathbf{r}, \mathbf{r}', \mathrm{g}_3) = B_0\, b^{-3} y^k \mid \mathbf{r}_b + \mathbf{r}_b' \mid^k r_b^{l_1} r_b'^{l_3}\, L_{n_1}^{l_1+\frac{1}{2}}(r_b^2)\, L_{n_3}^{l_3+\frac{1}{2}}(r_b'^2) \Omega^k_{l_1 l_3}(\omega_{rr'})$$
$$\times \exp\left[-\frac{1+2y}{2}(r_b^2 + r_b'^2)\right] \exp\left[\frac{y^2}{1+2y}(\mathbf{r}_b + \mathbf{r}_b')^2\right]$$
$$\times \sum_{w_2=0}^{n_2} \sum_{w_4=0}^{n_4} B_{w_2 w_4}(2y)\, L^{k+\frac{1}{2}}_{\frac{1}{2}(l_2+l_4-k)+w_2+w_4}\left(-\frac{y^2}{1+2y}(\mathbf{r}_b + \mathbf{r}_b')^2\right) \tag{29}$$

where $y = \beta b^2$ and $B_0 = \frac{1}{16\sqrt{\pi}}\left(\prod_{i=1}^{4} N_{n_i l_i}\right)$, and

$$B_{w_2 w_4}(z) = [\frac{l_2 + l_4 - k}{2} + w_2 + w_4]! \prod_{i=2,4} \frac{(-1)^{w_i}}{w_i!} \begin{pmatrix} n_i + l_i + \frac{1}{2} \\ n_i - w_i \end{pmatrix} (1+z)^{-\frac{1}{2}l_i - w_i - \frac{1}{2}(3+k)},$$

while the one corresponding to the factor g_2 can be found from (28) replacing $r_b \longleftrightarrow r_b'$ and $n_1 l_1 \longleftrightarrow n_3 l_3$.

The substitution of $A^{n_3 l_3 n_4 l_4, k}_{n_1 l_1 n_2 l_2}(\mathbf{r}, \mathbf{r}', \mathrm{g}_\ell)$ to the expression of $O_{22}(\mathbf{r}, \mathbf{r}', g_\ell)$ which is given by Eq. (20) leads to the analytical expression of the two-body term of the OBDM, which is of the form

$$O_{22}(\mathbf{r}_b,\mathbf{r}'_b) = f_1(r_b, r'_b, \cos\omega_{rr'}) \exp\left[-\frac{1+3y}{2(1+y)}r_b^2 - \frac{1}{2}r_b'^2\right] + f_1(r'_b, r_b, \cos\omega_{rr'}) \exp\left[-\frac{1+3y}{2(1+y)}r_b'^2 - \frac{1}{2}r_b^2\right] + f_3(r_b, r'_b, \cos\omega_{rr'}) \exp\left[-\frac{1+2y}{2}(r_b^2 + r_b'^2)\right] \exp\left[\frac{y^2}{1+2y}(\mathbf{r}_b + \mathbf{r}'_b)^2\right], \quad (30)$$

where $f_\ell(r_b, r'_b, \cos\omega_{rr'})$, $(\ell = 1, 3)$ are polynomials of r_b, r'_b and $\cos\omega_{rr'}$ which depend also on $y = \beta b^2$ and the occupation probabilities of the various states. The corresponding analytical expressions of the matrix elements $\tilde{A}^{n_3l_3n_4l_4,k}_{n_1l_1n_2l_2}(\mathrm{k}, \mathrm{g}_\ell)$ $(\ell = 1, 3)$ which contribute to the two-body term of the MD were found substituting $\phi_{nl}(r)$ with that of the HO wave function into Eq. (22). The expression of $\tilde{A}^{n_3l_3n_4l_4,k}_{n_1l_1n_2l_2}(\mathrm{k}, \mathrm{g}_1)$, which can be found easily, has the form

$$\tilde{A}^{n_3l_3n_4l_4,k}_{n_1l_1n_2l_2}(\mathrm{k}, \mathrm{g}_1) = B_0 b^3 (-1)^{n_3} k_b^{2l_3}\, L_{n_3}^{l_3+\frac{1}{2}}\left(k_b^2\right) \exp\left[-\frac{1+2y}{1+3y}k_b^2\right]$$
$$\times \sum_{w_1=0}^{n_1}\sum_{w_2=0}^{n_2}\sum_{w_4=0}^{n_4}\sum_{t=0}^{\frac{1}{2}(l_2+l_4-k)+w_2+w_4} \tilde{B}_{w_2w_4t}(y)\left[\frac{1}{2}(l_1 - l_3 + k) + w_1 + t\right]!$$
$$\times \frac{(-1)^{w_1}}{w_1!}\begin{pmatrix} n_1 + l_1 + \frac{1}{2} \\ n_1 - w_1 \end{pmatrix} 2^{\frac{1}{2}(l_1-l_3)+w_1}\,(1+y)^{\frac{1}{2}(l_1+l_3-l_2-l_4)+w_1-w_2-w_4}$$
$$\times (1+3y)^{-\frac{1}{2}(3+l_1+l_3+k)-w_1-t}\, L^{l_3+\frac{1}{2}}_{\frac{1}{2}(l_1-l_3+k)+w_1+t}\left(\frac{1+y}{2(1+3y)}k_b^2\right), \quad (31)$$

where

$$\tilde{B}_{w_2w_4t}(y) = (\sqrt{2}y)^{k+2t}\left[\frac{1}{2}(1+l_2+l_4+k)+w_2+w_4\right]!\left[\frac{k-l_2-l_4}{2} - w_2 - w_4\right]_t$$
$$\times \frac{1}{(k+t+\frac{1}{2})!}\prod_{i=2,4}\frac{(-1)^{w_i+t}}{w_i!t!}\begin{pmatrix} n_i + l_i + \frac{1}{2} \\ n_i - w_i \end{pmatrix}. \quad (32)$$

The expression of $\tilde{A}^{n_3l_3n_4l_4,k}_{n_1l_1n_2l_2}(\mathrm{k}, \mathrm{g}_3)$ is more complicated. It has the form

$$\tilde{A}^{n_3l_3n_4l_4,k}_{n_1l_1n_2l_2}(\mathrm{k}, \mathrm{g}_3) = \frac{2}{\sqrt{\pi}}B_0 b^3 \exp\left[-\frac{k_b^2}{1+2y}\right]\sum_{w_2=0}^{n_2}\sum_{w_4=0}^{n_4}\sum_{t=0}^{\frac{1}{2}(l_2+l_4-k)+w_2+w_4}\tilde{B}_{w_2w_4t}(y)$$
$$\times (1+2y)^{-\frac{1}{2}(3+l_2+l_4+k)-w_2-w_4-t} I^{n_3l_3,k}_{n_1l_1}(k_b)\,. \quad (33)$$

The general expression of the quantity $I^{n_3l_3,k}_{n_1l_1}(k_b)$ is quite complicated. For that reason we calculated it for various cases which are needed for the s-p and s-d shell nuclei (see Ref. [28]).

The substitution of $\tilde{A}^{n_3l_3n_4l_4,k}_{n_1l_1n_2l_2}(\mathbf{k}, g_\ell)$ to the expression of $\tilde{O}_{22}(\mathbf{k}, g_\ell)$ which is given by Eq. (20) leads to the analytical expression of the two-body term of the MD, which is of the form

$$\tilde{O}_{22}(k) = \tilde{f}_1(k_b^2)\exp\left[-\frac{1+2y}{1+3y}k_b^2\right] + \tilde{f}_3(k_b^2)\exp\left[-\frac{1}{1+2y}k_b^2\right], \tag{34}$$

where $\tilde{f}_\ell(k_b^2)$, $(\ell = 1, 3)$ are polynomials of k_b^2 which depend also on $y = \beta b^2$ and the occupation probabilities of the various states. Similar expressions have been found for the mean value of the kinetic energy.

4 RESULTS AND DISCUSSION

The calculations of the MD for the various s-p and s-d shell nuclei, with $N = Z$, have been carried out on the basis of Eq. (21) and the analytical expressions of the one- and two-body terms which were given in Sec. III. Two cases have been examined, named case 1 and case 2 corresponding to the analytical calculations with HO orbitals without and with SRC, respectively. The parameters b and β of the model in cases 1 and 2 have been determined by fit of the theoretical $F_{ch}(q)$, derived with the same cluster expansion, to the experimental one are given in Table 1. It is found that the inclusion of SRC's improves the fit of $F_{ch}(q)$ of the above mentioned nuclei and all the diffraction minima are reproduced in the correct place [24,28]. The values of the parameter β (see Fig. 1) is almost constant for the closed shell nuclei and takes larger values (less correlated system) in the open shell nuclei.

This behaviour has an effect on the MD of nuclei as it is seen from Fig. 2a, where the MD, of the various s-p and s-d shell nuclei calculated with the values of b and β of Table I for case 2, have been plotted. It is seen that the inclusion of SRC's increases considerably the high momentum component of $n(k)$, for all nuclei we have considered. Also, while the general structure of the high momentum component of the MD for $A = 4$, 12, 16, 24, 28, 32, 36, 40, is almost the same, in agreement with other studies [2,4,8,33], there is an A dependence of $n(k)$ both at small values of k and in the region 2 fm^{-1} $< k <$ 5 fm^{-1}. The A dependence of the high momentum

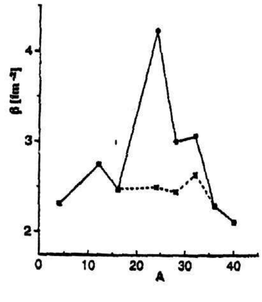

Fig. 1. The correlation parameter β versus the mass number A. The solid line correspond to the case when the nuclei ^{24}Mg,^{28}Si,^{32}S,^{36}Ar were treated as $1d$ shell nuclei while the dashed line to the case when these nuclei were treated as $1d$-$2s$ shell nuclei.

Table 1
The values of the parameters b and β, of the mean kinetic energy per nucleon, $\langle T \rangle$ and of the rms charge radii, $\langle r_{ch}^2 \rangle^{1/2}$, for various s-p and s-d shell nuclei, determined by fit to the experimental $F_{ch}(q)$. Case 1 refers to the HO wave function without SRC and case 2 when SRC are included. Case 2* is the same as case 2 but with the occupation probability of the state $2s$ taken to be as a free parameter. The experimental rms charge radii are from Ref. [34].

Case	Nucleus	b [fm]	β [fm^{-2}]	$\langle T \rangle$ [Mev]			$\langle r_{ch}^2 \rangle^{1/2}$ [fm]	
				HO	SRC	Total	Theor.	Expt.
1	^{4}He	1.4320	–	15.166	–	15.166	1.7651	1.676(8)
2	^{4}He	1.1732	2.3126	22.594	7.310	29.904	1.6234	
1	^{12}C	1.6251	–	17.010	–	17.010	2.4901	2.471(6)
2	^{12}C	1.5190	2.7468	19.469	6.111	25.580	2.4261	
1	^{16}O	1.7610	–	15.044	—	15.044	2.7377	2.730(25)
2	^{16}O	1.6507	2.4747	17.121	6.493	23.614	2.6802	
1	^{24}Mg	1.8495	–	16.162	–	16.162	3.1170	3.075(15)
2	^{24}Mg	1.8103	4.2275	16.870	4.239	21.109	3.0948	
2*	^{24}Mg	1.7473	2.4992	18.109	6.505	24.614	3.0638	
1	^{28}Si	1.8941	–	16.099	–	16.099	3.2570	3.086(18)
2	^{28}Si	1.8236	3.0020	17.369	5.564	22.933	3.2159	
2*	^{28}Si	1.7774	2.4440	18.283	6.922	25.205	3.1835	
1	^{32}S	2.0016	–	14.878	–	14.878	3.4830	3.248(11)
2	^{32}S	1.9368	3.0659	15.891	4.976	20.867	3.4425	
2*	^{32}S	1.8121	2.6398	18.154	6.761	24.915	3.2822	
1	^{36}Ar	1.8800	–	17.273	–	17.273	3.3270	3.327(15)
2	^{36}Ar	1.8007	2.2937	18.827	8.590	27.417	3.3343	
1	^{40}Ca	1.9453	–	16.437	–	16.437	3.4668	3.479(3)
2	^{40}Ca	1.8660	2.1127	17.863	8.754	26.617	3.5156	

component of $n(k)$ is larger in the open shell nuclei than in the closed shell nuclei. It is seen that the high momentum component is almost the same for the closed shell nuclei ^{4}He, ^{16}O and ^{40}Ca as expected from other studies [2,4,33].

In the previous analysis, the nuclei ^{24}Mg, ^{28}Si and ^{32}S were treated as $1d$ shell nuclei, that is, the occupation probability of the $2s$ state was taken to be zero. The formalism of the present work has the advantage that the occupation probabilities of the various states can be treated as free parameters in the fitting procedure of

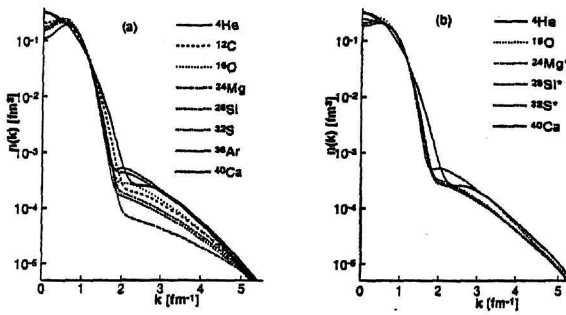

Fig. 2. (**a**) The correlated MD for various s-p and s-d shell nuclei calculated with the parameters b and β of the case when the nuclei ^{24}Mg, ^{28}Si, ^{32}S and ^{36}Ar were treated as $1d$ shell nuclei. The normalization is $\int n(\mathbf{k})d\mathbf{k} = 1$. (**b**) The same as in Fig. (**a**) but when the above mentioned nuclei were treated as $1d$-$2s$ shell nuclei.

$F_{ch}(q)$. Thus, the analysis can be made with more free parameters. For that reason we considered case 2* in which the occupation probability η_{2s} of the nuclei ^{24}Mg, ^{28}Si and ^{32}S was taken to be a free parameter together with the parameters b and β. We found that the χ^2 values become better, compared to those of case 2 and the A dependence of the parameter β is not so large as it was before. The new values of b and β are shown in Table 1. The values of the occupation probability η_{2s} of the above-mentioned three nuclei are 0.19982, 0.17988 and 0.50921 respectively, while the corresponding values of η_{1d} can be found from the values of η_{2s} through the relation $\eta_{1d} = [(Z-8) - 2\eta_{2s}]/10$. The MD of these three nuclei together with the closed shell nuclei ^{4}He, ^{16}O and ^{40}Ca found in case 2 are shown in Fig. 2b. It is seen that the A dependence of the high momentum component is now not so large as it was in case 2. As $F_{ch}(q)$ calculated in case 2* is closer to the experimental data than in case 2, we might say that this result is in the correct direction, that is the high momentum component of the MD of nuclei is almost the same. We would like to mention that experimental data for $n(k)$ are not directly measured but are obtained by means of y-scaling analysis [21] and only for ^{4}He and ^{12}C in s-p and s-d shell region. We expect that the above conclusion could be corroborated if new experimental data are obtained in the future for MD for several nuclei and we carry out a simultaneous fit both to MD and to form factors.

Finally, in table I we give the one and the two-body terms of the mean kinetic energy, $\langle T \rangle$, of the various s-p and s-d shell nuclei calculated on the basis of Eq. (23), as well as the rms charge radii, $\langle r^2_{ch} \rangle^{1/2}$ which are compared with the experimental values. It is seen that the introduction of SRC's (in case 2) increases the mean kinetic energy relative to case 1 $((\langle T_{case2} \rangle - \langle T_{case1} \rangle)/\langle T_{case2} \rangle)$ about 50% in ^{4}He and 23% in ^{24}Mg. This relative increase follows the fluctuation of the parameter β. Also the values of the kinetic energy in percents, $100\langle T_{SRC} \rangle / \langle T_{Total} \rangle$, as well as the ratio $< T_{Total} > / \langle T_{HO} \rangle$ follow the fluctuation of the parameter β. In closed shell nuclei there is an increase of the above values by the increasing of mass number.

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