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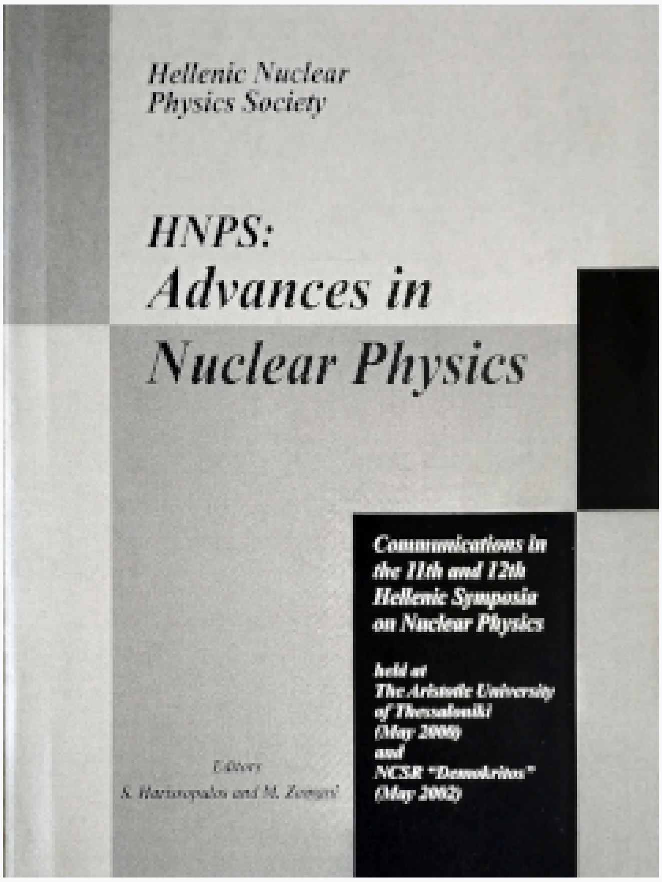

"Beat" Patterns of Odd-Even Staggering in Octupole Bands of Light Actinides

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“Beat” Patterns of Odd–Even Staggering in Octupole Bands of Light Actinides

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Abstract

The $\Delta I = 1$ staggering (odd–even staggering) in octupole bands of light actinides is found to exhibit a “beat” behaviour as a function of the angular momentum I, forcing us to revise the traditional belief that this staggering decreases gradually to zero and then remains at this zero value. Various algebraic models (spf-Interacting Boson Model, spdf-IBM, Vector Boson Model, Nuclear Vibron Model) predict in their su(3) limits constant staggering for this case, being thus unable to describe the “beat” behaviour. An explanation of the “beat” behaviour is given in terms of two Dunham expansions (expansions in terms of powers of $I(I+1)$) with slightly different sets of coefficients for the ground state band and the negative parity band, the difference in the values of the coefficients being attributed to Coriolis couplings to other negative parity bands.

1 Introduction

Rotational nuclear spectra have been long attributed to quadrupole deformations [1], corresponding to nuclear shapes produced by the revolution of an ellipsis around its maximum or minimum axis and rotating around an axis perpendicular to their axis of symmetry. In addition, it has been suggested that octupole deformation occurs in certain regions, most notably in the light actinides [2] and in the $A \approx 150$ mass region [3], corresponding to pear-like nuclear shapes [4,5]. In even nuclei exhibiting octupole deformation the ground state band, which contains energy levels with $I^\pi = 0^+, 2^+, 4^+, 6^+, \ldots$, is accompanied by a negative parity band containing energy levels with $I^\pi = 1^-$, 3^-, 5^-, 7^-, ... After the first few values of angular momentum I the two bands become interwoven, forming a single octupole band with levels characterized by $I^\pi = 0^+, 1^-, 2^+, 3^-, 4^+, 5^-, \ldots$ [2,3]. (It should be noted, however, that in the light actinides alternative interpretations of these bands in terms of alpha clustering have been proposed [6,7].)

It has been observed [8] that in octupole bands the levels with odd I and negative parity ($I^\pi = 1^-, 3^-, 5^-, \ldots$) are displaced relatively to the levels with even I and positive parity ($I^\pi = 0^+, 2^+, 4^+, \ldots$), i.e. the odd levels do not lie at the energies predicted by an $E(I) = AI(I+1)$ fit to the energy levels, but all of them lie systematically above or all of them lie systematically below the predicted energies. This is an example of *odd–even staggering* or $\Delta I = 1$ *staggering*, the latter term due to the fact that each energy level with angular momentum I is displaced relatively to its neighbours with angular momenta $I \pm 1$.

The $\Delta I = 1$ staggering effect is different from the $\Delta I = 2$ *staggering* effect recently observed [9,10] in superdeformed nuclear bands [11,12], since the $\Delta I = 2$ staggering effect refers to the systematic displacement of the levels with $I = 2, 6, 10, 14, \ldots$ relatively to the levels with $I = 0, 4, 8, 12, \ldots$, i.e. in this case the level with angular momentum I is displaced relatively to its neighbours with angular momenta $I \pm 2$.

It should be noted that the odd–even staggering in octupole bands is much larger than the relevant experimental errors, while the $\Delta I = 2$ staggering effect in superdeformed nuclear bands [9,10], is of the order of the experimental errors, with only one case (the (a) band of ^{149}Gd [9]) known to show an effect outside the limits of the experimental errors.

The dependence of the amplitude of the staggering effect on the angular momentum I presents much interest. The situation up to now has as follows:

1) Algebraic models of nuclear structure appropriate for the description of octupole bands, like the spf-Interacting Boson Model (spf-IBM) with u(11) symmetry [13], the spdf-IBM with u(16) symmetry [13,14], and the Vector Boson Model (VBM) with u(6) symmetry [15], predict in their su(3) limits $\Delta I = 1$ staggering of constant amplitude, i.e. all the odd levels are raised (or lowered) by the same amount of energy with respect to the even levels [16]. In other words, $\Delta I = 1$ staggering takes alternatively positive and negative values of equal absolute value as I increases.

2) Algebraic models of nuclear structure suitable for the description of alpha clustering effects, like the Nuclear Vibron Model (NVM) with u(6)⊗u(4) symmetry [6], also predict in the su(3) limit $\Delta I = 1$ staggering of constant

amplitude.

3) Older experimental work [2,3] on octupole nuclear bands suggests that $\Delta I = 1$ staggering starts from large values and its amplitude decreases with increasing I. These findings are in agreement with the interpretation that an octupole band is gradually formed as angular momentum increases [4].

Motivated by these recent findings, we make in the present work a systematic study in the light actinide region of all octupole bands for which at least 12 energy levels are known [17-23], taking advantage of recent detailed experimental work in this region. The questions to which we have hoped to provide answers are:

1) Which patterns of behaviour of the amplitude of the $\Delta I = 1$ staggering appear?

2) Can these patterns be interpreted in terms of the existing models [6,13-15], or in terms of any other theoretical description?

In Section 2 of the present paper the formalism of staggering is discussed, and is subsequently applied to the experimental data for octupole bands of light actinides in Section 3. Section 4 contains an interpretation of the experimental observations, while in Section 5 the conclusions reached, as well as plans for future work are given.

2. Formalism

Traditionally the odd–even staggering ($\Delta I = 1$ staggering) in octupole bands has been estimated quantitatively through use of the expression [8]

$$\delta E(I) = E(I) - \frac{(I+1)E(I-1) + IE(I+1)}{2I+1}, \tag{1}$$

where $E(I)$ denotes the energy of the level with angular momentum I. This quantity vanishes if the first two terms of the expression

$$E(I) = E_0 + AI(I+1) + B(I(I+1))^2 \tag{2}$$

are plugged into it, but it does not vanish if the third term of the above expression is substituted into it. Therefore it is suitable for measuring deviations from the pure rotational behaviour.

Recently, however, a new measure of the magnitude of staggering effects has been introduced [10] in the study of $\Delta I = 2$ staggering of nuclear superdeformed bands. In this case the experimentally determined quantities are the γ-ray transition energies between levels differing by two units of angular momentum ($\Delta I = 2$). For these the symbol

$$E_{2,\gamma}(I) = E(I+2) - E(I) \tag{3}$$

is used. The deviation of the γ-ray transition energies from the rigid rotator behavior is then measured by the quantity [10]

$$\Delta E_{2,\gamma}(I) = \frac{1}{16}(6E_{2,\gamma}(I) - 4E_{2,\gamma}(I-2) - 4E_{2,\gamma}(I+2)$$

$$+E_{2,\gamma}(I-4) + E_{2,\gamma}(I+4)). \tag{4}$$

Using the perturbed rigid rotator expression of Eq. (2) one can easily see that $\Delta E_{2,\gamma}(I)$ vanishes. This property is due to the fact that Eq. (4) is a (normalized) discrete approximation of the fourth derivative of the function $E_{2,\gamma}(I)$, i.e. essentially the fifth derivative of the function $E(I)$. Therefore we conclude that Eq. (4) is a more sensitive probe of deviations from rotational behaviour than Eq. (1).

By analogy, $\Delta I = 1$ staggering in nuclei can be measured by the quantity

$$\Delta E_{1,\gamma}(I) = \frac{1}{16}(6E_{1,\gamma}(I) - 4E_{1,\gamma}(I-1) - 4E_{1,\gamma}(I+1)$$

$$+E_{1,\gamma}(I-2) + E_{1,\gamma}(I+2)), \tag{5}$$

where

$$E_{1,\gamma}(I) = E(I+1) - E(I). \tag{6}$$

The transition energies $E_{1,\gamma}(I)$ are determined directly from experiment.

3. Analysis of experimental data

We have applied the formalism described above to all octupole bands of light actinides for which at least 12 energy levels are known [17-23] and which show no backbending (i.e. bandcrossing) [24] behaviour. Several of these nuclei ($^{222-226}$Ra, $^{224-228}$Th) are rotational or near-rotational (having $10/3 \geq R_4 \geq 2.7$), while others ($^{218-222}$Rn, ^{220}Ra, $^{220-222}$Th) are vibrational or near-vibrational (having $2.4 \geq R_4 \geq 2$), where the ratio $R_4 = \frac{E(4)}{E(2)}$ is a well known characteristic of collective behaviour. A special case is ^{218}Ra, for which it has been argued [18] that it is an example of a new type of transitional nuclei, in which the octupole deformation dominates over all other types of deformation.

The staggering results for $^{218-222}$Rn, $^{218-226}$Ra, and $^{220-228}$Th, have been given in Fig. 1, Fig. 2, and Fig. 3 of Ref. [25] respectively, which are not reproduced here because of space limitations. In all cases the experimental errors are of the size of the symbol used for the experimental point and therefore are not visible. The following observations can be made:

1) In all cases the shapes appearing are consistent with the following pattern: $\Delta I = 1$ staggering starts from large values at low I, it gradually decreases down to zero, then it starts increasing again, then it decreases down to zero and starts raising again. In other words, figures resembling beats appear. The most complete "beat" figures appear in the cases of ^{220}Ra, ^{224}Ra, ^{222}Th, as well as in the cases of ^{218}Ra, ^{222}Ra, ^{226}Ra.

2) In all cases within the first "beat" (from the beginning up to the first zero of $\Delta E_{1,\gamma}(I)$) the minima appear at odd I, indicating that in this region the odd levels are slightly raised in comparison to the even levels. Within the second "beat" (i.e. between the first and the second zero of $\Delta E_{1,\gamma}(I)$), the opposite holds: the minima appear at even I, indicating that in this region the odd levels are slightly lowered in comparison to the even levels. Within the third "beat" (after the second zero of $\Delta E_{1,\gamma}(I)$) the situation occuring within the first "beat" is repeated. (Notice that ^{220}Th is not an exception, since what is seen in the figure is the second "beat", starting from $I = 6$.)

3) In the case of ^{222}Rn the decrease of the staggering with increasing I, in the region for which experimental data exist, is very slow, giving the impression of almost constant staggering. One can get a similar impression from parts of the patterns shown, as, for example, in the cases of ^{220}Ra (in the region $I = 12 - 20$), ^{222}Ra (for $I = 9 - 17$), ^{224}Ra (for $I = 10 - 16$), ^{226}Ra (for $I = 14 - 20$), ^{222}Th (for $I = 10 - 18$).

The following comments are also in place:

1) In all cases bands not influenced by bandcrossing effects [24] have been considered, in order to make sure that the observed effects are "pure" single-band effects. The only exception is ^{220}Th, which shows signs of bandcrossings at 10^+ and 13^-, which, however, do not influence the relevant staggering pattern. A special case is ^{218}Ra, which shows a rather irregular dependence of $E(I)$ on I. As we have already mentioned, it has been argued [18] that this nucleus is an example of a new type of transitional nuclei in which the octupole deformation dominates over all other types of deformation.

2) The same "beat" pattern appears in both rotational and vibrational nuclei. The only slight difference which can be observed, is that the first vanishing of the staggering amplitude seems to occur at higher I for the rotational isotopes than for their vibrational counterparts. Indeed, within the Ra and Th series of isotopes under study, the I at which the first vanishing of the staggering amplitude occurs seems to be an increasing function of R_4, i.e. an increasing function of the quadrupole collectivity.

3) The present findings are partially consistent with older work [2,3]. The limited sets of data of that time were reaching only up to the I at which the first vanishing of the staggering amplitude occurs. It was then reasonable to assume that the staggering amplitude decreases down to zero and remains zero afterwards, since no experimental evidence for "beat" patterns existed at that time.

4. Interpretation of the experimental observations

Although the su(3) limits of the various algebraic models mentioned in the introduction are sufficient for providing an explanation for $\Delta I = 1$ staggering in the cases in which this appears as having almost constant amplitude [16], it is clear that some additional thinking is required for the many cases in which the experimental results show a "beat" pattern, as in Section 3 has been seen.

A simple explanation for the appearance of "beat" patterns can be given by the following assumptions:

1) It is clear that in each nucleus the even levels form the ground state band, which starts at zero energy, while the odd levels form a separate negative parity band, which starts at some higher energy. Let us call E_0 the bandhead energy of the negative parity band.

2) It is reasonable to try to describe the ground state band by an expression like

$$E_+(I) = AI(I+1) - B(I(I+1))^2 + C(I(I+1))^3 + \cdots \tag{7}$$

where the subscript + reminds us of the positive parity of these levels. Such expansions in terms of powers of $I(I+1)$ have been long used for the description of nuclear collective bands [26]. They also occur if one considers [27] Taylor expansions of the energy expressions provided by the Variable Moment of Inertia (VMI) model [28] and the su$_q$(2) model [29]. Notice that fits to experimental data [26] indicate that one always has $A > 0$, $B > 0$, $C > 0$, ..., while A is usually 3 orders of magnitude larger than B, B is 3 orders of magnitude larger than C, etc. Eq. (7) has been long used in molecular spectroscopy as well, under the name of Dunham expansion [30].

3) In a similar way, it is reasonable to try to describe the negative parity levels by an expression like

$$E_-(I) = E_0 + A'I(I+1) - B'(I(I+1))^2 + C'(I(I+1))^3 + \cdots \tag{8}$$

where the subscript $-$ reminds us of the negative parity of these levels, while E_0 is the above mentioned bandhead energy. In analogy to the previous case one expects to have $A' > 0$, $B' > 0$, $C' > 0$, ...

4) In the above expansions it is reasonable to assume that $A > A'$, $B > B'$, $C > C'$, ... This assumption is in agreement with earlier work [31,32], in which the Coriolis couplings between the lowest $K = 0$ negative parity band and higher negative parity bands with $K \neq 0$ are taken into account, resulting in an increase of the monent of inertia of the lowest $K = 0$ negative parity band [33]. This argument means that the coefficient A' in Eq. (8), which is inversely proportional to the moment of inertia of the negative parity band, should be smaller than the coefficient A in Eq. (7), which is inversely proportional to the moment of inertia of the positive parity band. In analogy to the relation $A > A'$, which we just justified, one can assume $B > B'$, $C > C'$, ... This last argument is admitedly a weak one, which is however driving to interesting results, as we shall soon see.

Using Eqs (7) and (8) in Eqs (5) and (6) we find the following results

$$\Delta E(I) = \pm E_0 \mp (A-A')(I^2+2I+2) \pm (B-B')\left(I^4+4I^3+13I^2+18I+\frac{23}{2}\right)$$

$$\mp(C-C')\left(I^6+6I^5+33I^4+92I^3+\frac{357}{2}I^2+\frac{333}{2}I+68\right)$$

$$\pm 45C'(I+1)+\cdots, \tag{9}$$

where the upper (lower) signs correspond to the case with I=even (I=odd). A sample staggering pattern drawn using these formulae has been given in Fig. 4 of Ref. [25]. On these results the following comments can be made:

1) The expression for odd I is the opposite of the expression with even I. This explains why in Fig. 4 of Ref. [25] the staggering points for even I and the staggering points for odd I form two lines which are reflection symmetric with respect to the horizontal axis.

2) For even I the behaviour of the staggering amplitude is as follows: At low I it starts from a positive value, because of the presence of E_0. As I increases, the second term, which is essentialy proportional to I^2, becomes important. (E_0 is expected to be much larger than $(A-A')$.) This term is negative (since $A > A'$), thus it decreases the amplitude down to negative values. At higher values of I the third term, which is essentially proportional to I^4, becomes important. (Remember that usually B is 3 orders of magnitude smaller than A [26].) This term is positive (since $B > B'$), thus it increases the amplitude up to positive values. (The behaviour up to this point can be seen in Fig. 4 of Ref. [25].) At even higher values of I the fourth term, which is essentially proportional to I^6, becomes important. (Remember that usually C is 3 orders of magnitude smaller than B [26].) This term is negative (since $C > C'$), thus it decreases the amplitude again down to negative values, and so on.

3) For odd I the behaviour of the staggering amplitude is exactly the opposite of the one described in 2) for even I. The amplitude starts from a negative value and then becomes consequently positive (because of the second term), negative (because of the third term), again positive (because of the fourth term), and so on. The first three steps of this behaviour can be seen in Fig. 4 of Ref. [25].

4) When drawing the staggering figure one jumps from an even I to an odd I, then back to an even I, then back to an odd I, and so on. It is clear therefore that a "beat" pattern appears, as it is seen in Fig. 4 of Ref. [25].

The following additional comments are also in place:

1) In the case of a single band (i.e. in the case of $A = A'$, $B = B'$, $C = C'$, etc), the first contribution to the staggering measure $\Delta E(I)$ is the last term in Eq. (9), which comes from the $C(I(I+1))^3$ term in the energy expansion (see Eqs (7), (8)). This is understandable: Since Eq. (5) is a discrete approximation of the fifth derivative of the function $E(I)$, as it has already been remarked, the terms up to $B(I(I+1))^2$ are "killed" by the derivative, while the $C(I(I+1))^3$ term gives a contribution linear in I.

2) The last term in Eq. (9) does not influence significantly the behaviour of the staggering pattern, since C is usually 6 orders of magnitude smaller than A and 3 orders of magnitude smaller than B [26].

3) This type of explanation of the staggering patterns seems to be outside the realm of the form of the su(3) limits of the algebraic models mentioned in the introduction. Even if one decides to include higher order terms of the type $(I(I+1))^2$, $(I(I+1))^3$, etc, in these models, by including in the Hamiltonian higher powers of the relevant Casimir operator, these terms will appear with the same coefficients for both the ground state band and the negative parity band, even though these two bands belong to different irreps. The only possible contributions to the staggering will then come from terms like the last term in Eq. (9), which comes from the term $(I(I+1))^3$, and similar terms coming from higher powers of $I(I+1)$. However, the term $(I(I+1))^3$ in the framework of the algebraic models already corresponds to 6-body interactions [34], which are usually avoided in nuclear structure studies.

We conclude therefore that the "beat" pattern can be explained in terms of two Dunham expansions with slightly different sets of coefficients, one for the ground state band with quadrupole deformation and another for the negative parity band in which in addition the octupole deformation appears. This is, however, a phenomenological finding, the microscopic origins of which should be searched for.

5. Discussion

We have demonstrated that octupole bands in the light actinides exhibit $\Delta I = 1$ staggering (odd–even staggering), the amplitude of which shows a "beat" behaviour. The same pattern appears in both vibrational and rotational nuclei, forcing us to modify the traditional belief that in octupole bands the staggering pattern is gradually falling down to zero as a function of the angular momentum I and then remains there.

The su(3) limits of various algebraic models, including octupole degrees of freedom [13-15] or based on the assumption that alpha clustering is important in this region [6], predict $\Delta I = 1$ staggering of amplitude constant as a function of the angular momentum I [16]. Although this description becomes reasonable in the rotational limit, it cannot explain the "beat" patterns appearing in both the rotational and the vibrational regions. The detailed study of limits other than the su(3) ones for these models remains an interesting open problem.

A simple explanatation of the "beat" behaviour has been given by describing the even I levels of the ground state band and the odd I levels of the negative parity band by two Dunham expansions [30] (expansions in powers of $I(I+1)$) with slightly different sets of coefficients, the difference in the coefficients being attributed to Coriolis couplings of the negative parity band to other negative parity bands. However, the microscopic origins of the "beat" behavior need further elucidation.

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