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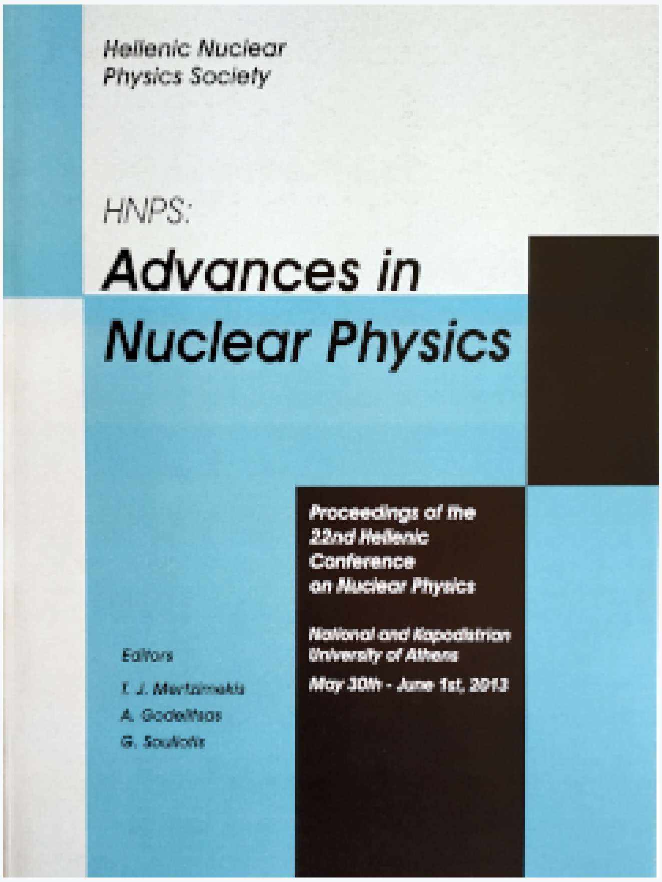

Electron-capture modes with realistic nuclear structure calculations

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Electron-capture modes with realistic nuclear structure calculations

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Abstract

Nuclear electron capture posses prominent position among other weak interaction processes occuring in explosive nucleosynthesis. In particular, this process plays important role in the core-colapse of massive stars by modifying the electron to baryon ratio Y_e. From a nuclear theory point of view, such processes may be studied by using the same nuclear methods (e.g. the quasi-particle random phase approximation, QRPA), employed in the present work with these used for the one-body charge changing nuclear reactions (β-decay modes, charged-current electron-neutrino absorption by nuclei, etc). In this work we calculate e^--capture cross sections on ^{56}Fe using two different approaches. At first, original cross section calculations are perfored by using the pn-QRPA method considering all the accessible transitions of the final nucleus ^{56}Mn. Secondly, we evaluate the Gamow-Teller strength distributions and obtain the cross sections at the limit of zero-momentum transfer. The agreement between the two methods is very good.

Keywords: semi-leptonic charge-curent reactions, electron capture, nuclear matrix elements, Quasi-Particle Random Phase Approximation, neutrino nucleosynthesis

1. Introduction

Weak interaction processes occuring in the presence of nuclei play crucial role in the late stages of the evolution of massive stars and in the presupernova stellar collapse. As it is known, the core of a massive star, at the end of hydrostatic burning is stabilized by electron degenerecy pressure as long as its mass does not exceed the appropriate Chandrasekhar mass M_{Ch} [1, 2] but if the core mass exceeds M_{Ch} electrons are captured by nuclei.

Electron capture on nuclei takes place in the very dense environment of the stellar core where the Fermi energy (or equivalently the chemical potential) of the degenerate electron gas is sufficiently large to overcome the threshold energy E_{thr} (E_{thr} is given by negative Q values of the reactions involved in the interior of the stars) [3]. This high Fermi energy of the degenerate electron gas leads to enormous e^--capture on nuclei and reduces the electron to baryon ratio Y_e [4, 5]. As a consequence, the nuclear composition is shifted to more neutron-rich and heavier nuclei (including those with $N > 40$) which dominate the matter composition for matter densities larger than about $10^{10} g\, cm^{-3}$ [6, 7].

In the early stage of collapse, for densities lower than a few $10^{10} g\, cm^{-3}$, the electron chemical potential is of the same order of magnitude as the nuclear Q value, and the e^- capture cross-sections are sensitive to the details of GT strength distributions in daughter nuclei. At these densities, electrons are captured on nuclei with mass number $A \leq 60$. For higher densities and tempratures, e^- capture occurs on heavier nuclei $A \geq 65$ [8].

In the present work e^--capture rates are discussed within a refined version of the Quasi-Particle Random Phase Approximation (QRPA) which is applied to construct all the accessible final (excited) states [9–11]. For the description of a correlated ground state we determine the single-particle occupation numbers, calculated in BCS theory as shown below [9, 10]. We also evaluate the Gamow-Teller strenght distributions and obtain the cross sections at the limit of zero-momentum transfer.

2. Construction of nuclear ground and excited states

Electrons of energy E_e are captured by nuclei interacting weakly with them via $W^{\pm}$ boson exchange as

$$(A, Z) + e^- \rightarrow (A, Z-1)^* + \nu_{e^-} \tag{1}$$

The outgoing neutrino carries energy E_ν while the daughter nucleus absorbs a part of the incident electron energy given by the difference between the initial and the final nuclear energies as $E = E_f - E_i$.

The nuclear calculations for the cross sections of the reaction (1) start by writing down the weak interaction Hamiltonian $\hat{\mathcal{H}}_w$ which is written as a product of the leptonic j_μ^{lept} and the hadronic current $\hat{\mathcal{J}}^\mu$ (current-current interaction) as

$$\hat{\mathcal{H}}_w = \frac{G}{\sqrt{2}} j_\mu^{lept} \hat{\mathcal{J}}^\mu \tag{2}$$

where $G = G_F cos\theta_c$ with G_F and θ_c being the well known weak interaction coupling constant and the Cabbibo angle, respectively [12].

Then, the transition matrix elements entering the cross section from an initial nuclear state $|i\rangle$ to a final $|f\rangle$ take the form

$$\langle f|\widehat{H_w}|i\rangle = \frac{G}{\sqrt{2}}\, \ell^\mu \int d^3x\, e^{-i\mathbf{qx}} \langle f|\widehat{\mathcal{J}_\mu}|i\rangle. \tag{3}$$

For the calculation of these transition matrix elements one takes advantage of the Donnelly-Walecka multipole decomposition which leads to a set of eight independent irreducible tensor multipole operators containing polar-vector and axial-vector components [12].

In Eq. (3) the ground state of the nucleus $|i\rangle$ is computed in the context of the BCS theory, by solving the relevant BCS equations which gives us the quasi-particle energies and the amplitudes V and U that determine the probability for each single particle level to be occupied or unoccupied, respectively [13]. In the nuclear ground state, nucleons are considered as independent particles interacting with the strong nuclear field, a Coulomb corrected Woods-Saxon potential with a spin orbit part for our description. In addition, the pairing correlations (known as residual two-body interaction) described by the Bonn C-D potential are also taken into consideration. The renormalization of this interaction is achieved through the two pairing parameters $g_{pair}^{p,n}$ the values of which are tabulated in Table 1.

Table 1: Parameters for the renormalization of the interaction of proton pairs, g_{pair}^p, and neutron pairs, g_{pair}^n. They have been fixed in such a way that the corresponding experimental gaps, Δ_p^{exp} and Δ_n^{exp}, are quite accurately reproduced.

Nucleus	g_{pair}^n	g_{pair}^p	Δ_n^{exp} (MeV)	Δ_n^{theor} (MeV)	Δ_p^{exp} (MeV)	Δ_p^{theor} (MeV)
^{56}Fe	0.9866	0,9756	1.3626	1.3626	1.5682	1.5683

The pairing parameters $g_{pair}^{p,n}$, are fitted by the reproduction of the energy gaps, $\Delta_{p,n}^{exp}$, from neighboring nuclei (3-point formula) as

$$\Delta_n^{exp} = -\frac{1}{4}\Big[S_n[(A-1, Z)] - 2S_n[(A, Z)] + S_n[(A+1, Z)]\Big] \tag{4}$$

$$\Delta_p^{exp} = -\frac{1}{4}\Big[S_p[(A-1, Z-1)] - 2S_p[(A, Z)] + S_p[(A+1, Z+1)]\Big] \tag{5}$$

Table 2: The experimental separation energies (in MeV) for protons S_p and neutrons S_n of the targets (A,Z), ^{56}Fe, and neighboring nuclei $(A \pm 1, Z \pm 1)$ and $(A \pm 1, Z)$.

$S_n(A-1,Z)$	9.298	$S_p(A-1,Z-1)$	8.067
$S_n(A,Z)$	11.197	$S_p(A,Z)$	10.184
$S_n(A+1,Z)$	7.646	$S_n(A+1,Z+1)$	6.028

where S_p and S_n are the experimental separation energies for protons and neutrons, respectively, of the target nucleus (A,Z) and the neighboring nuclei $(A \pm 1, Z \pm 1)$ and $(A \pm 1, Z)$. In Table 2, the values of experimental separation energies for the target ^{56}Fe and the neighboring nuclei ^{56}Mn, ^{56}Co are shown.

Subsequently, the excited states $|f\rangle$ of the studied daughter nucleus ^{56}Mn are constructed by solving the pn-QRPA equations [10]. Their solution is an eigenvalue problem, which provides the amplitudes for forward and backward scattering X and Y, respectively, as well as the QRPA excitation energies $\Omega^{\nu}_{J^\pi}$ [11]. In our method the solution of the QRPA equations is obtained separately for each multipole set of states $|J^\pi\rangle$.

For the renormalization of the residual interaction (Bonn C-D), the parameters g_{pp} and g_{ph} entering the QRPA matrices $\mathcal{A}$ and $\mathcal{B}$, are determined from the reproducibility of the low-lying experimental energy spectrum. The values of these parameters are listed in Table 3.

Table 3: Strength parameters for the particle-particle (g_{pp}) and particle-hole (g_{ph}) interaction for various multipolarities.

Positive Parity	g_{ph}	g_{pp}	Negative Parity	g_{ph}	g_{pp}
0^+	0.400	1.000	0^-	0.800	0.800
1^+	0.994	0.200	1^-	0.800	0.800
2^+	1.200	0.804	2^-	0.737	0.395
3^+	0.200	1.300	3^-	0.200	0.200
4^+	0.201	1.161	4^-	0.800	0.800
5^+	1.153	1.200	5^-	1.000	1.000

At this point, it is worth mentioning that the calculated pn-QRPA energy spectrum of each individual multipolarity J^π needs to be shifted in such a way that the first calculated value of each multipole state (i.e. $1^+_1, 2^+_1$...etc), to approach as close as possible the corresponding lowest experimental energy. Such a shifting is necessary whenever in the pn-QRPA a BCS ground state is used, a treatment adopted by other groups previously [1, 16–18]. Table 4 shows the shifting applied to our QRPA spectrum for each multipolarity of the daughter nucleus ^{56}Mn. The resulting low-energy spectrum with the use of the above parameters and the shifting shown in Table 4, agrees well with the experimental one (see Figure 1).

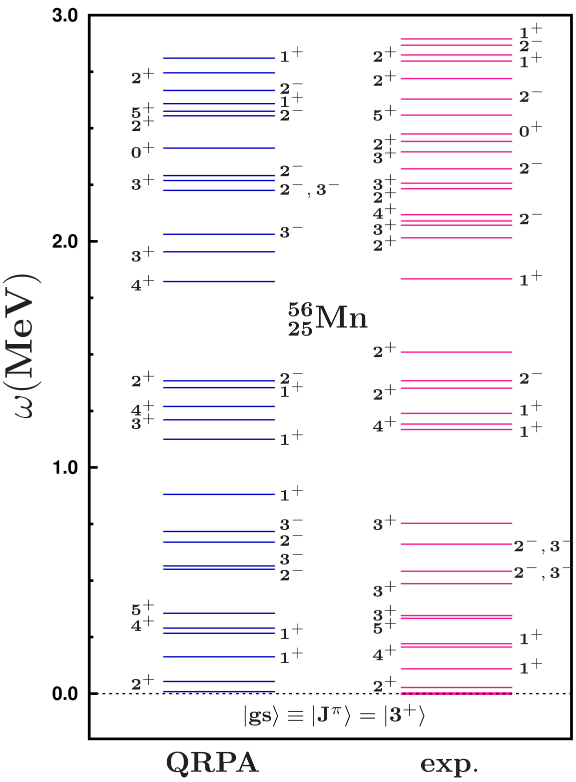
3. Calculations of e^--capture cross sections

In this work, we study the electron capture process in the ^{56}Fe isotope. The original cross section calculations are obtained by using the pn-QRPA method considering all the accessible transitions of the final nucleus ^{56}Mn.

Table 4: The shift (in MeV) of the spectrum applied seperately of each multipole set of states

Positive Parity States		Negative Parity States	
0^+	1.60	0^-	4.30
1^+	5.90	1^-	4.20
2^+	3.10	2^-	6.80
3^+	2.30	3^-	6.80
4^+	2.50	4^-	3.50
5^+	2.00	5^-	3.50

Figure 1: Comparison of the low-lying experimental spectrum of ^{56}Mn with the theoretical excitations resulting from the solution of the QRPA eigenvalue problem (up to about 3 MeV). The agreement is very good.



In the Donnelly-Walecka formalism the expression for the differential cross section in electron capture by nuclei in the interior of stars reads [8]

$$\begin{aligned}
\frac{d\sigma_{ec}}{d\Omega} &= \frac{G_F^2 cos^2\theta_c}{2\pi}\frac{F(Z,E_e)}{(2J_i+1)}\Big\{\sum_{J\geq 1}\mathcal{W}(E_e,E_\nu) \\
&\times \; \{[1-\alpha cos\Phi + bsin^2\Phi]\big[|\langle J_f\|\widehat{\mathcal{T}}_J^{mag}\|J_i\rangle|^2 + |\langle J_f\|\widehat{\mathcal{T}}_J^{el}\|J_i\rangle|^2\big] \\
&- \; \big[\frac{(\varepsilon_i+\varepsilon_f)}{q}(1-\alpha cos\Phi) - d\big]2Re\langle J_f\|\widehat{\mathcal{T}}_J^{mag}\|J_i\rangle\langle J_f\|\widehat{\mathcal{T}}_J^{el}\|J_i\rangle^*\} \\
&+ \; \sum_{J\geq 0}\mathcal{W}(E_e,E_\nu)\{(1+\alpha cos\Phi)|\langle J_f\|\widehat{\mathcal{M}}_J\|J_i\rangle|^2 \\
&+ \; (1+\alpha cos\Phi - 2bsin^2\Phi)|\langle J_f\|\widehat{\mathcal{L}}_J\|J_i\rangle|^2 \\
&- \; \big[\frac{\omega}{q}(1+\alpha cos\Phi)+d\big]2Re\langle J_f\|\widehat{\mathcal{L}}_J\|J_i\rangle\langle J_f\|\widehat{\mathcal{M}}_J\|J_i\rangle^*\}\Big\}
\end{aligned} \tag{6}$$

where $F(Z,E_e)$ is the well known Fermi function [14], $W(E_e,E_\nu)$ accounts the nuclear recoil [15], and the parameters α, b, d are given e.g. in Ref. [13].

For low momentum transfer, various authors use the approximation $q \to 0$. Then, the transitions of the Gamow-Teller operator ($GT_+ = \sum_i \tau_i^+\sigma_i$), provide the dominant contribution to the total cross section [1]. Under such assumptions the exclusive cross sections from a state $|i\rangle$ to a state $|f\rangle$ of e^--capture are given by

$$\sigma_{fi}(E_e) = \frac{6(E_e-E)^2 G_F^2 cos^2\theta_c}{\pi(2J_i+1)}|\langle J_f\|\widehat{\mathcal{L}}_1\|J_i\rangle|^2 \tag{7}$$

(G_F is the weak coupling constant and θ_c is the Cabbibo angle) where

$$\widehat{\mathcal{L}}_{1M} = \frac{i}{\sqrt{12\pi}}G_A\sum_{i=1}^{A}\tau_+(i)\sigma_{1M}(i) \tag{8}$$

For astrophysical environment, where the finite temprature and the matter density effects can not be ignored (the initial nucleus is at finite temprature), in general, the initial nuclear state needs to be a weighted sum over appropriate statistical factors. Then, assuming Maxwell-Boltzmann distribution of the initial state $|i\rangle$, the total cross section is given by the expression

$$\sigma(E_e) = \sum_{if}\frac{(2J_i+1)exp(-E_i/kT)}{Z_A}\sigma_{fi}(E_e) \tag{9}$$

where the temprature T is in MeV, Z_A denotes the partition function.

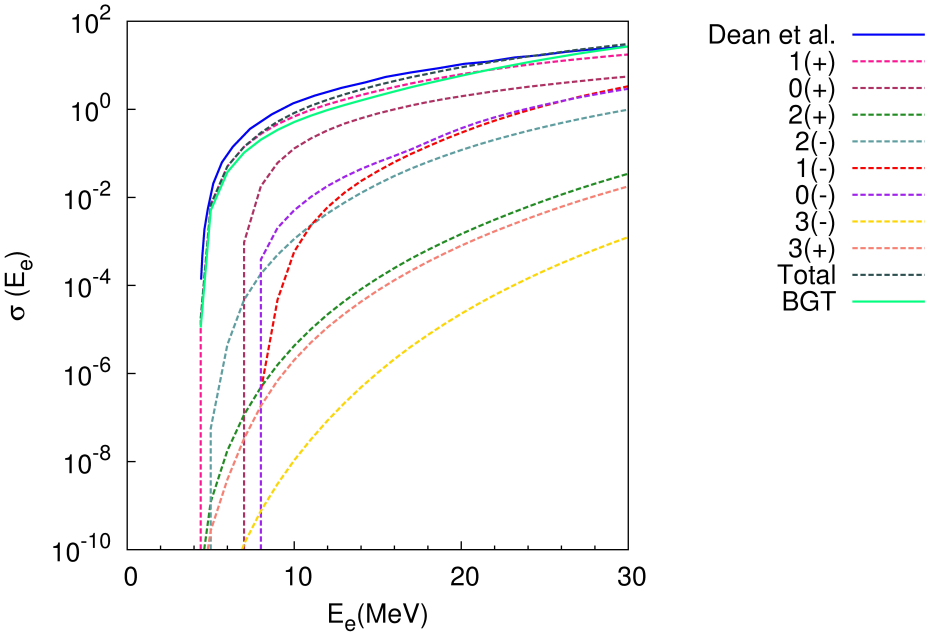
From the comparison of the results obtained with the above two methods we conclude that the aggrement is good (see Fig. 2).

At this point it should be mentioned that in both methods employed a quenched value of the free nucleon coupling constant is used [1].

4. Summary and Conclusions

As discussed in the Introduction, during the presupernova and collapse phase, electron capture on nuclei (and in the late stage also on free protons) plays an important role. Electron captures become increasingly possible as the density in the star's center is increased. It is accompanied by an increase of the chemical potential (Fermi energy) of the degenerate electron gas and it reduces the electron-to-baryon ratio Y_e of the matter composition.

Figure 2: Electron capture cross sections versus the incident electron energy E_e for ^{56}Fe nuclei. The individual contribution of each multipolarity and the total cross sections have been computed with state-by-state pn-QRPA calculations. For the sake of comparison we have plotted the results obtained from Ref. [1] using the G-T operator and our results using G-T operator.



In the present work, we use an advantageous numerical approach (constructed by our group recently) to calculate all basic multipole transition matrix elements needed for obtaining e-capture cross sections. The required nuclear wave functions are obtained within the context of the pn-QRPA using realistic two-body forces. Results for the cross sections are obtained by using two different methods: At first we perform state-by-state calculations by considering the momentum dependence of all transitions matrix elements in the Donnelly-Walecka method, and afterwards we neglected the momentum dependence and considered only the Gamow-Teller contribution to the total electron capture cross section as done by other authors previously. The agreement between the two methods is rather good.

Acknowledgments

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