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# Nuclear equation of state constraints from tidal deformability of neutron stars

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## Abstract

We study the effect of nuclear equation of state on the tidal polarizability of neutron stars. The predicted equations of state for the  $\beta$ -stable nuclear matter are parameterized by varying the slope  $L$  of the symmetry energy at saturation density on the interval  $65 \text{ MeV} \leq L \leq 115 \text{ MeV}$ . The effects of the density dependence of the nuclear symmetry energy on the neutron star tidal polarizability are presented and analyzed. A comparison of theoretical predictions with the recent observation predictions is also performed and analyzed.

*Keywords:* neutron stars, symmetry energy, tidal polarizability

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## 1. Introduction

Gravitational waves from the final stages of inspiraling binary neutron stars are expected to be one of the most important sources for ground-based gravitational wave detectors [1–6]. The masses of the component will be determined to moderate accuracy, especially if the neutron stars are slowly spinning, during the early part of the evolution. Flanagan and Hinderer [1] have recently pointed out that tidal effects are also potentially measurable during the early part of the evolution when the waveform is relatively clean. The tidal fields induce quadrupole moments on the neutron stars. The response of the neutron star is described by the dimensionless Love number  $k_2$ , which depends on the neutron star structure and consequently on the mass and the equation of state of the nuclear matter.

The motivation of the present work, in view of the above studies, is to study extensively the nuclear equation of state (EOS) effect on the dimensionless Love number  $k_2$ . We employ a phenomenological model for the energy per baryon of the asymmetric nuclear matter having the advantage of an analytical form. By suitably choosing the parametrization of the model we obtain different forms from the density dependence of the nuclear symmetry energy by varying the slope parameter  $L$  on the interval  $65 \text{ MeV} \leq L \leq 115 \text{ MeV}$ . The effects of the density dependence of the nuclear symmetry energy on the neutron star tidal polarizability are presented and analyzed while comparison of theoretical predictions with the recent observation predictions is also performed and analyzed.

## 2. Tidal polarizability and tidal Love numbers

The tidal Love numbers  $k_2$  is given by the ration of the induced quadrupole moment  $Q_{ij}$  and the applied tidal field  $E_{ij}$

$$Q_{ij} = -k_2 \frac{2R^5}{3G} E_{ij} \equiv \lambda E_{ij}, \quad (1)$$

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where  $R$  is the neutron star radius. The tidal Love number  $k_2$  is given by [2]

$$\begin{aligned}
k_2 &= \frac{1}{20} \left( \frac{R_s}{R} \right)^5 \left( 1 - \frac{R_s}{R} \right)^2 \left[ 2 - y_R + (y_R - 1) \frac{R_s}{R} \right] \\
&\times \left[ \frac{R_s}{R} \left( 6 - 3y_R + \frac{3R_s}{2R} (5y_R - 8) + \frac{1}{4} \left( \frac{R_s}{R} \right)^2 \right. \right. \\
&\times \left. \left. \left[ 26 - 22y_R + \left( \frac{R_s}{R} \right) (3y_R - 2) + \left( \frac{R_s}{R} \right)^2 (1 + y_R) \right] \right) \right. \\
&+ \left. 3 \left( 1 - \frac{R_s}{R} \right)^2 \left[ 2 - y_R + (y_R - 1) \frac{R_s}{R} \right] \ln \left( 1 - \frac{R_s}{R} \right) \right] - 1.
\end{aligned}$$

One first has to solve the following differential equation

$$r \frac{dy(r)}{dr} + y^2(r) + y(r)F(r) + r^2 Q(r) = 0, \quad y(0) = 2, \quad y_R \equiv y(R) \quad (2)$$

where

$$F(r) = \left[ 1 - \frac{4\pi r^2 G}{c^4} (\mathcal{E}(r) - P(r)) \right] \left( 1 - \frac{2M(r)G}{rc^2} \right)^{-1}, \quad (3)$$

and

$$\begin{aligned}
r^2 Q(r) &= \frac{4\pi r^2 G}{c^4} \left[ 5\mathcal{E}(r) + 9P(r) + \frac{\mathcal{E}(r) + P(r)}{\partial P(r)/\partial \mathcal{E}(r)} \right] \left( 1 - \frac{2M(r)G}{rc^2} \right)^{-1} \\
&- 6 \left( 1 - \frac{2M(r)G}{rc^2} \right)^{-1} \\
&- \frac{4M^2(r)G^2}{r^2 c^4} \left( 1 + \frac{4\pi r^3 P(r)}{M(r)c^2} \right)^2 \left( 1 - \frac{2M(r)G}{rc^2} \right)^{-2}.
\end{aligned} \quad (4)$$

The Schwarzschild radius  $R_S$  is given by

$$R_s = \frac{2GM}{c^2} = \frac{2GM_\odot}{c^2} \bar{M} = 2.948 \bar{M} (\text{Km}). \quad (5)$$

The equations (2), (3) and (5) must be integrated with the TOV equations

$$\frac{dP(r)}{dr} = - \frac{GM(r)\mathcal{E}(r)}{r^2 c^2} \left( 1 + \frac{P(r)}{\mathcal{E}(r)} \right) \left( 1 + \frac{4\pi r^3 P(r)}{M(r)c^2} \right) \left( 1 - \frac{2M(r)G}{rc^2} \right)^{-1}, \quad (6)$$

$$\frac{dM(r)}{dr} = \frac{4\pi^2 \mathcal{E}(r)}{c^2}, \quad (7)$$

with the boundary conditions  $y(0) = 2$ ,  $P(0) = P_c$  and  $M(0) = 0$ . Since the neutron stars in binaries have a broad mass distribution it is necessary to investigate the mass dependence of the tidal polarizability. Whereas what can be measured for a neutron star binary of mass  $M_1$  and  $M_2$  is the mass-weighted tidal polarizability [4, 6].

$$\tilde{\lambda} = \frac{1}{26} \left[ \frac{M_1 + 12M_2}{M_1} \lambda_1 + \frac{M_2 + 12M_1}{M_2} \lambda_2 \right].$$

In the present work we consider binaries consisting of two neutron stars with equal masses.

### 3. The nuclear model

The model used here, which has already been presented and analyzed in a previous paper [7], is designed to reproduce the results of the microscopic calculations of both nuclear and neutron-rich matter at zero temperature and can be extended to finite temperature. The energy per baryon at  $T = 0$ , is given by

$$\begin{aligned}
E_b(n, I) = & \frac{3}{10} E_F^0 u^{2/3} \left[ (1+I)^{5/3} + (1-I)^{5/3} \right] + \frac{1}{3} A \left[ \frac{3}{2} - \left( \frac{1}{2} + x_0 \right) I^2 \right] u \\
& + \frac{\frac{2}{3} B \left[ \frac{3}{2} - \left( \frac{1}{2} + x_3 \right) I^2 \right] u^\sigma}{1 + \frac{2}{3} B' \left[ \frac{3}{2} - \left( \frac{1}{2} + x_3 \right) I^2 \right] u^{\sigma-1}} \\
& + \frac{3}{2} \sum_{i=1,2} \left[ C_i + \frac{C_i - 8Z_i}{5} I \right] \left( \frac{\Lambda_i}{k_F^0} \right)^3 \left( \frac{((1+I)u)^{1/3}}{\frac{\Lambda_i}{k_F^0}} - \tan^{-1} \frac{((1+I)u)^{1/3}}{\frac{\Lambda_i}{k_F^0}} \right) \\
& + \frac{3}{2} \sum_{i=1,2} \left[ C_i - \frac{C_i - 8Z_i}{5} I \right] \left( \frac{\Lambda_i}{k_F^0} \right)^3 \left( \frac{((1-I)u)^{1/3}}{\frac{\Lambda_i}{k_F^0}} - \tan^{-1} \frac{((1-I)u)^{1/3}}{\frac{\Lambda_i}{k_F^0}} \right).
\end{aligned} \tag{8}$$

In Eq. (8),  $I$  is the asymmetry parameter ( $I = (n_n - n_p)/n$ ) and  $u = n/n_0$ , with  $n_0$  denoting the equilibrium symmetric nuclear matter density,  $n_0 = 0.16 \text{ fm}^{-3}$ . The parameters  $A$ ,  $B$ ,  $\sigma$ ,  $C_1$ ,  $C_2$  and  $B'$  which appear in the description of symmetric nuclear matter are determined in order that  $E_b(n = n_0, I = 0) = -16 \text{ MeV}$ ,  $n_0 = 0.16 \text{ fm}^{-3}$ , and the incompressibility is  $K = 240 \text{ MeV}$ . The baryon energy is written also as a function of the baryon density  $n$  and the proton fraction  $x$  ( $x = n_p/n$ ), that is  $E_b(n, x)$ , by replacing  $I = 1 - 2x$ . The additional parameters  $x_0$ ,  $x_3$ ,  $Z_1$ , and  $Z_2$  employed to determine the properties of asymmetric nuclear matter are treated as parameters constrained by empirical knowledge. The parameterizations used in the present model have only a modest microscopic foundation. Nonetheless, they have the merit of being able to closely approximate more physically motivated calculations.

#### 3.1. Symmetry energy

The energy  $E_b(n, I)$  can be expanded around  $I = 0$  as follows

$$E_b(n, I) = E_b(n, I = 0) + E_{sym,2}(n)I^2 + E_{sym,4}(n)I^4 + \dots + E_{sym,2k}(n)I^{2k} + \dots, \tag{9}$$

where the coefficients of the expansion are given by the expression

$$E_{sym,2k}(n) = \frac{1}{(2k)!} \left. \frac{\partial^{2k} E_b(n, I)}{\partial I^{2k}} \right|_{I=0}. \tag{10}$$

In (9), only even powers of  $I$  appear due to the fact that the strong interaction must be symmetric under exchange of neutrons with protons. The nuclear symmetry energy  $E_{sym}(n)$  is defined as the coefficient of the quadratic term, that is

$$E_{sym}(n) = E_{sym,2}(n) = \frac{1}{2!} \left. \frac{\partial^2 E_b(n, I)}{\partial I^2} \right|_{I=0}. \tag{11}$$

The slope of the symmetry energy  $L$  at nuclear saturation density  $n_0$ , which is correlated with the crust-core transition density  $n_t$  in a neutron star, is defined as

$$L = 3n_0 \left. \frac{\partial E_{sym}(n)}{\partial n} \right|_{n=n_0}. \tag{12}$$

By suitably choosing the parameters  $x_0$ ,  $x_3$ ,  $Z_1$ , and  $Z_2$ , it is possible to obtain different forms for the density dependence of the symmetry energy  $E_{sym}(n)$  as well as on the value of the slope parameter  $L$ . We take as a range of  $L$   $65 \text{ MeV} \leq L \leq 115 \text{ MeV}$  where the value of the symmetry energy at saturation density is fixed to be  $E_{sym}(n_0) = 30 \text{ MeV}$ . Actually, for each value of  $L$  the density dependence of the symmetry energy is adjusted so that the energy of pure neutron matter to be comparable with those of existing state-of-the-art calculations.

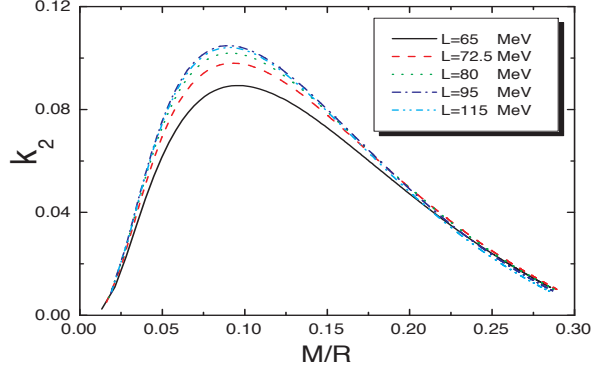


Figure 1: The dimensionless tidal Love number  $k_2$  as a function of the compactness  $M/R$  for the selected EOSs.

### 3.2. Proton fraction

The proton fraction  $x$  (as a function of the baryon density  $n$ ) in  $\beta$ -stable matter. In this case we have the processes

$$n \longrightarrow p + e^- + \bar{\nu}_e \quad p + e^- \longrightarrow n + \nu_e, \quad (13)$$

which take place simultaneously. We assume that neutrinos generated in these reactions have left the system. This implies that

$$\hat{\mu} = \mu_n - \mu_p = \mu_e. \quad (14)$$

The demand for equilibrium leads to equation

$$\frac{\partial}{\partial x} \left( E_b(n, x) + E_e(n, x) \right) = 0, \quad (15)$$

or

$$\left( \frac{\partial E_b}{\partial x} \right)_n = - \left( \frac{\partial E_e}{\partial x} \right)_n = -\mu_e. \quad (16)$$

Finally, considering that the chemical potential of the electron is given by the relation (relativistic electrons)

$$\mu_e = \sqrt{k_{F_e}^2 c^2 + m_e^2 c^4} \simeq k_{F_e} c = \hbar c (3\pi^2 n x)^{1/3}, \quad (17)$$

then Eq. (16) is written

$$\left( \frac{\partial E_b}{\partial x} \right)_n = -\hbar c (3\pi^2 n x)^{1/3}. \quad (18)$$

Eq. (18) determines the proton fraction of  $\beta$ -stable matter.

### 3.3. Nuclear equation of state for $\beta$ -stable matter

The total pressure  $P(n, x)$ , in the core of a neutron star, is decomposed into baryon and electron contributions

$$P(n, x) = P_b(n, x) + P_e(n, x), \quad (19)$$

where

$$P_b(n, x) = n^2 \frac{\partial E_b(n, x)}{\partial n}. \quad (20)$$

The electrons are considered as a non-interacting relativistic Fermi gas and their contribution to the total energy density  $\epsilon_e(n, x)$  and pressure  $P_e(n, x)$  reads

$$\epsilon_e(n, x) = \frac{\hbar c}{4\pi^2} (3\pi^2 x n)^{4/3}, \quad (21)$$

$$P_e(n, x) = \frac{\hbar c}{12\pi^2} (3\pi^2 x n)^{4/3}. \quad (22)$$

Now the total energy density  $\epsilon_{tot}$  and pressure  $P_{tot}$  of charge neutral and chemically equilibrium nuclear matter is

$$\epsilon_{tot} = \epsilon_b + \epsilon_e, \quad (23)$$

$$P_{tot} = P_b + P_e. \quad (24)$$

From Eqs. (23) and (24) we construct the equation of state in the form  $\epsilon = \epsilon(P)$ .

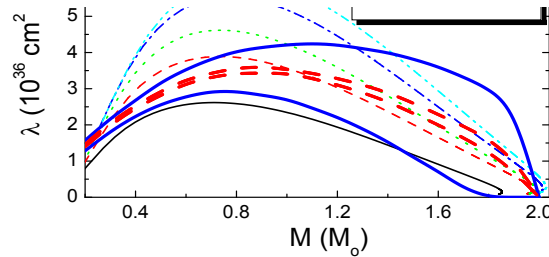


Figure 2: Tidal polarizability  $\lambda$  as a function of the the neutron star mass for the selected EOSs. An estimate of uncertainties in measuring  $\lambda$  for equal mass binaries at a distance of  $D = 100$  Mpc is shown for the Advanced LIGO detector and the Einstein Telescope.

#### 4. Results and Discussion

In Fig. 1 we plot the tidal Love number  $k_2$  as a function of the compactness  $M/R$  for the selected equations of state. The maximum of  $k_2$  occurs near to  $0.1 M/R$ . Actually the EOS effects are more pronounced on the interval  $0.05 - 0.15 M/R$ .

The tidal polarizability  $\lambda$  depends strongly on the equation of state as displayed in Fig. 2. In comparison an estimate of uncertainties in measuring  $\lambda$  for equal mass binaries at a distance of  $D = 100$  Mpc is shown for the Advanced LIGO detector and the Einstein Telescope. According to Fig. 2, a crude estimate of uncertainties in measuring  $\lambda$  excludes EOS with high values of  $L$  (especially for low values of neutron stars). Moreover, the narrow uncertain range for the proposed Einstein Telescope will enable it to tightly constrain the EOS compared to Advanced LIGO detector.

In any case additional work with a combination of theoretical, terrestrial experiments and observation data are necessary to constrain further the nuclear matter equation of state.

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