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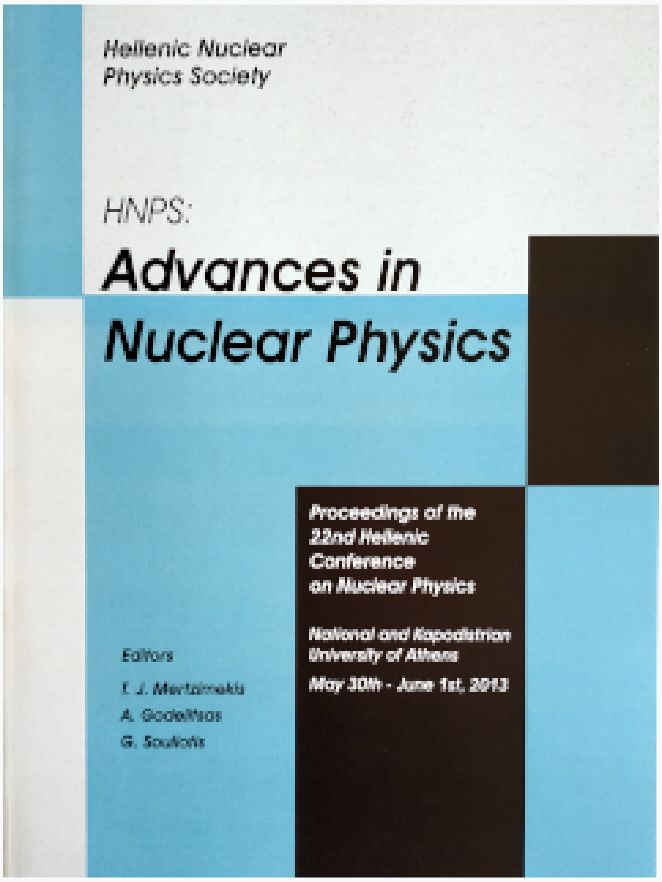

Critical point for the Deformation Dependent Mass model through a variational procedure

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Critical point for the Deformation Dependent Mass model through a variational procedure

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Abstract

The recently introduced Deformation-Dependent Mass model is combined with a variational approach to the Bohr Hamiltonian in order to describe transitional nuclei. The results of this procedure are demonstrated for the 'spherical to γ-unstable' and the 'spherical to deformed' transitional classes, which correspond to the E(5) and X(5) solutions.

Keywords: Deformation dependent mass, variational method

1. Introduction

Critical point symmetries, as the E(5) and X(5) [1, 2] solutions of the Bohr Hamiltonian became known, have for more than a decade described the properties of transitional nuclei and were followed by a variety of similar models giving successful, parameter-free predictions of energy ratios and B(E2) ratios. In one of these models [3], a Davidson potential with a variational parameter is used, instead of the infinite square well in β employed in both E(5) and X(5), and the critical point is determined by the procedure described in the next section. The results obtained resemble very closely those of E(5) and X(5).

In a recent publication [4], a variant of the Bohr Hamiltonian was proposed, where the mass term is allowed to depend on the β variable of nuclear deformation. Analytic solutions of this modified Hamiltonian were obtained using a Davidson potential in β and by employing techniques from supersymmetric quantum mechanics [5]. In addition to the new set of analytic solutions, the newly introduced Deformation-Dependent Mass (DDM) model offered a remedy to the problematic behaviour of the moment of inertia in the Bohr Hamiltonian, where it appears to increase proportionally to β^2. In the DDM model the moments of inertia increase at a much lower rate, in agreement with experimental data. Recently, a solution of the DDM model with a Kratzer potential has been obtained [6].

In what follows, the two approaches (the variational and the DDM with Davidson) are combined and the results for the ground band energy spectra are presented.

2. General description of the method

The method originally proposed in [3, 7] uses a Davidson potential ($u(\beta) = \beta^2 + \beta_0^4/\beta^2$) in the Bohr Hamiltonian and analytic expressions for the energies are obtained. The resulting expressions depend on the angular momentum L and the parameter β_0 of the potential, which is the position of the potential minimum and upon variation of β_0 (from 0 to sufficiently large values) the energy spectra change from those of the spherical type to the γ-unstable type and (with the addition of a harmonic oscillator potential term in γ) also from the spherical to the prolate-deformed.

Then, the critical point can be identified as the value of β_0 that maximizes the rate of change of each energy ratio ($R_L = E(L)/E(2)$), in accordance with the observation that in a phase transition certain characteristic quantities change most abruptly. Therefore, one looks for the value of β_0 for which the first derivative of R_L (for each separate value of L) with respect to β_0 ($dR_L/d\beta_0$), becomes maximum and subsequently uses these values to calculate the energy ratios. As shown below, the same method can be extended in the DDM framework for the two cases of shape transitions, with the addition of an extra parameter.

2.1. The spherical to γ-unstable transition

The energy spectrum of the ground band, in the DDM model with a Davidson potential as calculated in [4] is given by the expression:

$$\begin{aligned} E_0(\alpha, \beta_0, L) &= \alpha\frac{29}{4} + \frac{\sqrt{8+49\alpha^2+L(L+6)\alpha^2}}{2} + \alpha\frac{\sqrt{9+L(L+6)+8\beta_0^4}}{2} \\ &+ \frac{\sqrt{(8+49\alpha^2+L(L+6)\alpha^2)(9+L(L+6)+8\beta_0^4)}}{4} \end{aligned} \tag{1}$$

As can be seen, the energies, apart from the angular momentum L and β_0, depend also on α, which is a parameter that enters the formula of the mass depending on deformation [4], and for $\alpha = 0$ the results of the Bohr Hamiltonian with a Davidson potential are obtained [3]. Consequently, the energy is represented graphically by a surface, instead of a curve and the variational method described above is implemented by finding the pair of $(\alpha, \beta_0)_{crit}$ values that maximize the partial derivative of R_L with respect to β_0 (see fig. (1)). It should be noted that a proper rescaling of the potential, like the one followed in [6] may be necessary in order to lower the obtained β_0's to more physical values. Also, very recently, an interpretation of the role of the α parameter has been proposed in [8].

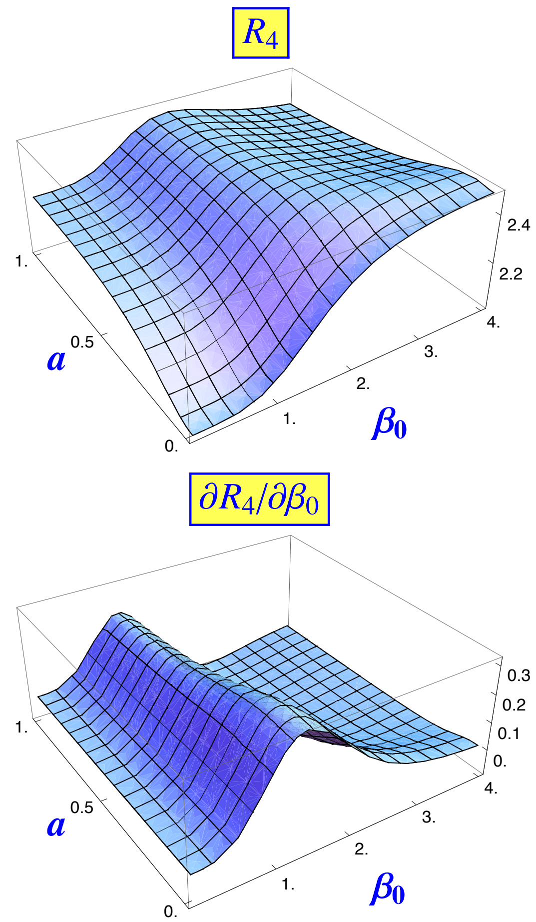

Figure 1: R_4 energy ratio surface (top) and its derivative with respect to β_0 (bottom) as functions of α and β_0 for the γ-unstable case.

Fig. 2 shows how the critical point 'migrates' for increasing L values, showing a stabilization in α for $L \geq 10$.

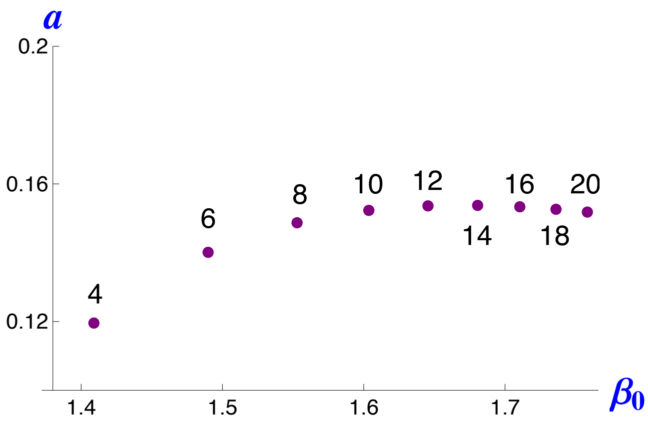

Figure 2: Critical values for α and β_0 for the various L values in the 'spherical to γ-unstable' transition.

The R_L ratios that correspond to the critical (α, β_0) values are shown in fig. 3. Although they follow the general trend of the E(5) results the variational R_L ratios tend more towards the O(6) limit.

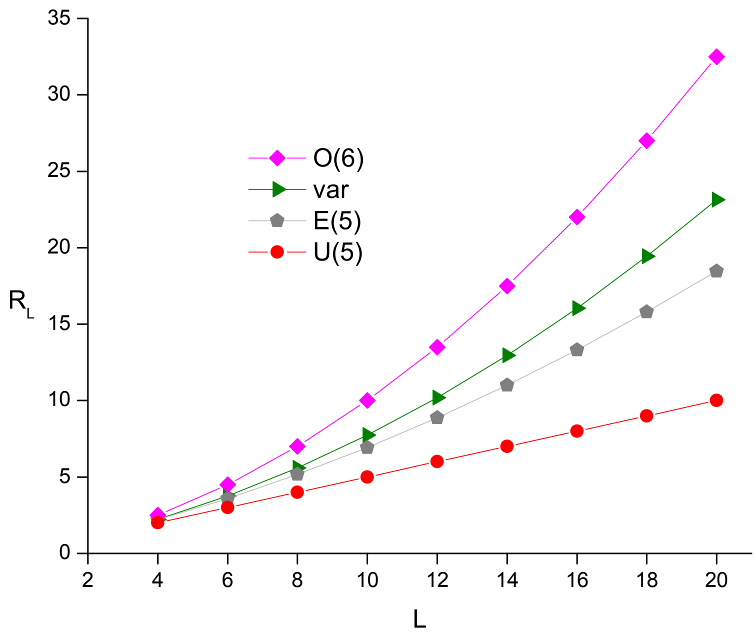

Figure 3: Critical R_L energy ratios (var) as functions of L for the ground state band and the U(5)-O(6) transition, as they compare to the E(5) results [1].

2.2. The spherical to deformed transition

The energies of the ground state band for the prolate deformed ($\gamma = 0°$) case in the DDM with Davidson are given by the expression:

$$\begin{aligned} E_0(\alpha, \beta_0, L) &= \alpha\frac{29}{4} + \frac{\sqrt{8+49\alpha^2+4L(L+1)\alpha^2/3}}{2} + \alpha\frac{\sqrt{9+4L(L+1)/3+8\beta_0^4}}{2} \\ &+ \frac{\sqrt{(8+49\alpha^2+4L(L+1)\alpha^2/3)(9+4L(L+1)/3+8\beta_0^4)}}{4} \end{aligned} \quad (2)$$

The R_4 ratio and its partial derivative with respect to β_0 are shown in fig. 4. As fig. 5 shows, the critical points for various L follow a similar path to that of the γ-unstable case with α showing even a slight decrease with increasing L, for $L \geq 10$.

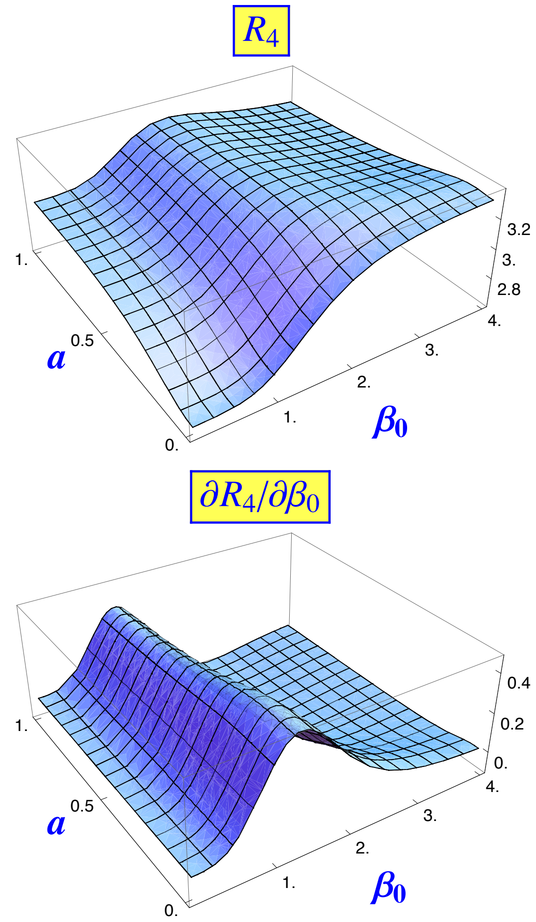

Figure 4: R_4 energy ratio surface (top) and its derivative with respect to β_0 (bottom) as functions of α and β_0 for the deformed case.

As in the γ-unstable case, the critical R_L energy ratios (fig. 6) follow the X(5) results, slightly shifted towards the SU(3) limit.

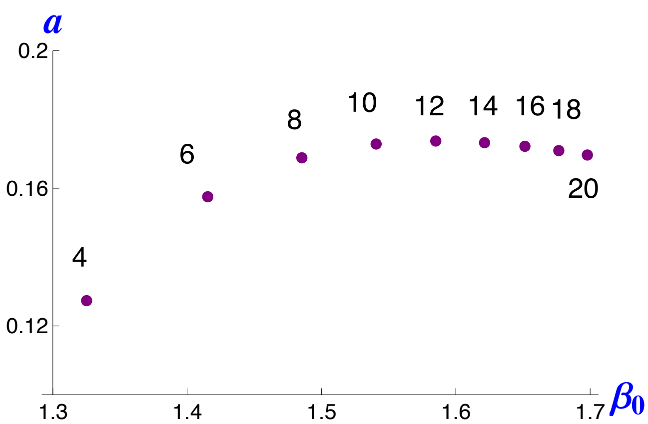

Figure 5: Critical values for α and β_0 for the various L values in the ‘spherical to deformed’ transition.

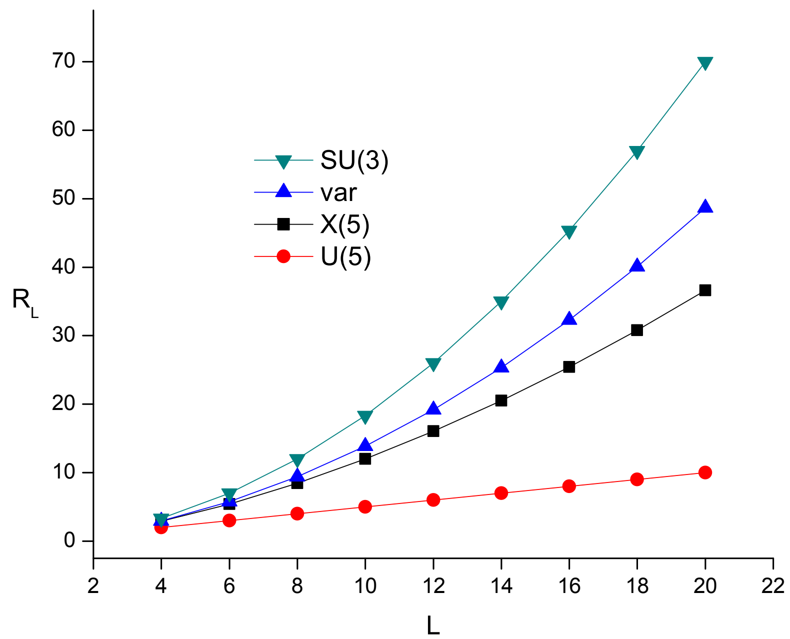

Figure 6: Critical R_L energy ratios (var) as functions of L for the ground state band and the U(5)-SU(3) transition, as they compare to the X(5) results [2].

3. Conclusion

A method, previously used to obtain the critical point with the Bohr Hamiltonian, was employed in the framework of the recently introduced Deformation-Dependent Mass (DDM) model. The results for the ground state band follow closely those of the E(5) and X(5) solutions, leaning though more towards the deformed limits.

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