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Nucleon Electromagnetic Form Factors in a constituent-quark model

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Abstract

The electromagnetic form factors of the nucleon are calculated in an extended chiral constituent-quark model where the effective interaction is described by the exchange of pseudoscalar and vector mesons belonging to the pseudoscalar and pseudovector octet, respectively, and a scalar meson. Two-body current-density operators, constructed consistently with the extended model Hamiltonian in order to preserve gauge invariance and current conservation, are found to give a significant contribution to the nucleon magnetic form factors and improve the estimates of the nucleon magnetic moments.

1 Introduction

Constituent-quark models (CQM) have been widely used to describe the spectroscopic properties of hadrons and have been rather successful in reproducing the gross features of hadron spectra within a nonrelativistic [1–4] and relativistic framework [5–8]. In all these models the effective interaction between the valence quarks is described by the one-gluon-exchange diagram and is identified with the hyperfine-like part of its nonrelativistic reduction. Various hybrid models have also been constructed including meson exchanges in addition to sizeable contributions coming from gluon exchanges [9–13].

Despite the overall success, none of these models has been able to explain the correct level orderings in light- and strange baryon spectra nor the flavour or spin content of the nucleon. This is mainly due to the inadequacy of interactions that do not take into account the implications of the spontaneous breaking of chiral symmetry (SB χ S). As a consequence of SB χ S, quarks acquire their dynamical masses related to $\langle q\bar{q} \rangle$ condensates and Goldstone bosons appear which couple directly to the

constituent quarks. Thus beyond the scale of $SB\chi S$ the effective degrees of freedom are constituent quarks and Goldstone-boson fields and baryons can be considered as systems of three constituent quarks that interact by Goldstone-boson exchange (GBE) and are subject to confinement [14,15]. The Goldstone bosons manifest themselves in the octet of pseudoscalar mesons (π , K , η).

In view of these considerations, a semirelativistic chiral CQM has been proposed [16–17] based on a semirelativistic Hamiltonian where the dynamical part consists of a linear confinement potential and a chiral potential derived from GBE and η' -exchange. The model is able to reproduce the correct level orderings of positive- and negative-parity excitations providing hence a unified description not only of the Nucleon and Delta spectra but also of all strange baryons.

A further, stringent test of the model would be to investigate its validity with regard to other observables. Such an attempt where the three- Q wavefunctions obtained from the pseudoscalar-exchange version of the semirelativistic chiral CQM were used to calculate the elastic electromagnetic form factors of the nucleon [18], has shown that the two-body current operator constructed consistently with the model Hamiltonian gives zero contributions to the elastic e.m. form factors of the nucleon. Furthermore, the semi-relativistic one-body charge- and current-density operators [18] underpredict the charge radii and the magnetic moments of the nucleon. However, the two-body currents arising in the pseudoscalar-exchange version of the model are due solely to the spin-spin component of the pseudoscalar-exchange interaction, whereas we would expect that the current operators obtained from the full pseudoscalar meson-exchange interaction including the tensor component would give non-zero contributions to the form factors altering the picture obtained in ref. [18].

The importance of two-body currents for the electromagnetic properties of baryons is therefore still not well understood and it is the aim of this contribution to gain further insight into the different Q - Q interactions and the relative contributions of the exchange-currents they give rise to within a semi-relativistic extended GBE CQM including spin-spin and tensor forces.

2 Extended GBE CQM

In the extended CQM the three-quark Hamiltonian is

$$H_0 = \sum_{i=1}^3 \sqrt{\vec{p}_i^2 + m_i^2} + \sum_{i<j}^3 V_{ij}, \quad (1)$$

with m_i the masses and \vec{p}_i the three-momenta of the constituent quarks. This form ensures that the average quark velocity be lower than the light velocity, a requirement that is not fulfilled by nonrelativistic models. The dynamical part consists of

a Q-Q interaction,

$$V_{ij} = V_{conf} + V_\chi, \quad (2)$$

where the confinement potential is in the extended CQM version taken to be

$$V_{conf}(\vec{r}_{ij}) = V_0 + Cr_0(1 + e^{-r_{ij}/r_0}), \quad (3)$$

depending on the interquark distance r_{ij} . The extended chiral potential V_χ^V is derived from GBE and multiple GBE [20] and consists of a pseudoscalar part describing pseudoscalar meson-exchange (π , K , η , η')

$$\begin{aligned} V_\chi^{PS}(\vec{r}_{ij}) = & \left[\sum_{a=1}^3 V_\pi^{SS}(\vec{r}_{ij}) \lambda_i^a \lambda_j^a + \sum_{a=4}^7 V_K^{SS}(\vec{r}_{ij}) \lambda_i^a \lambda_j^a \right. \\ & \left. + V_\eta^{SS}(\vec{r}_{ij}) \lambda_i^8 \lambda_j^8 + \frac{2}{3} V_{\eta'}^{SS}(\vec{r}_{ij}) \right] \vec{\sigma}_i \cdot \vec{\sigma}_j \\ & + \left[\sum_{a=1}^3 V_\pi^T(\vec{r}_{ij}) \lambda_i^a \lambda_j^a + \sum_{a=4}^7 V_K^T(\vec{r}_{ij}) \lambda_i^a \lambda_j^a \right. \\ & \left. + V_\eta^T(\vec{r}_{ij}) \lambda_i^8 \lambda_j^8 + \frac{2}{3} V_{\eta'}^T(\vec{r}_{ij}) \right] S_{ij} \end{aligned} \quad (4)$$

where S_{ij} is the tensor operator and a pseudovector part for vector meson-exchange (ρ , K^* , ω , ω')

$$\begin{aligned} V_\chi^{PV}(\vec{r}_{ij}) = & \left[\sum_{a=1}^3 V_\rho^{SS}(\vec{r}_{ij}) \lambda_i^a \lambda_j^a + \sum_{a=4}^7 V_{K^*}^{SS}(\vec{r}_{ij}) \lambda_i^a \lambda_j^a \right. \\ & \left. + V_\omega^{SS}(\vec{r}_{ij}) \lambda_i^8 \lambda_j^8 + \frac{2}{3} V_{\omega'}^{SS}(\vec{r}_{ij}) \right] \vec{\sigma}_i \cdot \vec{\sigma}_j \\ & + \left[\sum_{a=1}^3 V_\rho^T(\vec{r}_{ij}) \lambda_i^a \lambda_j^a + \sum_{a=4}^7 V_{K^*}^T(\vec{r}_{ij}) \lambda_i^a \lambda_j^a \right. \\ & \left. + V_\omega^T(\vec{r}_{ij}) \lambda_i^8 \lambda_j^8 + \frac{2}{3} V_{\omega'}^T(\vec{r}_{ij}) \right] S_{ij} \\ & + \left[\sum_{a=1}^3 V_\rho^C(\vec{r}_{ij}) \lambda_i^a \lambda_j^a + \sum_{a=4}^7 V_{K^*}^C(\vec{r}_{ij}) \lambda_i^a \lambda_j^a \right. \\ & \left. + V_\omega^C(\vec{r}_{ij}) \lambda_i^8 \lambda_j^8 + \frac{2}{3} V_{\omega'}^C(\vec{r}_{ij}) \right], \end{aligned} \quad (5)$$

where $\vec{\sigma}_i$ and λ_i^a represent the quark spin and flavor matrices, respectively. The scalar chiral potential for σ -meson-exchange is given by

$$V_\chi^S = V_\sigma^C(\vec{r}_{ij}). \quad (6)$$

In the static approximation one obtains the well-known spin-spin V^{SS} , tensor V^T and central V^C meson-exchange potentials. For example, the radial part of the spin-spin potential is given by

$$V_{\gamma}^{SS}(\vec{r}_{ij}) = \frac{g_{\gamma}^2}{4\pi} \frac{1}{12 m_i m_j} \left[\mu_{\gamma}^2 \frac{e^{-\mu_{\gamma} r_{ij}}}{r_{ij}} - 4\pi \delta(\vec{r}_{ij}) \right], \quad (7)$$

with μ_{γ} being the meson masses and g_{γ} the meson-quark coupling constants. For the pseudovector potentials ($\gamma = \rho, K^*, \omega, \omega'$) the $g_{\gamma}^2/4\pi$ is replaced by $2(g_{\gamma}^V + g_{\gamma}^T)^2/4\pi$ with $g_{\gamma}^{(V,T)}$ the meson-quark vector and tensor coupling constants respectively.

Since one is dealing with structured particles (constituent quarks and mesons) of finite extension, one has to smear out the δ function in Eq. (7).i.e.

$$4\pi \delta(\vec{r}_{ij}) \longrightarrow \left(\mu_{\gamma}^2 + \frac{\Lambda_{\gamma} (\Lambda_{\gamma}^2 - \mu_{\gamma}^2) r_{ij}}{2} \right) \frac{e^{-\Lambda_{\gamma} r_{ij}}}{r_{ij}}. \quad (8)$$

The corresponding expressions for the radial terms of the tensor and central potentials are obtained in similar fashion. The cut-off parameter Λ_{γ} in the above expressions varies linearly with the magnitudes of the meson masses μ_{γ}

$$\Lambda_{\gamma} = \Lambda_0 + \kappa \mu_{\gamma}. \quad (9)$$

The quark masses are fixed to the typical values $m_{u,d} = 340$ MeV and $m_s = 500$ MeV and the strength of the confinement potential to $C = 2.53 \text{ fm}^{-2}$. The remaining fitting parameters are the constant V_0 and the parameter r_0 in the confinement potential, and the parameters Λ_0 and κ defining Λ_{γ} . The meson-quark coupling constants were derived from the phenomenological meson-nucleon coupling constants. For the coupling constants g_0 of the pseudoscalar singlet (η') and g_{σ} of the scalar meson $g_8 = g_0 = g_{\sigma}$ was assumed [20].

The Schrödinger-type equation for the model is solved with the stochastic variational method [21] which is able to give results with an accuracy of better than 1%. The results show quite a satisfactory agreement with the spectra of all the low-lying light and strange baryons. The level orderings of the lowest positive- and negative-parity states in the nucleon spectrum are reproduced correctly and the same applies for the low-lying states in the Λ and Σ spectra, the only unresolved problem being the flavor singlet $\Lambda(1405)$.

3 Charge-exchange current operators

The relativistic form of the kinetic energy does not permit the use of the traditional one-body current density operator; so in order to be consistent with the model Hamiltonian the gauge invariant charge-current density operator is derived within

a functional derivative formalism [19]. It contains both one- and two-body terms. The one-body contribution includes the charge, the convective- and the spin-current operators. Their matrix elements between free particle states for a particle of charge e and mass m have been derived in momentum space [18] and with respect to the usual nonrelativistic expressions, only the spatial components of the charge-current density operator are affected, while the time component is simply given by the charge density.

The two-body current operator can be derived directly from the continuity equation consistently with the model Hamiltonian described in the previous section. In momentum space the continuity equation reads

$$\vec{q} \cdot \vec{J}_{[2]} = [\vec{J}_{[1]}^0, \bar{V}] \quad (10)$$

with $\vec{J}_{[1]}^0, \bar{V}$ the Fourier transforms of the one-body charge-density and Q-Q potential V_{ij} of the previous section, respectively. Due to the flavor-dependence of $J_{[1]}^0$ it turns out that the only non-vanishing exchange currents arise from π -, K -, ρ - and K^* -exchange. If we restrict ourselves to the non-strange baryon sector, then we have contributions only from the exchange of pions and rho-mesons and due to their isospin structure the exchange currents we finally obtain from Eq. (10) are the well-known pion(rho)-pair ($\pi(\rho)q\bar{q}$) currents and pion(rho)-in-flight ($\gamma\pi(\rho)\pi(\rho)$) currents

$$\vec{J}_{\pi q\bar{q}}(\vec{k}_i, \vec{k}_j) = ie \frac{g_\pi^2}{4m_i m_j} \left[\frac{\vec{\sigma}_i \cdot \vec{k}_i}{(\vec{k}_i^2 + \mu_\pi^2)} \left(\frac{\Lambda_\pi^2 - \mu_\pi^2}{\vec{k}_i^2 + \Lambda_\pi^2} \right)^2 \vec{\sigma}_j - (i \leftrightarrow j) \right] (\vec{\tau}_i \times \vec{\tau}_j)_z \quad (11)$$

$$\begin{aligned} \vec{J}_{\gamma\pi\pi}(\vec{k}_i, \vec{k}_j) = ie \frac{g_\pi^2}{4m_i m_j} \frac{\vec{\sigma}_i \cdot \vec{k}_i \vec{\sigma}_j \cdot \vec{k}_j}{(\vec{k}_i^2 + \mu_\pi^2)(\vec{k}_j^2 + \mu_\pi^2)} (\vec{k}_i - \vec{k}_j) \frac{(\Lambda_\pi^2 - \mu_\pi^2)^2}{(\vec{k}_i^2 + \Lambda_\pi^2)(\vec{k}_j^2 + \Lambda_\pi^2)} \\ \times \left(1 + \frac{\vec{k}_i^2 + \mu_\pi^2}{\vec{k}_i^2 + \Lambda_\pi^2} + \frac{\vec{k}_j^2 + \mu_\pi^2}{\vec{k}_j^2 + \Lambda_\pi^2} \right) (\vec{\tau}_i \times \vec{\tau}_j)_z, \end{aligned} \quad (12)$$

$$\begin{aligned} \vec{J}_{\rho q\bar{q}}(\vec{k}_i, \vec{k}_j) = ie \frac{(g_\rho^V + g_\rho^T)^2}{4m_i m_j} \left[\frac{\vec{\sigma}_i \times (\vec{\sigma}_j \times \vec{k}_j)}{(\vec{k}_j^2 + \mu_\rho^2)} \left(\frac{\Lambda_\rho^2 - \mu_\rho^2}{\vec{k}_j^2 + \Lambda_\rho^2} \right)^2 - (i \leftrightarrow j) \right] \\ \times (\vec{\tau}_i \times \vec{\tau}_j)_z, \end{aligned} \quad (13)$$

$$\begin{aligned} \vec{J}_{\gamma\rho\rho}(\vec{k}_i, \vec{k}_j) = ie \left[(g_\rho^V)^2 + \frac{(g_\rho^V + g_\rho^T)^2}{4m_i m_j} (\vec{\sigma}_i \times \vec{k}_i) \cdot (\vec{\sigma}_j \times \vec{k}_j) \right] \frac{(\vec{k}_i - \vec{k}_j)}{(\vec{k}_i^2 + \mu_\rho^2)(\vec{k}_j^2 + \mu_\rho^2)} \\ \times \frac{(\Lambda_\rho^2 - \mu_\rho^2)^2}{(\vec{k}_i^2 + \Lambda_\rho^2)(\vec{k}_j^2 + \Lambda_\rho^2)} \left(1 + \frac{\vec{k}_i^2 + \mu_\rho^2}{\vec{k}_i^2 + \Lambda_\rho^2} + \frac{\vec{k}_j^2 + \mu_\rho^2}{\vec{k}_j^2 + \Lambda_\rho^2} \right) (\vec{\tau}_i \times \vec{\tau}_j)_z \end{aligned} \quad (14)$$

where $\vec{q} = \vec{k}_i + \vec{k}_j$.

4 Results

The calculation of the elastic e.m. form factors involves the calculation of the matrix elements of the charge- and current-density operators presented in the previous section. The electric form factor consists of contributions from the one-body charge-density operator $J_{[1]}^0$ whereas the magnetic form factor consists of contributions from the one-body and two-body current-density operators. The full magnetic form factor can be written as the sum of the individual contributions

$$G_M(Q^2) = G_M^{[1]}(Q^2) + G_M^{\pi q \bar{q}}(Q^2) + G_M^{\gamma \pi \pi}(Q^2) + G_M^{\rho q \bar{q}} + G_M^{\gamma \rho \rho}(Q^2) \quad (15)$$

The calculations were performed without introducing any additional parameters and the results for the electric and magnetic form factors are plotted in Figs. 1 and 2. There is no modification in the calculated electric form factor since two-body effects do not appear in the charge-density operator. However, contrary to the case of a central pseudoscalar meson-exchange interaction where the contribution from two-body currents was found to be zero [18], we find that currents arising from the full pion- and rho-exchange interaction give a sizeable contribution to the magnetic form factors which gradually decreases with increasing Q^2 . Contributions from the pion pair-currents and pionic currents tend to cancel each other (the same applies to the rho-exchange currents) but the overall effect is the enhancement of the nucleon form factors. At $Q^2 = 0$ in particular, the contribution from two-body currents to the magnetic moment $\mu_N = G_M(Q^2 = 0)$ of the proton and neutron is significant, giving a much better agreement with the experimental values compared to the results with one-body currents only as shown in Table 1.

Table 1

Contributions to the magnetic moments of the proton and neutron from different currents. The experimental values are inserted in the last column.

N	$\mu_{[1]}$	$\mu_{\pi q \bar{q}}$	$\mu_{\gamma \pi \pi}$	$\mu_{\rho q \bar{q}}$	$\mu_{\gamma \rho \rho}$	μ^N
p	1.516	-0.126	0.735	-0.109	0.202	2.218
n	-0.993	0.126	-0.735	0.109	-0.202	-1.695

Despite the improvement at $Q^2 = 0$, the electric and magnetic form factors overestimate the experimental form factors at $Q^2 \neq 0$ and hence lead to an underestimation of the nucleon charge radii. This is a common feature of all CQM's and reflects the fact that constituent quarks are assumed to be pointlike particles. Any effects resulting from the collective excitations of sea quarks for example are unaccounted for. One way of incorporating these additional "sea quark" effects without spoiling the agreement with the observed baryon spectra is to consider that constituent quarks are effective degrees of freedom with some spatial extension.

Assuming that the up and down quarks are indistinguishable, a charge form factor $f(Q^2)$ of the Dirac type could be appended to the charge- and current-density operators (both one- and two-body ones). A rather good agreement with data can be obtained for $G_M^{p,n}$ at $Q^2 > 0.5 \text{ (GeV/c)}^2$ using a simple dipole form factor

$$f(Q^2) = \frac{1}{[1 + aQ^2]^2} \quad (16)$$

common to all quarks. Once constituent quarks are treated as extended objects, it is not unreasonable to introduce an anomalous magnetic moment κ . Thus, besides a dipole form for $f(Q^2)$, the following form for $g(Q^2)$ has been considered:

$$g(Q^2) = f(Q^2) + \kappa \frac{1}{[1 + bQ^2]^3} \quad (17)$$

for the magnetic current-density operator. The actual value of κ has been fixed in order to obtain the experimental value of the proton magnetic moment. For a quark mass $m = 340$ one obtains $\kappa = 0.379$. Correspondingly, the neutron magnetic moment turns out to be -2.073 n.m. in good agreement with experiment. The other two parameters a and b in Eqs. (15) and (16) are then fixed by fitting the Q^2 dependence of G_M^p . The resulting value for the quark charge radius is: $r_c = 0.7 \text{ fm}$. It is worth noting that the extracted value of the quark charge radius is fairly close to the value predicted by the Vector Meson Dominance model. Without any additional free parameter one can then calculate the other nucleon form factors. The results are shown in Figs. 1 and 2 by the solid lines. A satisfactory agreement is obtained for both G_E^p and G_M^p .

5 Conclusions

A completely consistent calculation of the nucleon elastic electromagnetic form factors has been performed within the extended chiral constituent model [20]. We find that the two-body currents derived from the complete pseudoscalar and pseudovector meson-exchange potentials by means of the continuity equation give rise to significant contributions to the proton and neutron magnetic moments, improving the agreement with the experimental values.

However, both the electric and magnetic form factors predicted by the model overestimate the observed nucleon form factors for $Q^2 > 0$ reflecting thus the inadequacy of the assumption of pointlike constituent quarks. A satisfactory agreement is obtained when treating the constituent quarks as extended particles with anomalous magnetic moment using suitable Dirac- and Pauli-type form factors. The resulting quark charge radius is consistent with the prediction of the Vector Meson Dominance model.

Acknowledgements

References

- [1] A. De Rújula, H. Georgi and S. L. Glashow, *Phys. Rev. D* 12 (1975) 147.
- [2] N. Isgur and G. Karl, *Phys. Rev. D* 18 (1978) 4187.
- [3] M.M. Giannini, *Rep. Prog. Phys.* 54 (1990) 453.
- [4] G. Karl, *J. Mod. Phys.* 1 (1992) 491.
- [5] M. Warns, H. Schröder, W. Pfeil and H. Rollnik, *Zeit. Phys. C* 45 (1990) 613.
- [6] S. Capstick, *Phys. Rev. D* 46 (1992) 2864.
- [7] F. Cardarelli, E. Pace, G. Salmè and S. Simula, *Phys. Lett.* 357 (1995) 267.
- [8] A. Szczepaniak, C.-R. Ji and S.R. Cotanch, *Phys. Rev. C* 52 (1995) 2738.
- [9] I.T. Obukhovskiy and A.M. Kusainov, *Phys. Lett. B* 238 (1990) 142.
- [10] A. Buchmann, E. Hernandez and A. Faessler, *Phys. Rev. C* 55 (1997) 448.
- [11] A. Valcarce, P. Gonzáles, F. Fernández and V. Vento, *Phys. Lett. B* 367 (1996) 35.
- [12] Z. Dziembowski, M. Fabre de la Ripelle and G.A. Miller, *Phys. Rev. C* 53 (1996) R2038.
- [13] P.N. Shen, Y.B. Dong, Z.Y. Zhang, Y.W. Yu and T.-S.H. Lee, *Phys. Rev. C* 55 (1997) 2024.
- [14] A. Manohar and H. Georgi, *Nucl. Phys. B* 234 (1984) 189.
- [15] L. Ya. Glozman and D. O. Riska, *Phys. Rep.* 268 (1996) 1.
- [16] L. Ya. Glozman, Z. Papp, W. Plessas, K. Varga and R. F. Wagenbrunn, *Phys. Rev. C* 57 (1998) 3406.
- [17] L. Ya. Glozman, W. Plessas, K. Varga and R. F. Wagenbrunn, *Phys. Rev. D* 58 (1998) 094030.
- [18] S. Boffi, P. Demetriou, M. Radici and R. F. Wagenbrunn, *Phys. Rev. C* 60 (1999) 025206.
- [19] S. Boffi, F. Capuzzi, P. Demetriou and M. Radici, *Nucl. Phys. A* 637 (1998) 585.
- [20] L. Ya. Glozman, W. Plessas, K. Varga and R. F. Wagenbrunn, in *Proc. of Conf. on Perspectives in Hadronic Physics, I.C.T.P. Trieste, 10-14 May 1999*, (ed. World Scientific).

- [21] K. Varga and Y. Suzuki, Phys. Rev. C 52 (1995) 2885.
- [22] R.C. Walker et al., Phys. Lett. B 224 (1989) 353.
- [23] W. Bartel et al., Nucl. Phys. B 58 (1973) 429.
- [24] P. Bosted et al., Phys. Rev. Lett. 68 (1992) 3841.
- [25] K.M. Hanson et al., Phys. Rev. D 8 (1973) 753.
- [26] C.W. Akerlof et al., Phys. Rev. 135 (1964) B810.
- [27] A. Lung et al., Phys. Rev. Lett. 70 (1993) 718.

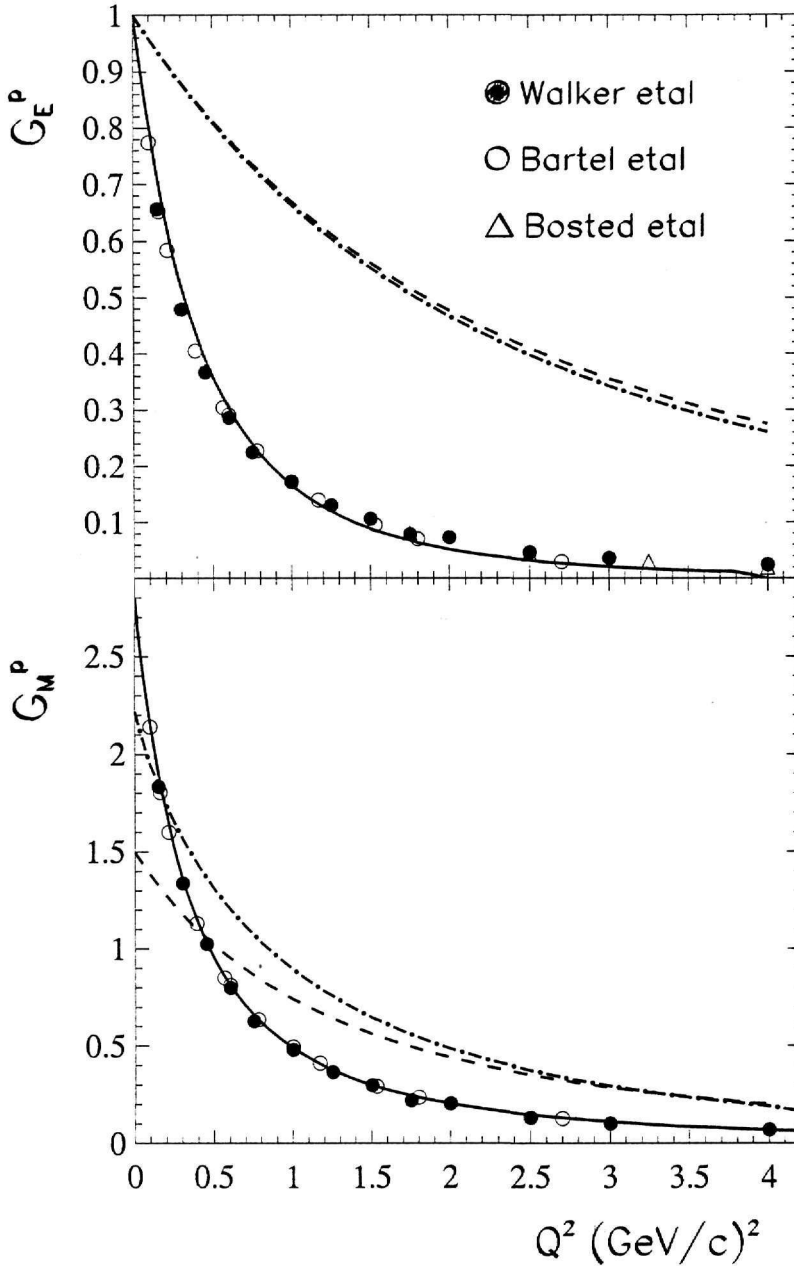


Fig. 1. The electric (G_E^p) and magnetic (G_M^p) form factors of the proton as a function of the four-momentum squared Q^2 . The dashed, dot-dashed and solid lines refer to the results of the GBE CQM, extended GBE CQM and extended GBE CQM with quark form factors, respectively. Experimental points are from [22] (solid circles), [23] (open circles) and [24] (triangles).

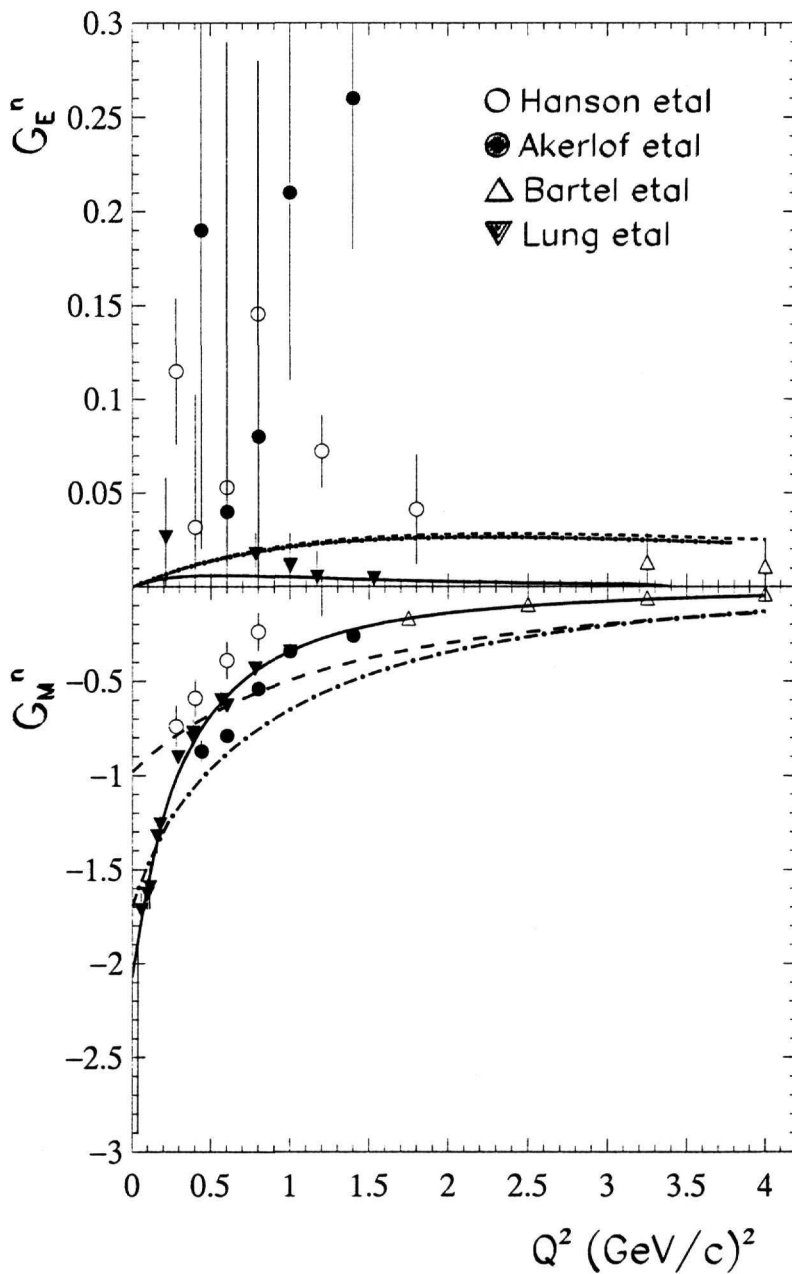


Fig. 2. The same as in Fig.1 but for the neutron. Experimental points are from [25] (open circles), [26] (solid circles), [23] (open triangles) and [27] (solid triangles).