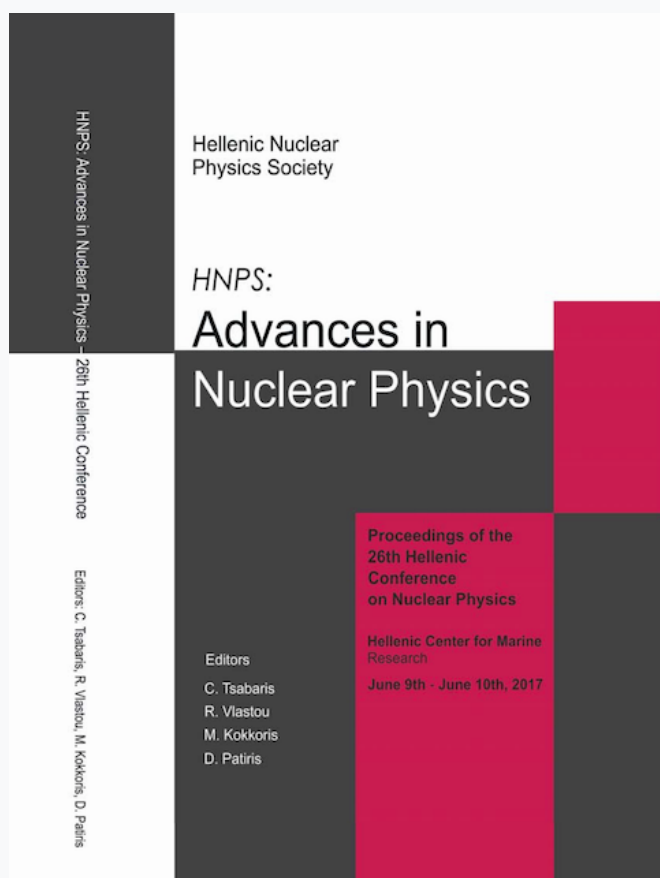


## HNPS Advances in Nuclear Physics

Vol 25 (2017)

HNPS2017



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doi: [10.12681/hnps.1966](https://doi.org/10.12681/hnps.1966)

### To cite this article:

Mertzimekis, T. J. (2019). Isospin symmetry as a master mason for paving nuclear structure experiments along the proton dripline. *HNPS Advances in Nuclear Physics*, 25, 149–153. <https://doi.org/10.12681/hnps.1966>

# Isospin symmetry as a master mason for paving nuclear structure experiments along the proton drip line

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**Abstract** The charge independence of the strong nuclear interaction has been in the core of nuclear physics from very early. Isospin symmetry plays a decisive role in shaping nuclear structure along the proton drip line, where Coulomb repulsion is disproportionately enhanced with respect to the nuclear attraction, as the atomic number increases. This interplay is strongly pronounced in the values of magnetic dipole moments known in the mass region. Distinct symmetries have been observed for several  $T=1/2$  and  $3/2$  mirror pairs, as recorded in their spin expectation values, while remarkable similarities exist between systematics for those two isospin values. A scheme originally developed by Buck and Perez has been revisited recently to: a) evaluate data of nuclear magnetic moments; b) provide understanding of the existing isospin symmetry in terms of mirror nuclei ground-state wavefunctions; c) extend systematics to  $T=5/2$  mirror pairs for the first time; and d) predict magnetic moments for hard-to-reach experimentally, neutron-deficient nuclei lying on the west border of nuclear chart. The results are expected to provide invaluable input to designing future experiments at the extreme.

**Keywords** magnetic moments, isospin symmetry, Buck-Perez

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## INTRODUCTION

As an observable, the magnetic dipole moment can offer invaluable insights to nuclear structure due its one-body nature and its strong sensitivity on the orbital and spin components of the state wavefunction. Consequently, measurements of magnetic moments can submit nuclear models to stringent tests. Regarding mirror nuclei, experimental data on magnetic moments can check the validity of essential symmetries, such as isospin conservation.

The isospin formalism is useful in expressing the magnetic moment operator in terms of the orbital and spin components. Sugimoto [1] expressed the magnetic moment,  $\mu$ , as the expectation value for the state with  $M=J$ , where  $J$  is the nuclear spin and  $M$  is the magnetic quantum number:

$$\mu = \left\langle \sum_i \left[ (1 + \tau_3^i)(l_z^i) + \sigma_z^i \mu_p + \frac{1}{2}(1 - \tau_3^i)\sigma_z^i \mu_n \right] \right\rangle_{M=J} \quad (1)$$

If the isospin is considered as a good quantum number, the spin expectation value involving the magnetic moments of a pair of mirror nuclei can be written as [2]:

$$\langle \sum \sigma_z \rangle = \frac{\mu(T_z = +T) + \mu(T_z = -T) - J}{\mu_p + \mu_n - \frac{1}{2}} \quad (2)$$

since the total spin is  $J = \langle \sum_i l_z^i \rangle + \frac{1}{2} \langle \sum_i \sigma_z^i \rangle$ .

Several works by Buck, Perez and collaborators [3-5], and others [2,6] have illustrated the significance of this relation mainly due to the sensitivity of the spin expectation value to

small changes in the magnetic moments of the mirror nuclei. This property offers an advantage when looking at experimental data of mirror nuclei, in particular magnetic moments of  $T=1/2$  mirror pairs that have all been measured in mass range  $A=3\sim 43$  and  $A=57,59$  [5]. Abandoning the extreme odd-nucleon model (Schmidt moments) and considering potential contributions by all the odd nucleons in forming the observed values, a linear relation between the proton magnetic moment,  $\mu_p$ , and the neutron magnetic moment,  $\mu_n$ , can be established [3]. Expanding this relation into a more general perspective, Buck and Perez transformed it to a relation between g factors:

$$\gamma_p = \alpha\gamma_n + \beta \quad (3)$$

where

$$\begin{aligned} \gamma_{p,n} &= \mu_{p,n} / J & g_l^p &= 1 & g_s^p &= +5.586 \\ \alpha &= (g_s^p - g_l^p) / (g_s^n - g_l^n) & g_l^n &= 0 & g_s^n &= -3.826 \\ \beta &= g_s^p - \alpha g_l^n \end{aligned}$$

Using Eq. 3 to fit all available experimental data of magnetic moments can provide a relation between mirror pair magnetic moments. In Ref. [3], the results of the fit on  $T=1/2$  data produced  $\alpha$  and  $\beta$  values that deviated from theoretical expectations significantly ( $\alpha=-1.199$ ,  $\beta=1$  for the free nucleon), but not immensely. Due to more experimental data accumulated in exotic beam factories, especially on the neutron-deficient side of the nuclear chart, this relation has been revisited a few more times. In addition to  $T=1/2$ ,  $3/2$  mirror pairs have been examined and compared with the  $T=1/2$  data, hinting at a rather universal behavior of mirror pairs when examined under the so-called Buck-Perez scheme. It would be interesting to expand this analysis to  $T=5/2$  mirror pairs and check if mirror nuclei with large neutron-proton number differences, and very close to the proton dripline, still conform with the initial assumption of charge symmetry as in the cases of  $T=1/2$  and  $3/2$ .

The Buck-Perez analysis can also be very effective in evaluating magnetic moments, mainly in two directions: (a) assigning the proper sign in the magnetic moment in case the sign has not been determined unambiguously in the experiment and (b) promoting the credibility of a particular measurement over competing ones in an evaluation, avoiding expensive theoretical calculations.

## METHODOLOGY

All data for  $T=1/2$ ,  $3/2$ , and  $5/2$  mirror partners employed in the present work have been resourced from the online nuclear moments database maintained at the University of Athens [7]. For Buck-Perez analysis, the  $\gamma_p$  vs  $\gamma_n$  data were fitted with a linear model using least squares regression. Each fit produced a slope, an intercept, and a correlation coefficient. The corresponding linear curves have been drawn in the  $\gamma_p$ - $\gamma_n$  plots included below.

In case a missing sign existed in the magnetic moment of the odd-proton or odd-neutron partner in the mirror pair, an assignment was decided based on the following criterion: The corresponding point ( $\gamma_n, \gamma_p$ ) should be as close as possible to the fitted curve. In all cases under consideration in this part, alternative sign assignments created points on the plots that deviated largely from what was expected. Those options were not considered further and a

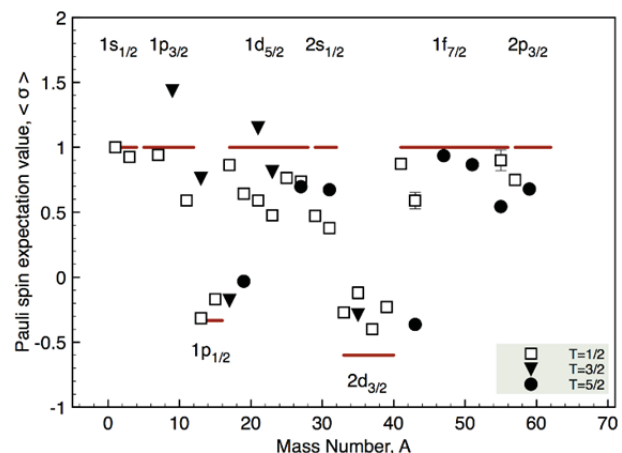
final evaluation of the sign was completed accordingly. Predicted values of magnetic moments for incomplete mirror pairs, having just one of the two nuclei with a measured value, have been estimated similarly. The sign of the moment was automatically assigned without further considerations. It has to be noted, however, that all such cases produced signs that agreed with what expected for the odd-proton or odd-neutron in the nuclear shell examined.

A significant portion of the data ( $\approx 50\%$  for  $T=1/2$ ) is drawn from single measurements. Weighted averages and corresponding errors have been calculated for all nuclei having multiple entries in the database. An exception to the latter case was  $^{57}\text{Cu}$ , which will be discussed below. Regarding the currently eight available  $T=5/2$  mirror pairs, no measurements exist for both partners. For those pairs, the  $\alpha$  and  $\beta$  deduced by  $T=1/2$  data have been adopted, mainly because there is a strong similarity between  $T=1/2$  and  $T=3/2$  linear curves with slopes and intercepts being essentially the same within statistical errors, while the charge symmetry seems to be valid for pairs of mirror nuclei. Furthermore:

1.  $T=1/2$  mirror pairs up to  $A=63$  have at least one partner nucleus with a known experimental magnetic moment. The sole exception is the  $A=5$  pair ( $^5\text{Li}$  -  $^5\text{He}$ ) with none.
2. Missing values for one partner in  $T=1/2$  pairs have been predicted for  $A=45,49,51,53,59,63$
3. Six complete mirror pairs exist for  $T=3/2$ , i.e.  $A=9,13,17,21,23,35$
4. Predicted moments for  $T=3/2$  correspond to  $A=25,27,33,37,39,41,43,45,47,49,51,53,57,61,63$
5.  $T=5/2$  predictions refer to  $A=7,31,43,47,51,55,59$ , all corresponding to the odd-neutron partner.

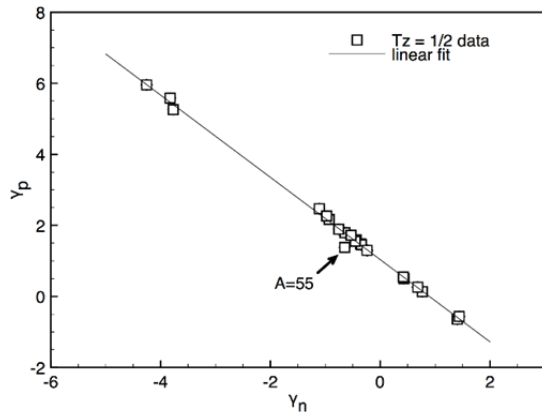
## RESULTS AND DISCUSSION

The spin expectation values for all mirror pairs are plotted in Fig. 1.  $T=1/2,3/2$  groups have been updated with values found in [7]. There are eight data points for  $T=5/2$  (solid circles) that have been estimated using the slope and intercept from the case of  $T=1/2$ . The slope and intercept values are  $\alpha=-1.1582(164)$  and  $\beta=1.0344(266)$  for  $T=1/2$  and  $\alpha=-1.1717(333)$  and  $\beta=1.0957(304)$  for  $T=3/2$ , respectively. All spin expectation values are grouped together and fall between extreme-particle limits (solid lines in the plot). The exception of the mirror pairs for  $A=9$  and 21 that violate this trend has been discussed elsewhere [2] as has the deviation of the  $A=55$  mirror pair from the straight line shown in

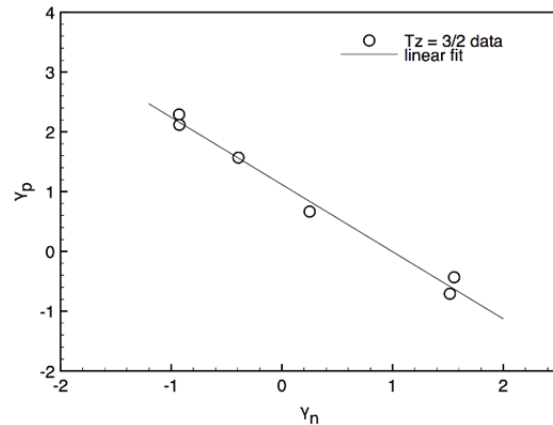


**Fig. 1** Spin expectation values for mirror pairs of mass  $A$ . The straight lines represent the extreme shell-model predictions

Fig. 2 corresponding to the best fit of  $T=1/2$  data. The fit to the  $T=3/2$  data is shown in Fig. 3.



**Fig. 2** Odd–proton vs. odd–neutron gyromagnetic factor for  $T = 1/2$  mirror nuclei.



**Fig. 3** Odd–proton vs. odd–neutron gyromagnetic factor for  $T = 3/2$  mirror nuclei.

Using the Buck-Perez scheme the sign of existing moments can be evaluated and/or the magnetic moment value of a mirror partner can be estimated if the other partner's magnetic moments is known. In Table 1, signs for several  $T=1/2$  and  $T=3/2$  nuclei have been evaluated, while in Table 2 a full list of predictions for unknown magnetic moments of mirror partners have been calculated.

$T=1/2$		$T=3/2$	
Isotope	$\mu$	Isotope	$\mu$
$^{13}\text{N}$	$-0.3222 \pm 0.0004$	$^9\text{C}$	$-1.3914 \pm 0.0005$
$^{23}\text{Mg}$	$-0.5364 \pm 0.0003$	$^9\text{Li}$	$+3.43680 \pm 0.00006$
$^{25}\text{Al}$	$+3.6455 \pm 0.0012$	$^{13}\text{O}$	$-1.3891 \pm 0.0003$
$^{27}\text{Si}$	$-0.8627 \pm 0.0002$	$^{17}\text{N}$	$-0.3550 \pm 0.0004$
$^{29}\text{P}$	$+1.2348 \pm 0.0002$	$^{21}\text{F}$	$+3.9194 \pm 0.0012$
$^{31}\text{S}$	$-0.48793 \pm 0.00008$	$^{35}\text{S}$	$+1.00 \pm 0.04$
$^{39}\text{Ca}$	$+1.02168 \pm 0.00012$	$^{35}\text{K}$	$+0.390 \pm 0.007$
$^{43}\text{Ti}$	$-0.85 \pm 0.02$		
$^{55}\text{Ni}$	$-0.98 \pm 0.03$		

**Table 1** Sign evaluation in mirror nuclei using the Buck–Perez analysis.

Some mirror partners, such as  $^9\text{B}$ ,  $^{11}\text{N}$  and  $^{15}\text{F}$ ,  $^{19}\text{Na}$  and  $^{19}\text{Mg}$ , are either unbound or too short-lived to be reached experimentally; consequently, they are not included in the table. In addition,  $^{59}\text{Ge}$ , a potential candidate for  $2p$  decay, has been recently found to undergo a  $\beta$ -decay instead, with a measured half-life of  $t_{1/2}=13.3(17)$  ms [8]. This half-life is in the same order of magnitude with those from the other  $T=5/2$  nuclei listed in the same table. It has to be noted that  $^{59}\text{Ge}$  is the heaviest bound nucleus with  $T_z=-5/2$  having a mirror partner with a known magnetic moment.

Isotope	$J^\pi$	$\mu$	Isotope	$J^\pi$	$\mu$
$T=1/2$					
$^{45}\text{V}$	7/2-	$3.510 \pm 0.027$	$^{49}\text{Mn}$	5/2-	$2.035 \pm 0.027$
$^{51}\text{Fe}$	5/2-	$-0.849 \pm 0.043$	$^{53}\text{Co}$	7/2-	$1.999 \pm 0.029$
$^{59}\text{Zn}$	3/2-	$-0.294 \pm 0.026$	$^{63}\text{Ge}$	3/2-	$1.213 \pm 0.026$
$T=3/2$					
$^{25}\text{Si}$	5/2+	$-0.805 \pm 0.048$	$^{27}\text{P}$	1/2+	$1.029 \pm 0.041$
$^{33}\text{P}$	1/2+	$1.395 \pm 0.059$	$^{37}\text{Ca}$	3/2+	$0.819 \pm 0.030$
$^{39}\text{Sc}$	7/2-	$5.695 \pm 0.034$	$^{41}\text{Ti}$	3/2+	$1.219 \pm 0.027$
$^{43}\text{V}$	7/2-	$5.379 \pm 0.033$	$^{45}\text{Cr}$	7/2-	$-0.786 \pm 0.052$
$^{47}\text{Mn}$	5/2-	$3.663 \pm 0.032$	$^{49}\text{Fe}$	7/2-	$-0.542 \pm 0.046$
$^{51}\text{Co}$	7/2-	$4.929 \pm 0.032$	$^{53}\text{Ni}$	7/2-	$-1.023 \pm 0.047$
$^{57}\text{Zn}$	7/2-	$-0.756 \pm 0.045$	$^{61}\text{Ge}$	3/2+	$-0.397 \pm 0.046$
$^{63}\text{As}$	3/2-	$1.974 \pm 0.031$			
$T=5/2$					
$^{27}\text{S}$	5/2+	$-1.130 \pm 0.060$	$^{31}\text{Ar}$	5/2+	$-1.074 \pm 0.060$
$^{43}\text{Cr}$	3/2+	$1.199 \pm 0.040$	$^{47}\text{Fe}$	7/2-	$-1.485 \pm 0.084$
$^{51}\text{Ni}$	7/2-	$-1.320 \pm 0.083$	$^{55}\text{Zn}$	5/2-	$-0.762 \pm 0.059$
$^{59}\text{Ge}$	7/2-	$-0.869 \pm 0.082$			

**Table 2** Predicted ground state<sup>[1]</sup> nuclei with  $T= 1/2, 3/2$  and  $5/2$ , based on the Buck–Perez analysis. Spin and parity values for  $^{43}\text{V}$  and  $^{59}\text{Ge}$  have not been deduced experimentally yet, so they have been adopted from their corresponding mirror partners.

## CONCLUSIONS

The Buck–Perez analysis was performed on updated sets of magnetic moments for  $T=1/2$  and  $3/2$  mirror nuclei. Almost identical linear trends have been resulted in both cases. Using the linear relation parameters deduced for  $T=1/2$  nuclei, the scheme was further extended to eight  $T=5/2$  mirror pairs, for the first time, as a quick way to predict ground–state magnetic moments of mirror partners located very close to the proton dripline. Overall, the empirical Buck–Perez analysis, expressing the underlying charge symmetry of nuclear forces, seems to be particularly effective in evaluating magnetic moments in mirror nuclei, while it proves itself as a valuable tool in planning future experimental projects.

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