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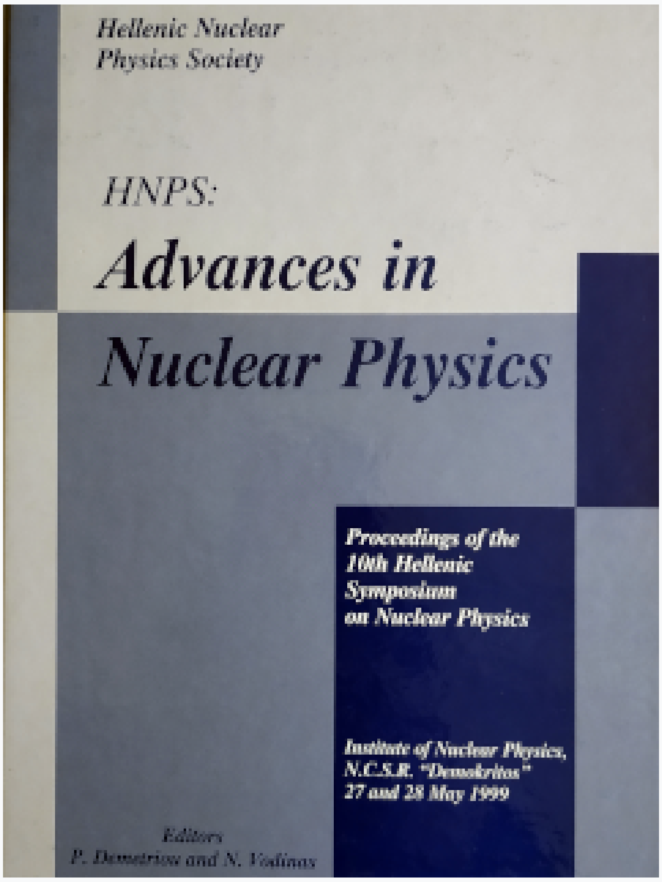

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An analytical formula for proton momentum distribution in nuclei with A≥4

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Abstract

A successful analytical formula for the proton momentum distribution in all nuclei with A≥4 accounting for nucleon-nucleon correlation effects, is presented. In this formula the Isomorphic Shell Model wave functions are employed, which are readily available for all nuclei all the way up to ^{208}Pb. However, other wave functions (e.g., shell model or Hartree-Fock) could be used with almost equivalent results. Available experimental data for ^{4}He, ^{12}C and ^{56}Fe and predictions of other theories, e.g., for ^{40}Ca, are used for comparison of the predictions of the present formula. A reservation is kept concerning the validity of this formula for the momentum distribution of exotic nuclei.

1 Introduction

In a recent publication [1] it was pointed out that the knowledge of the experimental proton momentum distribution for ^{4}He is enough information for an estimation of the proton momentum distribution (including high momentum components) of nuclei beyond ^{4}He. This is possible due to the fact that proton momentum distributions of nuclei with $k<2$ fm^{-1} follow, in a good approximation, the shell model predictions [1-3], while these distributions with $k \geq 2$ fm^{-1} (i.e., their high momentum components) are approximately the same with that of ^{4}He [4-6]. This possibility is very valuable since the experimental momentum distribution is known only for a few nuclei [4], i.e., for ^{4}He, ^{12}C, and ^{56}Fe, and the theoretical calculations are limited as well [7-17]. At the same time, the knowledge of momentum distribution is important for calculations of cross sections for various kinds of nuclear reactions.

The present work constitutes a substantial improvement of Ref.[1] in the sense that, in contrast to what happens in this reference, no experimental information is needed for an estimation of the proton momentum distribution of any nucleus with A$\geq$4. In other words, a parameter-free analytical formula is provided, valid for the proton momentum distribution of any nucleus beyond ^{4}He. This formula could be approximated by using Ref.[18], where an analytical expression for the approximation of the proton momentum distribution of ^{4}He was obtained simultaneously with an analytical expression for the approximation of the proton density distribution of this nucleons. However, it has here been found that, for a much better approximation of the experimental momentum distribution alone of ^{4}He, another set of parameters is needed in the analytical expression of the momentum distribution of ^{4}He introduced in Ref.[18]. Specifically, the parameters (i.e.,$\lambda_{1s} = 0.93$, $\eta\omega_{1s} = 15$ MeV, $\eta\omega_{1d}$ = 72 MeV, where $2\lambda_{1s} + 10\lambda_{1d} = 2$) used here lead to results for the momentum distribution alone of ^{4}He much better than those coming from the parameters used in Ref.[18] (i.e.,$\lambda_{1s} = 0.86$, $\eta\omega_{1s} = 22{,}5$ MeV, $\eta\omega_{1d} = 150$ MeV, where $2\lambda_{1s} + 10\lambda_{1d}$ = 2) which aimed to a simultaneous reproduction of both the momentum distribution and the density distribution of ^{4}He. Thus, as will become apparent shortly, the parameter-free analytical formula here provided comes combining Ref.[1] and the analytical expression for the proton momentum distribution of ^{4}He corresponding to the aforementioned new parameters, instead of those from Ref.[18].

The formula suggested in the present work uses the transparency of the single-particle picture, being within the framework of a given correlation method by means of the natural orbital representation [19]. Specifically, in the applications here provided, the wave functions of the Isomorphic Shell Model [1, 20-24] are employed which have been proved very successful in predicting several observables in many nuclei.

2 The Model

We start from the natural orbital representation [19], where the proton momentum distribution of a nucleus with Z protons, normalized to unity, has the form [3]

$$n(k) = \frac{1}{4\pi Z}\sum_{n\ell j}(2j+1)\lambda_{n\ell j}|\tilde{R}_{n\ell j}(k)|^2, \tag{1}$$

where $\lambda_{n\ell j}$ is the natural orbital occupation number for the state with quantum numbers (n,ℓ,j) and

$$\sum_{n\ell j}(2j+1)\lambda_{n\ell j} = Z. \tag{2}$$

We call hole-state (h) natural orbitals those natural orbitals for which the numbers are significantly larger than the remaining ones called particle-state (p) natural orbitals [25]. It has been shown by the Jastrow correlation method [12,13] that the high-momentum components of the total , caused by the short-range correlations, are almost completely determined by the contributions of the particle-state natural orbitals. That is,

$$n(k) = n_{\rm h}(k) + n_{\rm p}(k), \tag{3}$$

where the equation

$$n_{\rm h}(k) = \frac{1}{4\pi Z}\sum_{n\ell j}^{F.L.}(2j+1)\lambda_{n\ell j}|\tilde{R}_{n\ell j}(k)|^2, \tag{4}$$

with F.L. meaning Fermi Level, stands (almost totally for the Hartree-Fock or, similarly, for the shell model case) for the hole-state wave functions contribution and the equation

$$n_{\rm p}(k) = \frac{1}{4\pi Z}\sum_{n\ell j}^{F.L.}(2j+1)\lambda_{n\ell j}|\tilde{R}_{n\ell j}(k)|^2, \tag{5}$$

stands (almost totally) for the high-momentum components due to particle-state wave functions contribution.

From Ref.[18] for ^{4}He we have for $n(k)$

$$\begin{aligned} n^{^4\rm He}(k) &= n_{\rm h}^{^4\rm He}(k) + n_{\rm p}^{^4\rm He}(k) \\ &= \frac{1}{4\pi(2)}[2\lambda_{1s}^{^4\rm He}|\tilde{R}_{1s}^{^4\rm He}(k)|^2 + 10\lambda_{1d}^{^4\rm He}|\tilde{R}_{1d}^{^4\rm He}(k)|^2]. \end{aligned} \tag{6}$$

Approximating the particle part of the momentum distribution of (A, Z)-nucleus, Eq.(5), by the particle part $n_{\rm p}^{^4\rm He}(k)$ from Eq.(6), Eq.(1) for the correlated proton momentum distribution of any nucleus may be written as

$$\begin{aligned} n(k) = \frac{N_0}{4\pi Z}(&2\lambda_{1s}^{^4\rm He}|\tilde{R}_{1s}^{^4\rm He}(k)|^2 + \frac{Z}{2}10\lambda_{1d}^{^4\rm He}|\tilde{R}_{1d}^{^4\rm He}(k)|^2 \\ &+ \sum_{n\ell(\neq 1s)}^{F.L.(A,Z)} 2(2\ell+1)\lambda_{n\ell}^{A,Z}|\tilde{R}_{n\ell}^{A,Z}(k)|^2), \end{aligned} \tag{7}$$

where $\lambda_{n\ell}^{A,Z} \sim 1$ and

$$N_0 = Z/[2\lambda_{1s}^{^4\mathrm{He}} + \frac{Z}{2}10\lambda_{1d}^{^4\mathrm{He}} + \sum_{n\ell(\neq 1s)}^{F.L.(A,Z)} 2(2\ell+1)\lambda_{n\ell}^{A,Z}]. \tag{8}$$

Eq.(7) expresses the proton momentum distribution for any nucleus with A≥4 under the assumption (experimentally verified [4-6]) that beyond ^{4}He all nuclei have similar (almost equal) tail of their proton momentum distribution [4-6].

As shown in Refs.[12-13], the hole-state natural orbitals are almost unaffected by the short-range correlations and, therefore, the functions $\tilde{R}_{n\ell}(k)$ in Eq.(7) can be replaced by the corresponding Hartree-Fock single-particle wave functions or by any shell-model wave functions $\tilde{R}_{n\ell j}(k)$ [1].

3 Calculations and discussion

In Fig. 1(a)-(d) the proton momentum distributions for the nuclei ^{4}He, ^{12}C, ^{40}Ca and ^{56}Fe coming from Eq.(7) are shown. For the proton momentum distribution of ^{4}He the parameters given in the introduction [namely, $\lambda_{1s}^{^4\mathrm{He}} = 0.930$, $\lambda_{1d}^{^4\mathrm{He}} = 0.014$ (where $2\lambda_{1s}^{^4\mathrm{He}} + 10\lambda_{1d}^{^4\mathrm{He}} = 2$), and $\eta\omega_{1s} = 15$ MeV, $\eta\omega_{1d} = 72$ MeV] and not those from Ref.[18], are employed. That is, the parameters leading to the best fit of the momentum distribution alone of ^{4}He are utilized, instead of those leading to the best simultaneous fit of momentum and density distribution of this nucleus as in Ref.[18]. Semi-experimental data (called so since their analysis is based on certain assumptions) for comparison (available only [4] for ^{4}He, ^{12}C and ^{56}Fe) are also shown, together with the predictions of other models (where available) and with those of Ref.[1]. The comparison for ^{40}Ca, where no experimental data exist, is made solely with respect to existing calculations of other correlated methods [10-16]. As seen from this figure our predictions for ^{4}He, ^{12}C and ^{56}Fe are very good and better than the predictions by any other method [1]. For ^{40}Ca no similar comment can be made, since there is no experimental data for comparison. In our calculations of Fig. 1(a)-(d) we use the multiharmonic oscillator potential single-particle wave functions from Ref.[21] as it was the case in Ref. [1].

From Fig. 1(a)-(d) it is apparent that the results of the present formula [Eq.(7)] are better than those of Ref.[1], and those obtained by using the parameters for the momentum distribution of ^{4}He employed in Ref.[18] (see Fig.1(b) in this reference), particularly near the origin. In addition to the quality of fitting, of course, one should notice that Eq. (7) makes no use of semi-experimental data, while in Ref. [1] those of ^{4}He are employed. In both cases (Eq. (7) and Ref.[1]) no adjustable parameters are involved.

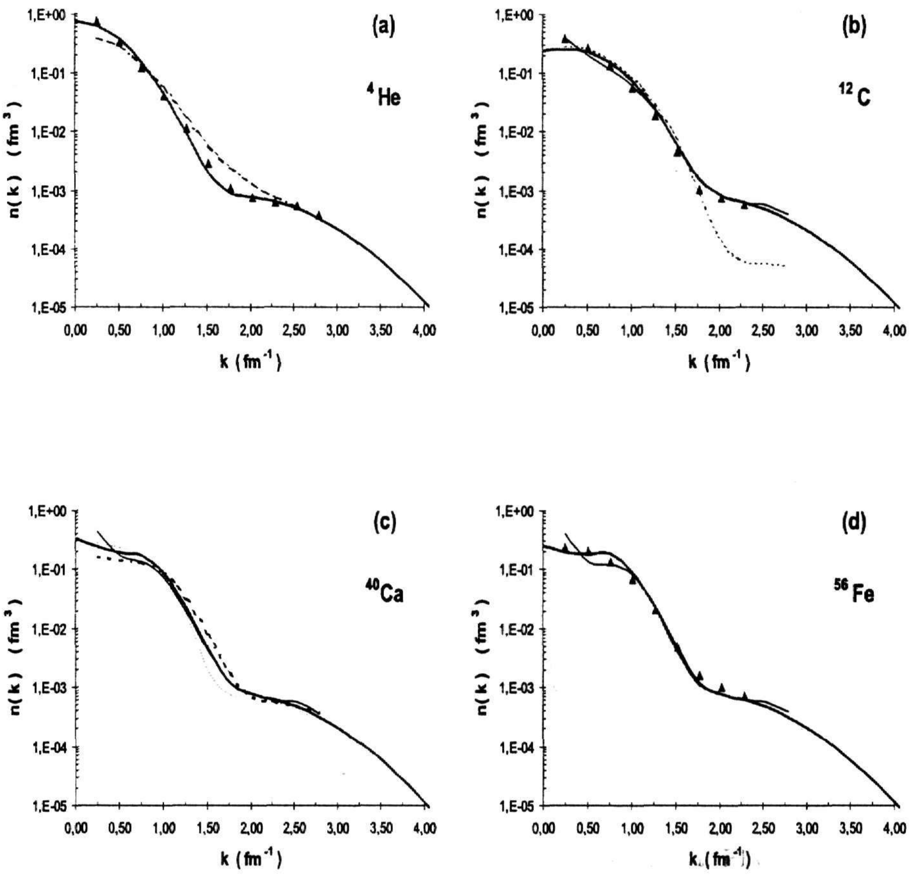

Fig. 1. Proton momentum distributions $n(k)$ versus k. (a) of the ^{4}He, (b) of the ^{12}C, (c) of the ^{40}Ca, and (d) of the ^{56}Fe. In all parts of the figure triangles stand for semi-experimental data from [4], thick solid lines for results coming from Eq.(7) with no use of experimental data, thin solid lines for results from [1] with use of experimental data for ^{4}He, dash-dot lines for results of calculations from [9], dotted line for results from [12,13] based on the Jastrow correlation method, and dashed line for results of calculations from [15].

4 Conclusions

In the present work a parameter-free analytical formula, Eq.(7), for calculating the proton momentum distribution of any nucleus with $A \geq 4$ is introduced making no use of available semi-experimental data.. This formula combines the mean-field part of momentum distribution with its correlated part which is taken to be identical to that part of ^{4}He [1,4-6]. The universal parameters in Eq.(7) describing the momentum distribution of ^{4}He are those given in the introduction which are different from those of Ref.[18].

While different single-particle models (e.g., Hartree-Fock or Shell Model) can be used for an estimation of the hole-state wave functions entering our formula, the use of the Isomorphic Shell Model wave functions [1, 20-24] employed here, besides the excellent results with respect to the data or the results from other correlated methods as shown in Fig. 1(a)-(d), has the additional advantages of being adjustable-parameters free and well specified in advance (readily available) for any nucleus up to ^{208}Pb [21].

The formula here proposed offers an easy calculation of $n(k)$ for any nucleus with $A \geq 4$ and thus allows us to describe (as in Ref.[27]) quantities involved in processes of particle scattering by nuclei.

It is apparent that the present work is a substantial improvement of Ref.[1] for two reasons. First, no experimental information is necessary for an application of the formula here proposed which has an analytical form and, second, the results provided are better both in the tail section and particularly in the region near the origin of the proton momentum distribution (Fig. 1(a),(b),(d)).

Finally, a reservation could be kept concerning the validity of Eq. (7) in exotic nuclei, where at the moment no semi-experimental data are available, according to the following reasoning. Ref. [18] deals with a possible explanation of the high momentum components of ^{4}He as a result of collective internal rotation of certain nucleons at the ground state in addition to the usual nucleon internal rotation of shell model type. That is, this additional internal rotation modifies (specifically increases) the high components of the momentum distribution. Also, according to Ref. [28] the appearance of the halo phenomenon in exotic nuclei could be understood as a result of collective internal rotation at the ground state of a group of nucleons, as in the aforementioned case of high momentum components in ^{4}He. Hence, in exotic nuclei one could expect an additional increase of the high momentum components of their momentum distribution, due to an additional internal collective rotation of certain nucleons in their ground states. Thus, a deviation of the predictions of formula (7) in exotic nuclei could be expected.

An effort, to search whether a parameter-free analytical formula for calculating the proton and the neutron density distributions of any nucleus (making no use of experimental data) can be obtained, is under way.

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